PART FOUR

STRESS, STRAIN AND DISPLACEMENT RELATIONSHIPS FOR OPEN AND CLOSED SINGLE CELL THIN WALLED BEAMS

Assumptions:

1) Axial constraint effects are negligible

2) Shear stresses normal to surface can be neglected

3) Direct and shear stresses on planes normal to surface are constant across thickness

4) Beams have uniform section, with skin thickness varying around the section but constant along length of the beam

The parameter *'s'* in this analysis is the distance measured around the cross-section of the tube/beam from some convenient origin.

For a loaded beam, look at an element of size $\delta s \times \delta z \times t$ of its wall showing all shear and direct stresses necessary to keep it in equilibrium.

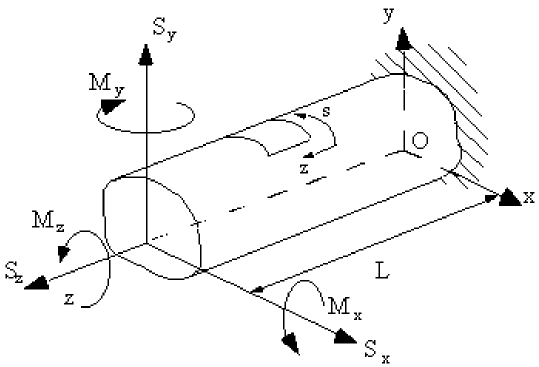

Figure 23: Loaded beam structure.

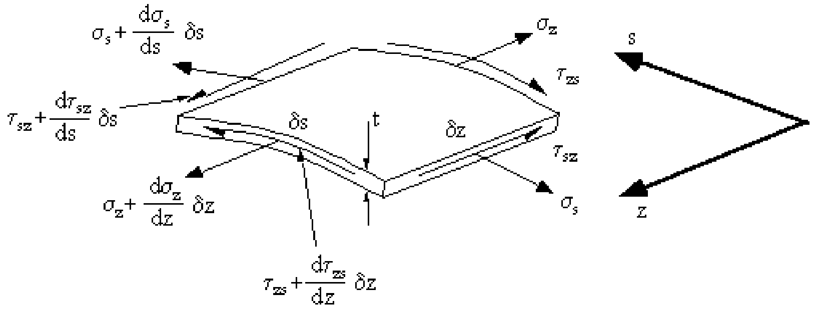

Figure 24: General stress system on element of closed or open beam section.

These shear and direct stresses are produced by the bending moments, shear loads and internal pressures. Although 't' can vary with 's' for each element of length δs, we can assume that this length is small enough to make 't' constant over this length.

From Elasticity we have that:

$$\tau_{zs} = \tau_{sz} = \tau$$

Instead of using shear stress, the analysis will become easier if we introduce the term ***Shear Flow q*** , which is the shear force per unit length rather than shear stress. It is represented by:

$$q = \tau t \quad \textbf{(4.1)}$$

Shear flow is defined positive if it is in the same direction as increasing 's'.

What we are attempting to determine in this analysis are two things, firstly a relationship between the shear and direct stresses, and secondly a relationship between shear strain and the deformation of this element. So we may then use these relationships to determine a shear flow equation and an angle of twist equation.

We now replace the shear stress values in the element with those of shear flow:

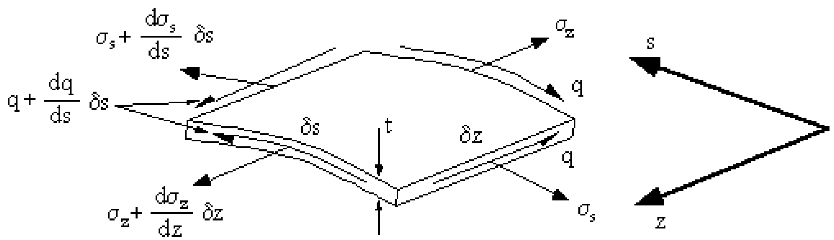

Figure 25: Element showing the direct stresses and shear flow.

Using equilibrium about the z-axis and neglecting body forces gives:

$$\sum F_z = \left(\sigma_z + \frac{\partial \sigma_z}{\partial z}\delta z\right)t\,\delta s - \sigma_z t\,\delta s + \left(q + \frac{\partial q}{\partial s}\delta s\right)\delta z - q\,\delta z = 0$$

Dividing by δz and in the limit as δz→0, it simplifies to:

$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0 \quad \textbf{(4.2)}$$

and using equilibrium about the 's' direction Equation **(4.3)** is similarly derived.

$$\frac{\partial q}{\partial z} + t\frac{\partial \sigma_s}{\partial s} = 0 \quad \textbf{(4.3)}$$

We now need to look at strain relationships. Starting by defining the three components of displacement at a point on the tube wall.

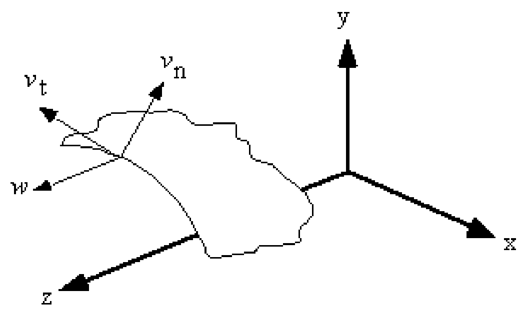

Figure 26: Axial, tangential and normal components of displacement of a point in beam wall.

