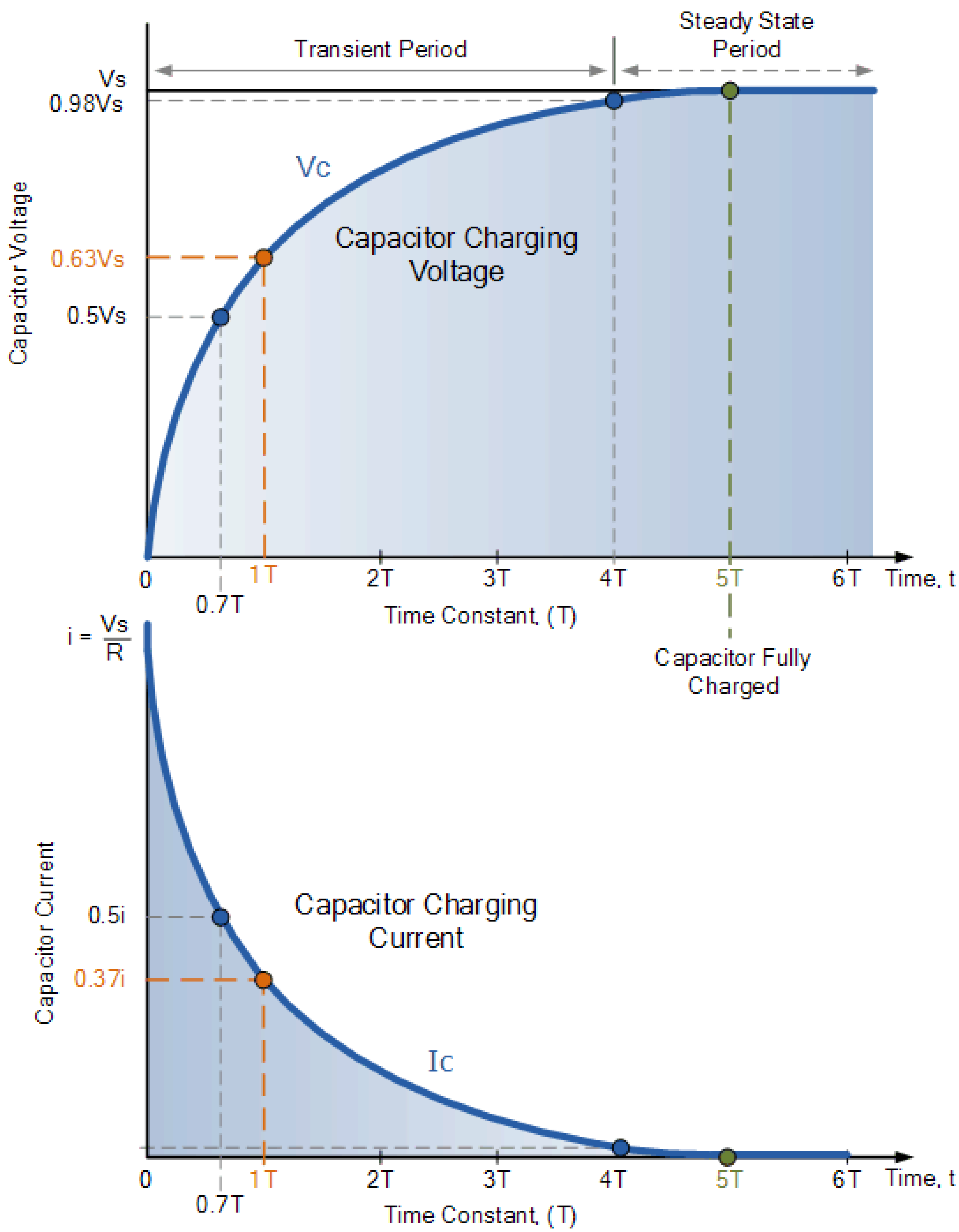
RC Charging Circuit Curves



The capacitor (C), charges up at a rate shown by the graph. The rise in the RC charging curve is much steeper at the beginning because the charging rate is fastest at the start of charge but soon tapers off exponentially as the capacitor takes on additional charge at a slower rate.

As the capacitor charges up, the potential difference across its plates begins to increase with the actual time taken for the charge on the capacitor to reach 63% of its maximum possible fully charged voltage, in our curve 0.63Vs, being known as one full Time Constant, (T).

This 0.63Vs voltage point is given the abbreviation of 1T, (one time constant).

