Tishk International University
Mechatronics Engineering Department
Fluid Mechanics
Week ;15/02/2024

Viscosity

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Properties of fluids

- DENSITY
- VISCOSITY
- CAUSES OF VISCOSITY IN GASES
- CAUSES OF VISCOSITY IN A LIQUID
- SURFACE TENSION
- CAPILLARITY
- CAVITATION
- COMPRESSIBILITY AND THE BULK MODULUS

Properties of fluids

- EQUATION OF STATE OF A PERFECT GAS
- THE UNIVERSAL GAS CONSTANT
- SPECIFIC HEATS OF A GAS
- EXPANSION OF A GAS

Density

- **Problem 1:** You have a rock with a volume of $15cm^3$ and a mass of 45 g. What is its density?

1.8.1 Mass density

Mass density ρ is defined as the mass of the substance per unit volume. As mentioned above, we are concerned, in considering this and other properties, with the substance as a continuum and not with the properties of individual molecules. The mass density at a point is determined by considering the mass δm of a very small volume δV surrounding the point. In order to preserve the concept of the continuum, δV cannot be made smaller than x^3, where x is a linear dimension which is large compared with the mean distance between molecules. The density at a point is the limiting value as δV tends to x^3:

$$\rho = \lim_{\delta V \to x^3} \frac{\delta m}{\delta V}.$$

Units: kilograms per cubic metre (kg m^{-3}).

Dimensions: ML^{-3}.

Typical values at $p = 1.013 \times 10^5$ N m^{-2}, $T = 288.15$ K: water, 1000 kg m^{-3}; air, 1.23 kg m^{-3}.

1.8.2 Specific weight

Specific weight w is defined as the weight per unit volume. Since weight is dependent on gravitational attraction, the specific weight will vary from point to point, according to the local value of gravitational acceleration g. The relationship between w and ρ can be deduced from Newton's second law, since

$$\text{Weight per unit volume} = \text{Mass per unit volume} \times g$$

$$w = \rho g.$$

Units: newtons per cubic metre (N m^{-3}).
Dimensions: $ML^{-2}T^{-2}$.
Typical values: water, 9.81×10^3 N m^{-3}; air, 12.07 N m^{-3}.

1.8.3 Relative density

Relative density (or specific gravity) σ is defined as the ratio of the mass density of a substance to some standard mass density. For solids and liquids, the standard mass density chosen is the maximum density of water (which occurs at 4 °C at atmospheric pressure):

$$\sigma = \rho_{\text{substance}}/\rho_{\text{H}_2\text{O at 4 °C}}.$$

For gases, the standard density may be that of air or of hydrogen at a specified temperature and pressure, but the term is not used frequently.

Units: since relative density is a ratio of two quantities of the same kind, it is a pure number having no units.

Dimensions: as a pure number, its dimensions are $M^0L^0T^0 = 1$.

Typical values: water, 1.0; oil, 0.9.

1.8.4 Specific volume

In addition to these measures of density, the quantity specific volume is sometimes used, being defined as the reciprocal of mass density, i.e. it is used to mean volume per unit mass.

Solution

Density is mass divided by volume, so that the density is 45 g divided by $15cm^3$, which is **3.0 g/cm^3**.

$$Density = \frac{Mass}{Volume}$$

$$Density = \frac{45g}{15cm^3} = 3.0\frac{g}{cm^3}$$

Density

- **Problem 2:** You have a different rock with a volume of $30cm^3$ and a mass of 60g. What is its density?

Solution

Density is mass divided by volume, so that the density is 60 g divided by $30cm^3$, which is **2.0 g/cm^3**.

$$Density = \frac{Mass}{Volume}$$

$$Density = \frac{60g}{30cm^3} = 2.0\frac{g}{cm^3}$$

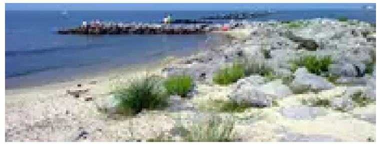
Density

- **Problem 3:** In the above two examples which rock is heavier? Which is lighter?

solution

The question is asking about *heavier* and *lighter*, which refers to mass or weight. Therefore, all you care about is the mass in grams and so **the 60 g rock in the second problem is heavier**and the **45 g rock (in the first question) is lighter**.

- **Problem 6:** Rocks are sometimes used along coasts to prevent erosion. If a rock needs to weigh 2,000 kilograms (about 2 tons) in order not to be shifted by waves, how big (what volume) does it need to be? You are using basalt, which has a typical density of 3200 kg/m^3



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In this problem you need a volume, so you will need to rearrange the density equation to get volume.

$$Density = \frac{Mass}{Volume}$$

By multiplying both sides by volume, we can get volume out of the numerator (the bottom).

$$Volume \times Density = Mass$$

You can then divide both sides by density to get volume alone:

$$Volume = \frac{Mass}{Density}$$

By substituting in the values listed above,

$$Volume = \frac{2000\ kg}{3200\ \frac{kg}{m^3}}$$

So the volume will be **0.625 m^3**

Note that the above problem shows that densities can be in units other than grams and cubic centimeters. To avoid the potential problems of different units, many geologists use specific gravity (SG), explored in problems 8 and 9, below.

Calculate the specific weight, density and specific gravity of 1 litre of petrol which weighs 7 N.