Where :

w = displacement in the z axis

v_t = tangential displacement, positive with increasing 's'

v_n = normal displacement, positive outwards

From Elasticity :

$$\varepsilon_z = \frac{\partial w}{\partial z} \quad \textbf{(4.4)}$$

To define Shear strain look at how the element is distorted by shear:

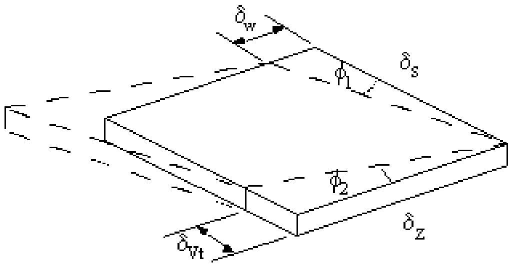

Figure 27: Element distorted due to shear.

Shear strain is then defined as the addition of the two angles of rotation of the sides, such that:

$$\gamma = \phi_1 + \phi_2$$

where in the limits as the element size goes to zero:

$$\gamma = \frac{\partial w}{\partial s} + \frac{\partial v_t}{\partial z} \quad \textbf{(4.5)}$$

It is now necessary to define the term v_t as a function of displacements u and v (in x and y axis) and angle of twist of the section θ. In order to do this it is necessary to assume that the ribs are able to hold the cross section rigidly enough so that when it twists it holds its cross sectional shape. However the ribs have no strength in a plane normal to them, allowing the section to warp or deform in the z axis.

Define

ψ = Angle between the tangent to the surface of the beam's cross section and the x-axis.

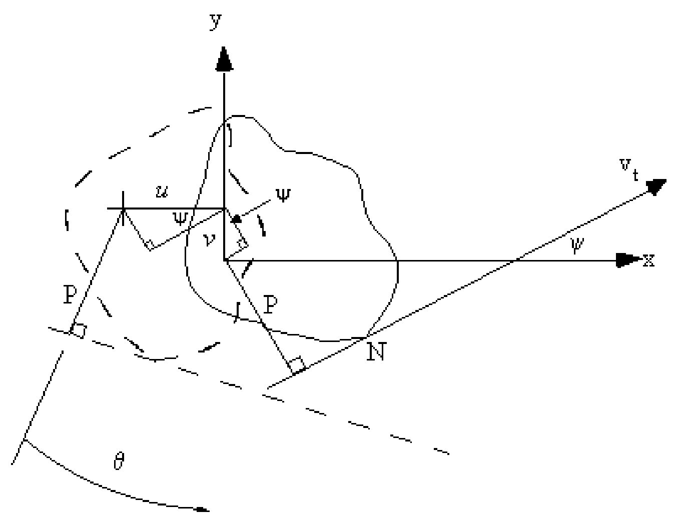

Figure 28: Beam cross section rotated by angle θ, showing the rotation of the normal to surface.

Tangential displacement at any point N on the tube is :

$$v_t = p\theta + u\cos(\psi) + v\sin(\psi) \quad \textbf{(4.6)}$$

Relating these displacements about a point R, which is the centre of twist gives:

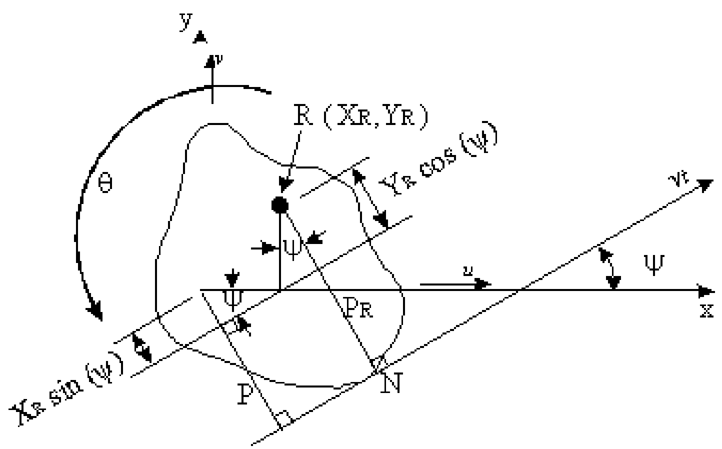

Figure 29: Rotation of beam section about centre of twist

the displacement v_t is

$$v_t = p_R \theta$$

and

$$p_R = p - x_R \sin(\psi) + y_R \cos(\psi)$$

which when combined give:

$$v_t = p\theta - x_R\theta\sin(\psi) + y_R\theta\cos(\psi) \qquad \textbf{(4.7)}$$

These two equations which describe the tangential displacement of the tube. In order to determine the second term of Equation **(4.5)** it is necessary to differentiate Equations **(4.6)** and **(4.7)** with respect to z.

$$\frac{\partial v_t}{\partial z} = p\frac{d\theta}{dz} + \frac{du}{dz}\cos(\psi) + \frac{dv}{dz}\sin(\psi) \qquad \textbf{(4.8)}$$

and

$$\frac{\partial v_t}{\partial z} = p\frac{d\theta}{dz} - x_R\sin(\psi)\frac{d\theta}{dz} + y_R\cos(\psi)\frac{d\theta}{dz} \qquad \textbf{(4.9)}$$

These two equations represent the same value, therefore the centre of rotation is:

$$x_R = -\frac{dv/dz}{d\theta/dz}, \quad y_R = \frac{du/dz}{d\theta/dz} \qquad \textbf{(4.10)}$$

These equations will be to use to determine the shear stress distribution in a thin walled open or closed tube, as well as the displacement, warping and angle of twist of the section due to *these shear loads.*

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