The capacitor continues charging up and the voltage difference between Vs and Vc reduces, so too does the circuit current, i. Then at its final condition greater than five time constants (5T) when the capacitor is said to be fully charged, t = ∞, i = 0, q = Q = CV. At infinity the charging current finally diminishes to zero and the capacitor acts like an open circuit with the supply voltage value entirely across the capacitor as Vc = Vs.

So mathematically we can say that the time required for a capacitor to charge up to one time constant, (1T) is given as:

RC Time Constant, Tau

$$\tau \equiv R \times C$$

This RC time constant only specifies a rate of charge where, R is in Ω and C in Farads.

Since voltage V is related to charge on a capacitor given by the equation, Vc = Q/C, the voltage across the capacitor (Vc) at any instant in time during the charging period is given as:

$$V_C = V_S\left(1 - e^{(-t/RC)}\right)$$

Where:

- Vc is the voltage across the capacitor
- Vs is the supply voltage
- e is an irrational number presented by Euler as: 2.7182
- t is the elapsed time since the application of the supply voltage
- RC is the *time constant* of the RC charging circuit

After a period equivalent to 4 time constants, (4T) the capacitor in this RC charging circuit is said to be virtually fully charged as the voltage developed across the capacitors plates has now reached 98% of its maximum value, 0.98Vs. The time period taken for the capacitor to reach this 4T point is known as the **Transient Period**.

After a time of 5T the capacitor is now said to be fully charged with the voltage across the capacitor, (Vc) being aproximately equal to the supply voltage, (Vs). As the capacitor is therefore fully charged, no more charging current flows in the circuit so $I_C = 0$. The time period after this 5T time period is commonly known as the **Steady State Period**.

Then we can show in the following table the percentage voltage and current values for the capacitor in a RC charging circuit for a given time constant.

RC Charging Table

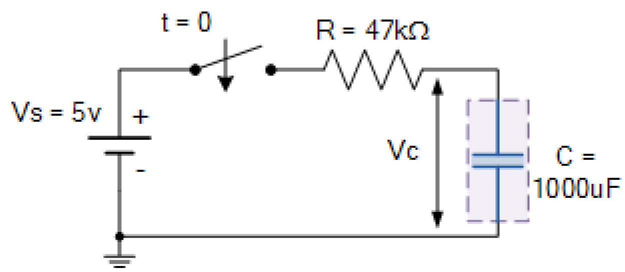
Time Constant	RC Value	Percentage of Maximum	
		Voltage	Current
0.5 time constant	0.5T = 0.5RC	39.3%	60.7%
0.7 time constant	0.7T = 0.7RC	50.3%	49.7%
1.0 time constant	1T = 1RC	63.2%	36.8%
2.0 time constants	2T = 2RC	86.5%	13.5%
3.0 time constants	3T = 3RC	95.0%	5.0%
4.0 time constants	4T = 4RC	98.2%	1.8%
5.0 time constants	5T = 5RC	99.3%	0.7%

Notice that the charging curve for a RC charging circuit is exponential and not linear. This means that in reality the capacitor never reaches 100% fully charged. So for all practical purposes, after five time constants (5T) it reaches 99.3% charge, so at this point the capacitor is considered to be fully charged.

As the voltage across the capacitor Vc changes with time, and is therefore a different value at each time constant up to 5T, we can calculate the value of capacitor voltage, Vc at any given point, for example.

RC Charging Circuit Example No1

Calculate the RC time constant, τ of the following circuit.



The time constant, τ is found using the formula T = R x C in seconds.

Therefore the time constant τ is given as: T = R x C = 47k x 1000uF = 47 Secs

a) **What will be the value of the voltage across the capacitors plates at exactly 0.7 time constants?**

At 0.7 time constants (0.7T) Vc = 0.5Vs. Therefore, Vc = 0.5 x 5V = 2.5V

b) **What value will be the voltage across the capacitor at 1 time constant?**

At 1 time constant (1T) Vc = 0.63Vs. Therefore, Vc = 0.63 x 5V = 3.15V

c) **How long will it take to "fully charge" the capacitor from the supply?**

We have learnt that the capacitor will be fully charged after 5 time constants, (5T).

