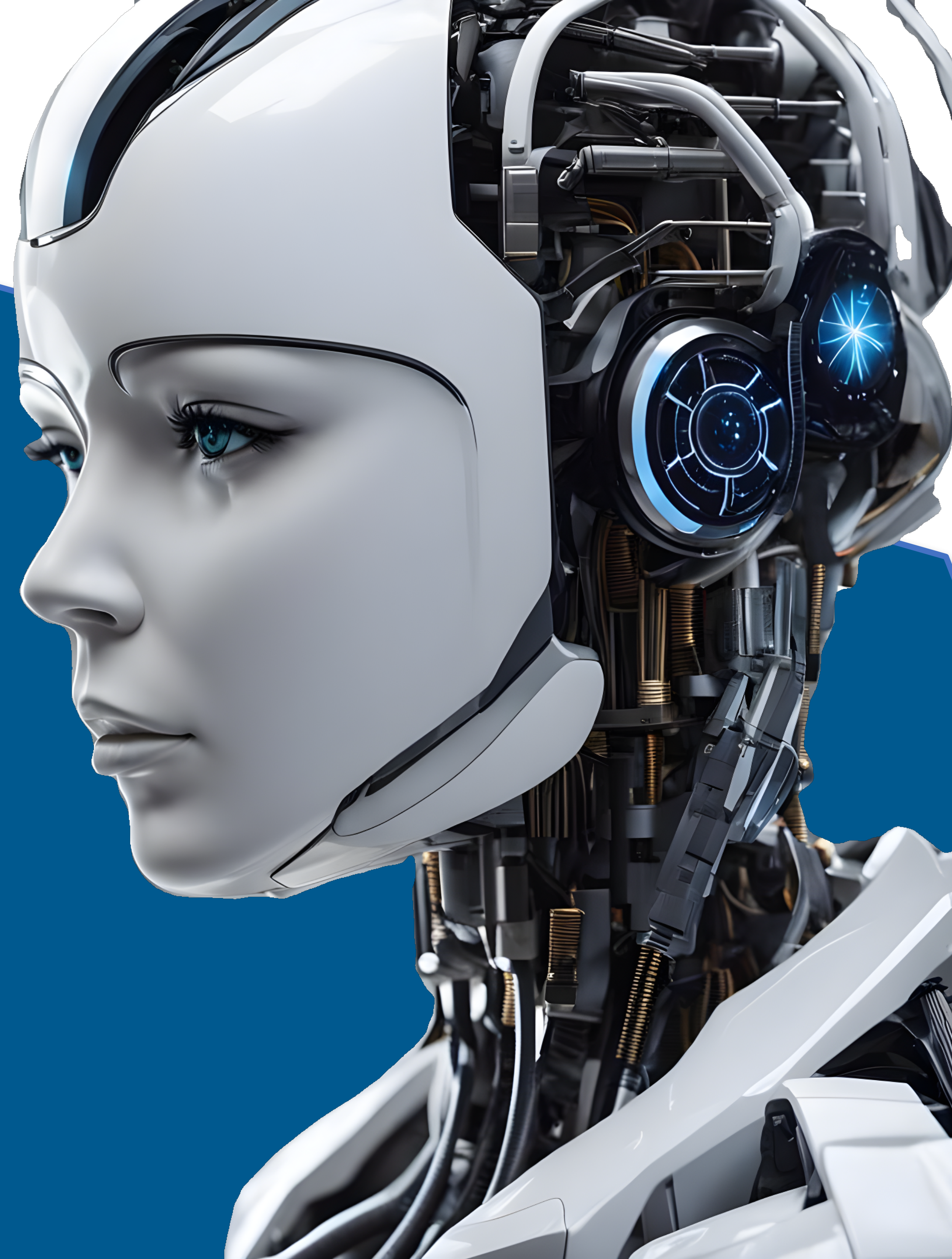




Tishk International University  
IT Department  
Course Code: IT-344/A



# Introduction to Machine Learning

## Classifications (Bayes Classifier)

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## Lecture 5



# Outline



- Bayes Classifier
- Bayes' Theorem
- Types of Bayes Classifiers
- Examples

# Objectives



- Understand the concept of a Bayes classifier and how it uses Bayes' theorem to classify data based on probability.
- Learn the mathematical formula for Bayes' theorem and how it relates prior and conditional probabilities.
- Analyze real-world examples and use cases where Bayes classifiers are applied, such as spam filtering, sentiment analysis, or document classification



