Tishk International University
Faculty of Science
Information technology Department

Chapter TWO
Electric Potential and Potential Difference

Dr. Bestoon Mustafa

GENERAL PHYSICS II
Weeks 3-4
Fall 2023/24
Date

Outline

1. Electric potential energy
2. Electric potential energy of point charges
3. Electric potential
4. Potential difference (voltage)
5. Relationship between potential difference and electric field
6. Motion of charge in a uniform electric field

Objectives

- Understanding Electric potential energy
- Understanding the concept of electric potential and potential difference
- Potential difference in daily life
- Motion of charges is affected inside electric field

1. Electric potential energy

- When a charged particle moves in an electric field, the field exerts a force that can do *work on the particle.*
- The force on a particle moves the charges from point *a to point b, the work* done by the force is given by a *line integral*

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b F\cos\phi\, dl \quad \text{(work done by a force)}$$

- *This work can always be expressed in **terms***

of **electric potential energy.** Mathematically, we can say:

$$\boxed{W = -\Delta E_P}$$

- Note: when the charge moves, the potential energy converts to kinetic energy.

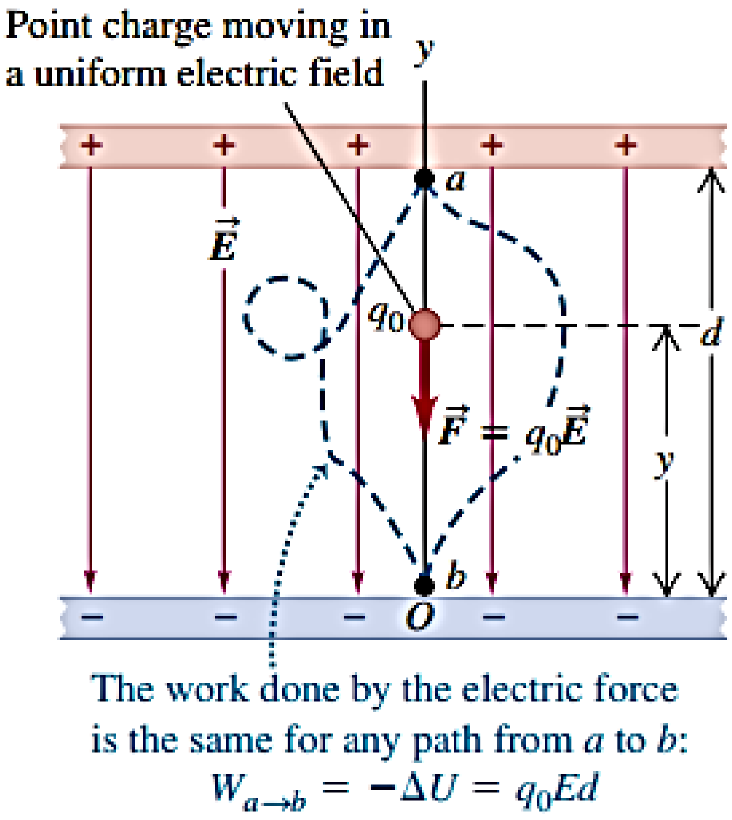

Figure 1 a charged particle is displaced along the electric field direction. Work is done by the field.

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- Suppose a positive charge of +q is placed in a uniform electric field as in figure 2.1. The charge +q has its maximum electric potential energy just near the positive plate at point 1 and after it is released, Its electric potential energy decreases as it changes into kinetic energy.

$$W = -\Delta E_p$$

$$\boxed{W = F_e d \quad and \quad F_e = qE}$$

$$qEd = -(E_{P2} - E_{P1})$$

$$qE\,(d_1 - d_2) = -(E_{P2} - E_{P1})$$

$$qE\,d_1 - qEd_2 = E_{P1} - E_{P2}$$

Hence **the electric potential energy (E_p)** of a charge q in a uniform electric field is;

$$\boldsymbol{E_P = qEd}$$

$\boldsymbol{E_P}$ is measured by joule (J).

2. Electric potential energy of point charges

- The electric potential energy of a system of two point charges, of the same sign, is equal to the work done by the electric force to move the charge q from r to infinity.
- **We said**

$$E_P = qEd$$

- By substituting the value of E in the above formular, we obtain,

$$E_P = k\frac{q_1 q_2}{r}$$

Figure 2 point charges have potential energy

- **Note: this quantity can be either positive or negative depending on the sign of q.**

Example 1

Find the potential energy of a system of point charges of q1 =20 μC and q2 = -10 μC when they are r_1 = 20 cm apart. How does the potential energy changes if we separate charges to a distance of $r_2 = 40\ cm$.