1 time constant (1T) = 47 seconds, (from above). Therefore, 5T = 5 x 47 = 235 secs

d) **The voltage across the Capacitor after 100 seconds?**

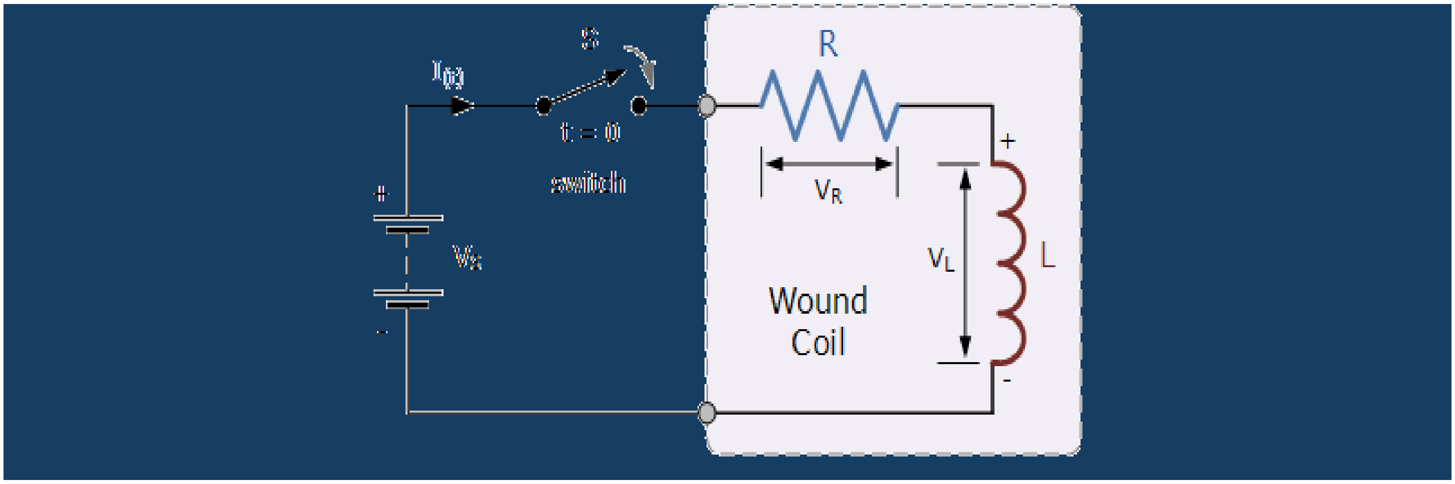
The voltage formula is given as $Vc = V(1 - e^{(-t/RC)})$ so this becomes: $Vc = 5(1 - e^{(-100/47)})$

Where: V = 5 volts, t = 100 seconds, and RC = 47 seconds from above.

Therefore, $Vc = 5(1 - e^{(-100/47)}) = 5(1 - e^{-2.1277}) = 5(1 - 0.1191) =$ 4.4 volts

We have seen here that the charge on a capacitor is given by the expression: Q = CV, where C is its fixed capacitance value, and V is the applied voltage. We have also learnt that when a voltage is firstly applied to the plates of the capacitor it charges up at a rate determined by its RC time constant, τ and will be considered fully charged after five time constsants, or 5T.

In the next tutorial we will examine the current-voltage relationship of a discharging capacitor and look at the discharging curves associated with it when the capacitors plates are effectively shorted together.



LR Series Circuit

All coils, inductors, chokes and transformers create a magnetic field around themselves consist of an Inductance in series with a Resistance forming an LR Series Circuit

The first tutorial in this section about Inductors, we looked briefly at the time constant of an inductor stating that the current flowing through an inductor could not change instantaneously, but would increase at a constant rate determined by the self-induced emf in the inductor.

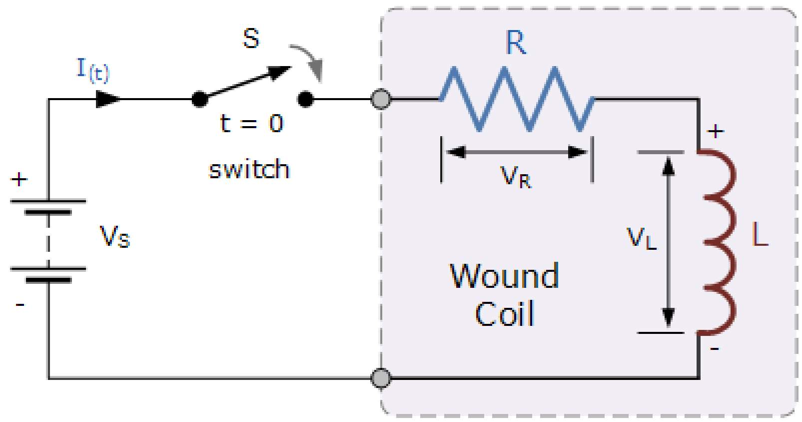
In other words, an inductor in an electrical circuit opposes the flow of current, (i) through it. While this is perfectly correct, we made the assumption in the tutorial that it was an ideal inductor which had no resistance or capacitance associated with its coil windings.