# Bayes Classifier

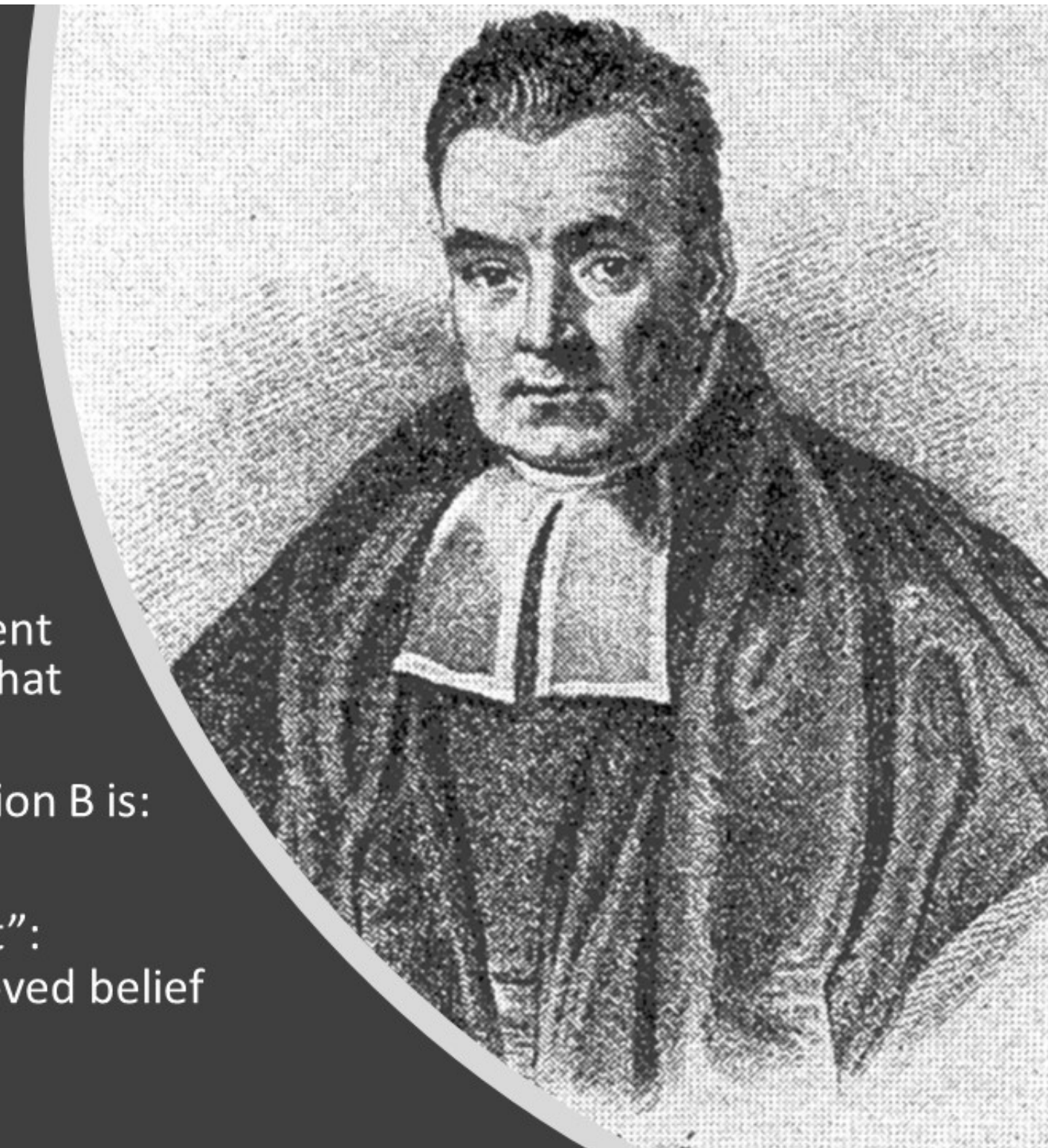


**Bayes Classifier** is a probabilistic model based on Bayes' theorem used for classification tasks.

- It serves as a foundational concept in probability theory and statistics.
- It calculates the probability of each class given a set of features

Rev. Thomas Bayes  
(c. 1702 – 1761)