Given:

Volume= 1 litre =0.001 m^3

Weight=7 N

Specific Weight[w]= $\frac{Weight}{Volume} = \frac{7}{0.001}$ =7000 $\frac{N}{m^3}$

w.k.t Sp. Wt.[w]= ρ g

Density ρ = $\frac{w}{g} = \frac{7000}{9.81}$ =713.56 kg/m^3

Specific gravity S = $\frac{Density\ of\ the\ liquid}{Density\ of\ water} = \frac{713.56}{1000}$ = 0.714

Calculate the density, specific weight and weight of 1 litre of liquid of specific gravity 0.7

Given:

Volume, V= 1 litre =0.001 m^3

Specific gravity, S=0.7

w.k.t Specific gravity S = $\frac{Density\ of\ the\ liquid}{Density\ of\ water}$

$\therefore Density\ of\ the\ liquid$ ρ= S x 1000 = 0.7 x 1000 = 700 kg/m^3

$Specific\ weight, w$= ρ x g = 700 x 9.81 = 6867 N/m^3

w.k.t $Specific\ weight$ w = $\frac{Weight}{Volume}$

$\therefore Weight$ = w x Volume = 6867 x 0.001 = 6.867 N

Viscosity

- resistance of a fluid (liquid or gas) to a change in shape, or movement of neighboring portions relative to one another.

Mathematically, $$\tau \propto \frac{du}{dy}$$

or $$\tau = \mu \frac{du}{dy}$$

$$\mu = \frac{\text{Shear stress}}{\frac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\frac{\text{Length}}{\text{Time}}\right) \times \frac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(Length)}^2}{\frac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

$\therefore$ SI unit of viscosity = Ns/m^2 = Pa s.

SI unit of viscosity $= \frac{\text{Newton-sec}}{m^2} = \frac{Ns}{m^2}$

Kinematic viscosity

- kinematic viscosity refers to the ratio of dynamic viscosity to density. Based on the expression above,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

The units of kinematic viscosity is obtained as

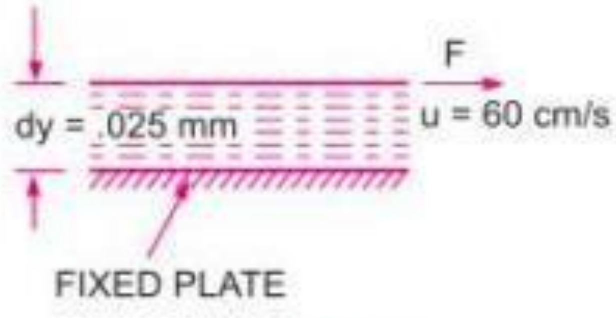
$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\frac{\text{Mass}}{\text{Length}}\right)} \qquad \left\{\because \text{Force} = \text{Mass} \times \text{Acc.} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2}\right\}$$

$$= \frac{(\text{Length})^2}{\text{Time}}.$$

Example 1

A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates.



Solution

Solution. Given :

Distance between plates, $dy = .025$ mm

$= .025 \times 10^{-3}$ m

Velocity of upper plate, $u = 60$ cm/s $= 0.6$ m/s

Force on upper plate, $F = 2.0 \dfrac{\text{N}}{\text{m}^2}$.

Using the equation $$\tau = \mu \frac{du}{dy}.$$

where du = Change of velocity $= u - 0 = u = 0.60$ m/s

dy = Change of distance $= .025 \times 10^{-3}$ m

τ = Force per unit area $= 2.0 \dfrac{\text{N}}{\text{m}^2}$

$$\therefore \quad 2.0 = \mu \frac{0.60}{.025 \times 10^{-3}} \quad \therefore \quad \mu = \frac{2.0 \times .025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \frac{\text{Ns}}{\text{m}^2}$$

$$= 8.33 \times 10^{-5} \times 10 \text{ poise} = \mathbf{8.33 \times 10^{-4} \text{ poise}}.$$ **Ans.**

Example 2

Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m × 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.

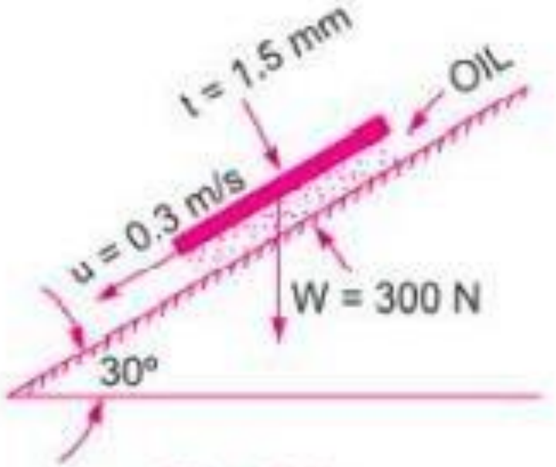


Fig. 1.4

Solution

Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300 \text{ N}$

Velocity of plate, $u = 0.3 \text{ m/s}$

and shear stress, $\tau = \frac{F}{\text{Area}} = \frac{150}{0.64} \text{ N/m}^2$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

Solution

where du = change of velocity = $u - 0 = u = 0.3$ m/s

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = \mathbf{11.7 \text{ poise.}} \text{ Ans.}$$

Class Activity

Example 3

A flat plate of area 1.5×10^6 mm^2 is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise.

$pa.s = 10\,poise$

r=1 poise=0.1 pa.s

$r = \mu \frac{du}{dy}$ r=0.1*$\frac{0.4}{0.15*10^{-3}}$=266.66 N/m^2

(i) ∴ Shear force, $F = \tau \times \text{area} = 266.66 \times 1.5 =$ **400 N. Ans.**

(ii) Power* required to move the plate at the speed 0.4 m/sec

$= F \times u = 400 \times 0.4 =$ **160 W. Ans.**

Next lecture

- Example on kinematic viscosity
- Surface tension