However, in the real world "ALL" coils whether they are chokes, solenoids, relays or any wound component will always have a certain amount of resistance no matter how small. This is because the actual coils turns of wire being used to make it uses copper wire which has a resistive value.

Then for real world purposes we can consider our simple coil as being an "Inductance", **L** in series with a "Resistance", **R**. In other words forming an **LR Series Circuit**.

A **LR Series Circuit** consists basically of an inductor of inductance, L connected in series with a resistor of resistance, R. The resistance "R" is the DC resistive value of the wire turns or loops that goes into making up the inductors coil. Consider the LR series circuit below.

The LR Series Circuit



The above *LR series circuit* is connected across a constant voltage source, (the battery) and a switch. Assume that the switch, S is open until it is closed at a time t = 0, and then remains permanently closed producing a "step response" type voltage input. The current, i begins to flow through the circuit but does not rise rapidly to its maximum value of Imax as determined by the ratio of V / R (Ohms Law).

This limiting factor is due to the presence of the self induced emf within the inductor as a result of the growth of magnetic flux, (Lenz's Law). After a time the voltage source neutralizes the effect of the self induced emf, the current flow becomes constant and the induced current and field are reduced to zero.

We can use Kirchhoff's Voltage Law, (KVL) to define the individual voltage drops that exist around the circuit and then hopefully use it to give us an expression for the flow of current.

Kirchhoff's voltage law (KVL) gives us:

$$V_{(t)} - (V_R + V_L) = 0$$

The voltage drop across the resistor, R is I*R (Ohms Law).

$$V_R = I \times R$$

The voltage drop across the inductor, L is by now our familiar expression L(di/dt)

$$V_L = L\frac{di}{dt}$$

Then the final expression for the individual voltage drops around the LR series circuit can be given as:

$$V_{(t)} = I \times R + L\frac{di}{dt}$$

We can see that the voltage drop across the resistor depends upon the current, i, while the voltage drop across the inductor depends upon the rate of change of the current, di/dt. When the current is equal to zero, (i = 0) at time t = 0 the above expression, which is also a first order differential equation, can be rewritten to give the value of the current at any instant of time as:

Expression for the Current in an LR Series Circuit

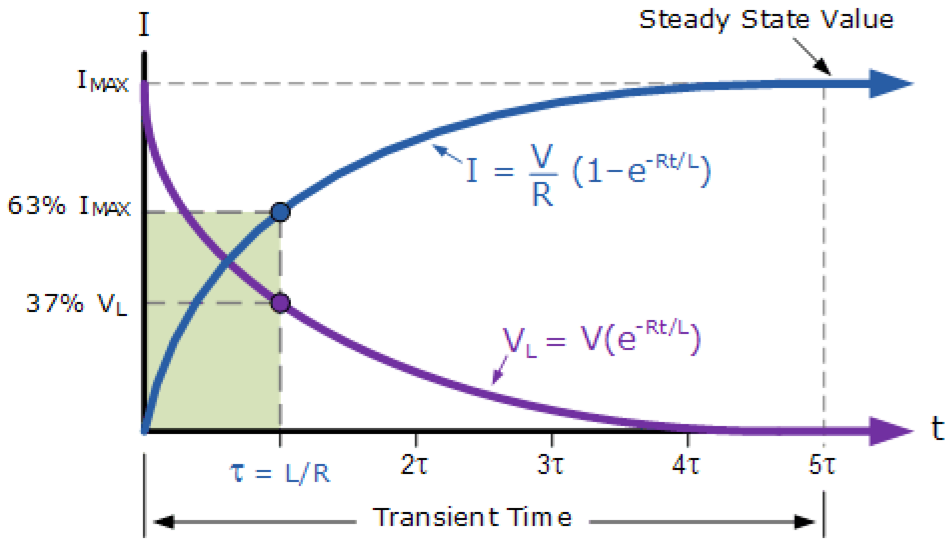
$$I_{(t)} = \frac{V}{R}\left(1 - e^{-Rt/L}\right) \ \ (A)$$

- Where:
- V is in Volts
- R is in Ohms
- L is in Henries
- t is in Seconds
- e is the base of the Natural Logarithm = 2.71828

The **Time Constant**, (τ) of the LR series circuit is given as L/R and in which V/R represents the final steady state current value after five time constant values. Once the current reaches this maximum steady state value at 5τ, the inductance of the coil has reduced to zero acting more like a short circuit and effectively removing it from the circuit.

Therefore the current flowing through the coil is limited only by the resistive element in Ohms of the coils windings. A graphical representation of the current growth representing the voltage/time characteristics of the circuit can be presented as.