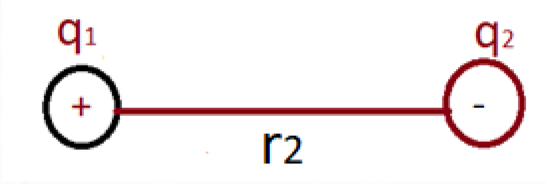
Solution

Their potential energy when they are $r_1 = 20$ cm apart is

$$E_p = k\frac{q_1 q_2}{r_1} = 9\times10^9\ \frac{N\cdot m^2}{C^2}\ \frac{(20\times10^{-6}\ C)(-10\times10^{-6}\ C)}{(0.2\ m)}$$

$$E_p = -9\ J$$

their potential energy when they are $r_2 = 40$ cm apart is



$$E_p = k\frac{q_1 q_2}{r_2} = 9\times10^9\ \frac{N\cdot m^2}{C^2}\ \frac{(20\times10^{-6}\ C)(-10\times10^{-6}\ C)}{(0.4\ m)} = -4.5\ J$$

The potential energy has increased $(Ep_2 > Ep_1)$ because the potential energy of unlike charges increases as we separate them apart.

3. Electric potential

- Electric potential, V , is defined as: the electric potential energy per unit charge. And it is equal to the **work done** by an **electric field** in carrying a unit positive charge from infinity to point a.
- Mathematically, we can say:

$$\boxed{V = \frac{E_P}{q_0}}$$

- For a point charge at point a, by substituting the value of E from above equation, we obtain:

$$V_a = \frac{E_P}{q_0} = \frac{k\frac{qq_0}{r}}{q_0}$$

thus,

$$\boxed{V_a = k\frac{q}{r}}$$

V is measured by volt, and

$$1Volt = 1\,J/C$$

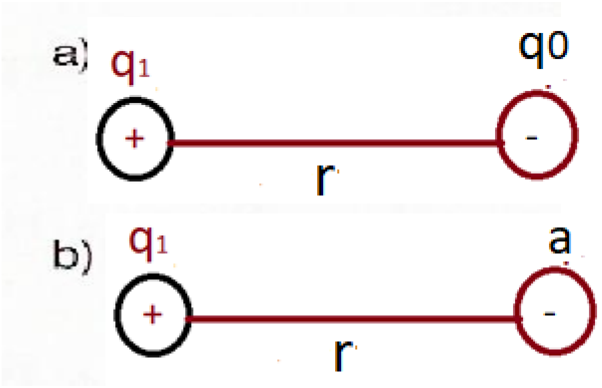


Fig.3 Electric potential of charge q at point a.

Example 2

Two charges of 5μC and -10μC are placed at the corners of an equilateral triangle of single length of 10 cm. Determine the electric potential at point a.

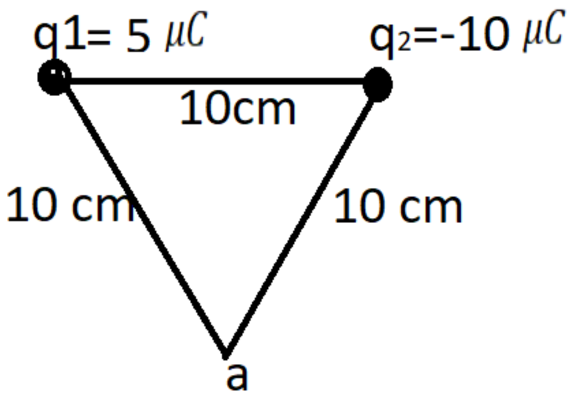
Solution:
The net electric potential at point a is the algebraic sum of electric potential due to both charges, then.

$$V_{a_1} = k\frac{q_1}{r_1} = \left(9\times10^9\ \frac{N\cdot m^2}{C^2}\right)\frac{5\times10^{-6}\ C}{10^{-1}\ m} = 4.5\times10^5\ V$$

$$V_{a_2} = k\frac{q_2}{r_2} = \left(9\times10^9\ \frac{N\cdot m^2}{C^2}\right)\frac{-10\times10^{-6}\ C}{10^{-1}\ m} = -9\times10^5\ V$$

$$V_a = V_{a_1} + V_{a_2} = (4.5\times10^5\ V) + (-9\times10^5\ V) = -4.5\times10^5\ V$$

$$V_a = -4.5\times10^5\ V$$



4. Potential difference (voltage)

- Potential difference is the work that has to be done to transfer a unit positive charge from one point (for example A) to the other (example B), denoted by ΔV and measure by volts.
- Mathematically, potential difference between two points is defined as the work done on a positive charge by external force dividing by the magnitude of the charge.

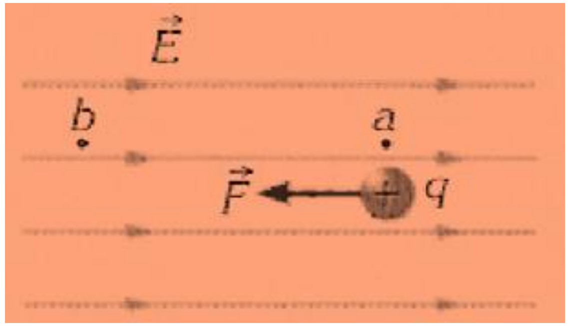


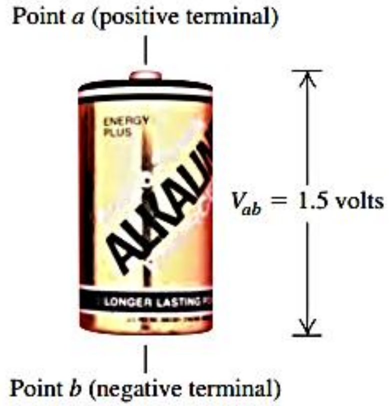
Figure 4 a point charge, q, is displaced by an external force F, from point a to point b.

- So potential difference is: $\boxed{\Delta V = V_b - V_a}$

And $\boxed{V_b = \frac{E_{Pb}}{q} \quad and \quad V_a = \frac{E_{Pa}}{q}}$

$$V_b - V_a = \frac{E_{Pb}}{q} - \frac{E_{Pa}}{q} = \frac{\Delta E_P}{q} = \frac{W_{ab}}{q}$$

$V_b - V_a = \frac{W_{ab}}{q}$ Thus $\boxed{W_{ab} = q(V_b - V_a)}$



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- We said, $V = \frac{E}{q}$

By substituting the E value, we get $\boxed{V = k\frac{q}{r}}$

<u>Example 3</u>

A particle of charge q = 5 μC is displaced from a point at potential 700 V to a point of potential 50 V. What work is done by the external force, W_{ext} and the electric force W_e?

Solution

The charge is displaced from a point of high potential to a point of low potential. The work done by the external force is

$$W_{ext} = q(V_b - V_a)$$

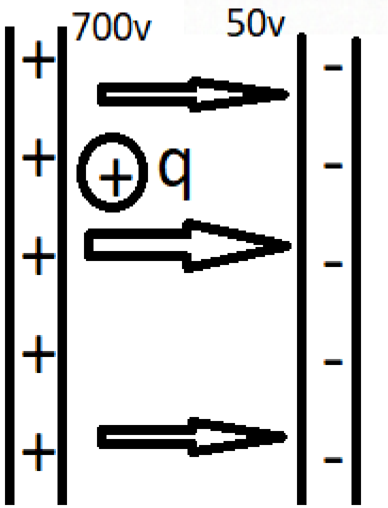
$$W_{ext} = (5\times10^{-6}\ \text{C})\ (50\ \text{V} - 700\ \text{V})$$

$$W_{ext} = -3.25\times10^{-3}\ \text{J}.$$

Thus, the work done by the electric force is the opposite of W_{ext}

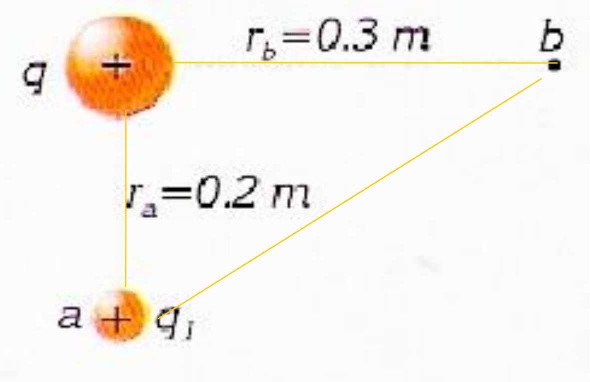
$$W_e = 3.25\times10^{-3}\ \text{J}$$

This means that the potential energy of the charge decreases, if it is displaced parallel to the direction of the electric field, as is shown in the figure.