Transient Curves for an LR Series Circuit



Since the voltage drop across the resistor, V_R is equal to I*R (Ohms Law), it will have the same exponential growth and shape as the current. However, the voltage drop across the inductor, V_L will have a value equal to: $Ve^{(-Rt/L)}$. Then the voltage across the inductor, V_L will have an initial value equal to the battery voltage at time t = 0 or when the switch is first closed and then decays exponentially to zero as represented in the above curves.

The time required for the current flowing in the LR series circuit to reach its maximum steady state value is equivalent to about **5 time constants** or 5τ. This time constant τ, is measured by τ = L/R, in seconds, where R is the value of the resistor in ohms and L is the value of the inductor in Henries. This then forms the basis of an RL charging circuit were 5τ can also be thought of as "5*(L/R)" or the *transient time* of the circuit.

The transient time of any inductive circuit is determined by the relationship between the inductance and the resistance. For example, for a fixed value resistance the larger the inductance the slower will be the transient time and therefore a longer time constant for the LR series circuit. Likewise, for a fixed value inductance the smaller the resistance value the longer the transient time.

However, for a fixed value inductance, by increasing the resistance value the transient time and therefore the time constant of the circuit becomes shorter. This is because as the resistance increases the circuit becomes more and more resistive as the value of the inductance becomes negligible compared to the resistance. If the value of the resistance is increased sufficiently large compared to the inductance the transient time would effectively be reduced to almost zero.

LR Series Circuit Example No1

A coil which has an inductance of 40mH and a resistance of 2Ω is connected together to form a LR series circuit. If they are connected to a 20V DC supply.

a). What will be the final steady state value of the current.

$$\text{Steady State Current, } I = \frac{V}{R} = \frac{20}{2} = 10A$$

b) What will be the time constant of the RL series circuit.

$$\text{Time Constant, } \tau = \frac{L}{R} = \frac{0.04}{2} = 0.02s \text{ or } 20mS$$

c) What will be the transient time of the RL series circuit.

$$\text{Transient Time, } 5\tau = 5 \times 0.02s = 100mS$$

d) What will be the value of the induced emf after 10ms.

$$\text{Induced emf, } V_L = Ve^{(-Rt/L)} = 20e^{(-2\times0.01/0.04)}$$

$$V_L = 20 \times 0.6065 = 12.13V$$

e) What will be the value of the circuit current one time constant after the switch is closed.

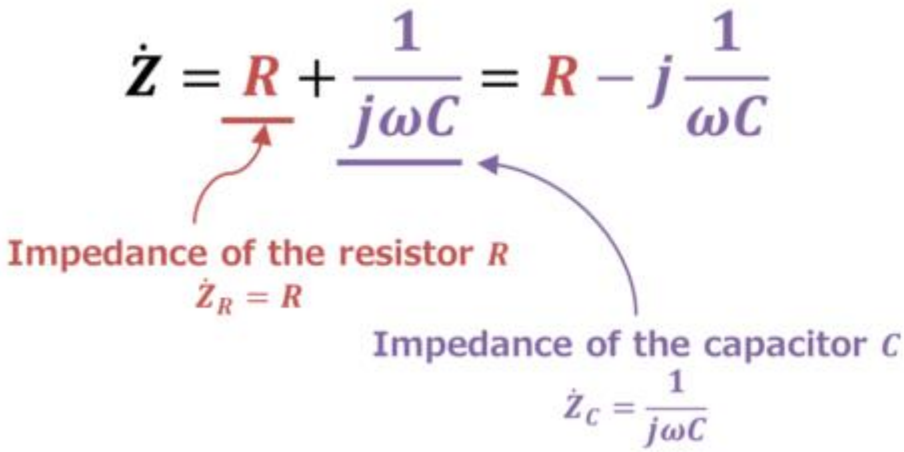
$$\text{Instantaneous Current, } I_{(t)} = \frac{V_S}{R}(1-e^{-Rt/L})$$

The Time Constant, τ of the circuit was calculated in question b) as being 20ms. Then the circuit current at this time is given as:

$$I_{(t)} = \frac{20}{2}(1 - e^{-2\times0.02/0.04})$$

$$I_{(t)} = 10(1-0.368) = 6.32A$$

You may have noticed that the answer for question (e) which gives a value of 6.32 Amps at one time constant, is equal to 63.2% of the final steady state current value of 10 Amps we calculated in question (a). This value of 63.2% or 0.632 x I_{MAX} also corresponds with the transient curves shown above.



$Z=R+jX_L$

Here R is the resistance, and X_L is the inductive reactance.

$X_L=\omega L$

$\Rightarrow Z=R+j\omega L$