Example 4

Calceolate the work done when a charge of q1= 2μC is moved from point a to point b under the effect of charge q= 50 μC as shown below:



<u>Solution</u>:

$$W_{ba} = q_1 U = (V_b - V_a)$$

$$W_{ba} = q_1\left(k\frac{q}{r_b} - k\frac{q}{r_a}\right) = kq_1q\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$W_{ba} = \left(9\times10^9\ \frac{N\cdot m^2}{C^2}\right)(2\times10^{-6}\ C)\ (50\times10^{-6}\ C)\left(\frac{1}{0.3\,m} - \frac{1}{0.2\,m}\right)$$

$$W_{ba} = -1.5\ J$$

The negative sign means the work done by the electric field is positive, so that the potential energy f the system decreased

5. Relationship between potential difference and electric field

- We can make a relation between E and V.
- This principle occurs for any two plates bring together connected to a source (battery).

Mathematically, electric field can be represented as following,

$$E = \frac{V}{d}$$

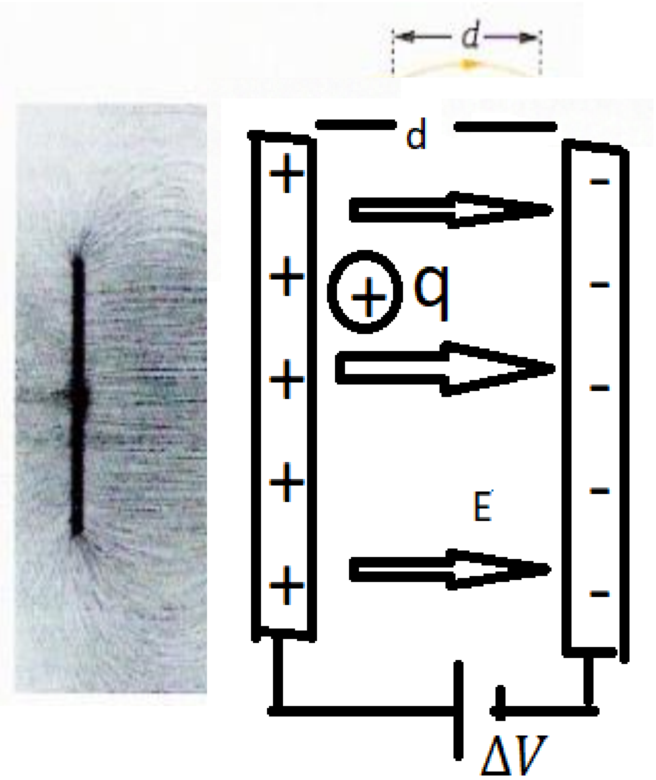
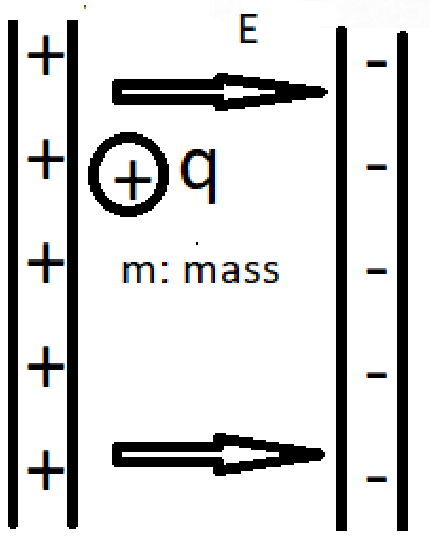


Figure 5 the direction if the uniform electric field due to a charged parallel-plate capacitor is directed from the positive plate towards the negative plate.

6. Motion of charge in a uniform electric field

- when a particle of charge q and mass m is released in a uniform electric field, it experiences an electric force as shown in figure below, if the only force acting on it is the electric force we can use Newton's law in the form:

$$F_e = m\vec{a}$$



Substituting the value if Fe, we obtain,

$$|q|\vec{E} = m\vec{a} \qquad \boldsymbol{\vec{a} = \frac{|q|\vec{E}}{m}}$$

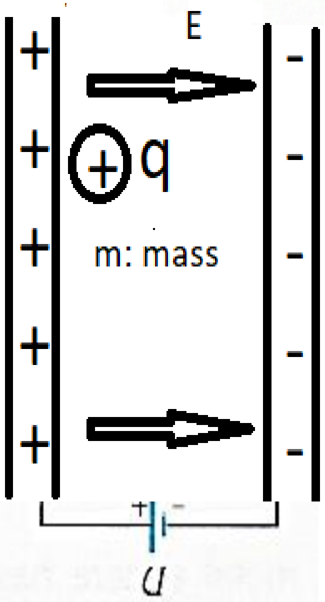
$\vec{a}$ is the magnitude of the acceleration of the particle.

- **Note**: we have ignored the gravitational force acting in the particle. In some cases where gravitational force is not be small compared to electric force, we should take this force into account too.

Its direction is the same as the electric field direction if the charge is positive as its direction is opposite the electric field direction if the charge is negative.

Example 5

A positive charge of $q = 10\ \mu C$ and mass, $m = 10^{-6}$ kg is placed near the positive plate, as shown in the figure. What is the acceleration and velocity of the particle just before striking the negative plate? Assume the electric field is uniform, the voltage of the battery is $U = 10^4$ V and the distance between the plates is 5 cm (ignore the effects of gravitation).



Solution :

Ignoring the gravitational force acting on the charge, the acceleration is given by

$$a = \frac{F}{m} = \frac{|q|E}{m} \quad \text{where} \quad E = \frac{U}{d} \quad \text{then}$$

$$a = \frac{|q|U}{md} = \frac{(10^{-6}\ C)(10^4\ V)}{(10^{-6}\ kg)(5\times 10^{-2}\ m)}$$

$$a = 2\times 10^5\ m/s^2$$

The velocity can be found using

$$\upsilon^2 = \upsilon_0^2 + 2ad \quad \text{where} \quad \upsilon_0 = 0$$

$$\upsilon = \sqrt{2ad} = \sqrt{2(2\times 10^5\ m/s)(5\times 10^{-2}\ m)}$$

$$\upsilon = 1.4\times 10^2\ m/s$$

Example 6

An object of charge -10μc and mass 50 mg remains in equilibrium in an uniform electric field. Find the magnitude and direction of the electric field.

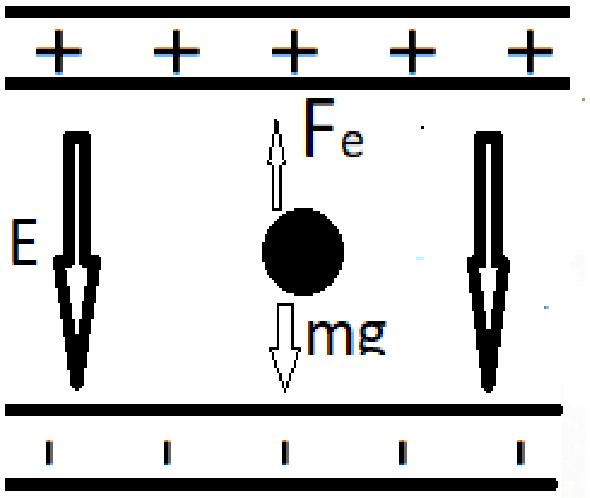
Solution

The gravitational force acting on the object is downwards. So the electric force must be upwards. The electric field is directed downwards (see the figure). We can write

$$F_e = F_g \qquad |q|E = mg$$

$$E = \frac{mg}{|q|} = \frac{(50\times10^{-6}\ \text{kg})\ (10\ \text{N/kg})}{|-10\times10^{-6}\ \text{C}|}$$

$$E = 50\ \text{N/C}$$



Conclusion

- We explored the idea of electric potential energy in the context of point charges, understanding how it relates to electric potential and the concept of potential difference or voltage.
- The relationship between potential difference and electric fields was elucidated, demonstrating how these factors influence the motion of charges in uniform electric fields.

References (in APA style)

- ✓ Alexander, C. K. (2013). Fundamentals of electric circuits. McGraw-Hill,.
- ✓ Young, H. D., Freedman, R. A., & Ford, A. L. (2014). University physics with modern physics (p. 822). New York: Pearson.