- English theologian and mathematician
- Bayes' Theorem: the probability of an event based on prior knowledge of conditions that are related to the event
- i.e., the probability of A under the condition B is:  
$$P(A|B) = P(B|A) \cdot P(A) / P(B)$$
- i.e., “In (Conditional) Probability We Trust”:  
Initial belief + New data = Adjusted improved belief



# Bayes' Theorem



$$P(A | B) = \frac{P(B | A) \cdot P(A)}{P(B)}$$

- $P(A|B)$  is the **posterior** probability of event A given event B has occurred.
- $P(B|A)$  is the **likelihood** of event B occurring given that event A has occurred.
- $P(A)$  is the **prior probability** of event A before considering any new evidence.
- $P(B)$  is the probability of event B occurring, also known as the **marginal likelihood** or **evidence**.



# Bayes' Theorem



$$P(A | B) = \frac{P(B | A) \cdot P(A)}{P(B)}$$

Diagram illustrating Bayes' Theorem with labels:

- Posterior**:  $P(A | B)$
- Likelihood**:  $P(B | A)$
- Prior**:  $P(A)$
- Evidence**:  $P(B)$

- Posterior Probability: Updated probability of classes after considering the evidence.
- Likelihood: Probability of observing data given class.
- Prior Probability: Initial belief about the probability of classes.
- Evidence: Probability of observing the data.

# Types of Bayes Classifiers



- **Naive Bayes Classifier:** Assumes independence between features.
- **Gaussian Naive Bayes:** Assumes Gaussian distribution for continuous features.
- **Multinomial Naive Bayes:** Suitable for discrete features with a multinomial distribution.

# Example



## Normal Message

1. **Hello**, how are you today, **friend**?
2. **Hello** there! Long time no see, **friend**!
3. **Hello**! Would you like to grab food tonight?
4. **Hello friend**, let's catch up over **dinner** soon.
5. **Hello**, it's been a while since we last had dinner together.
6. **Hello friend**, want to join me for **dinner** tomorrow?
7. **Hello friend**, let's plan a **dinner** outing this weekend.
8. **Hello** there, would you like to avail a **discount** on your next time?

8 messages (Normal)	
Word	Count
Hello	8
Friend	5
Dinner	3
Discount	1
Total	17



# Example



## Spam Messages:

1. **Hello!** Congratulations, you've won a **discount** voucher!
2. Hey there, **friend!** Check out our exclusive **discount** offers!
3. Get ready for amazing **discounts** on our food specials!
4. **Hello!** Don't miss out on our limited-time **discount** deals!

4 messages (Spam)	
Word	Count
Hello	2
Friend	1
Dinner	0
Discount	4
Total	7

# Example



8 messages (Normal)		
Word	Count	P(word   N)
Hello	8	$p(\text{Hello}   N) = 8/17 = 0.47$
Friend	5	$p(\text{Friend}   N) = 5/17 = 0.29$
Dinner	3	$p(\text{Dinner}   N) = 3/17 = 0.18$
Discount	1	$p(\text{Discount}   N) = 1/17 = 0.06$
Total	17	

8 messages (Normal)	
Word	Count
Hello	8
Friend	5
Dinner	3
Discount	1
Total	17

$$p(\text{Normal}) = (\text{Normal messages}) / (\text{Total messages})$$

$$p(N) = 8 / (8 + 4) = 0.67$$



# Example



4 messages (Spam)		
SpamWord	Count	P(SpamWord   S)
Hello	2	$P(\text{Hello}   S) = 2/7 = 0.29$
Friend	1	$P(\text{Hello}   S) = 1/7 = 0.14$
Dinner	0	$P(\text{Hello}   S) = 0/7 = 0$
Discount	4	$P(\text{Hello}   S) = 4/7 = 0.57$
Total		7

4 messages (Spam)	
Word	Count
Hello	2
Friend	1
Dinner	0
Discount	4
Total	7

$$p(\text{Spam}) = (\text{Spam messages}) / (\text{Total messages})$$

$$p(S) = 4 / (8 + 4) = 0.33$$

# Example



New Message: “Hello Friend”

## Normal Class

- $p(\text{Hello}|\text{N})=0.47$
- $p(\text{Friend}|\text{N})=0.29$
- $p(\text{Dinner}|\text{N})=0.18$
- $p(\text{Discount}|\text{N})=0.06$
- $p(\text{N}) = 0.67$

$$p(\text{N} | \text{Hello, Friend}) = p(\text{Hello, Friend} | \text{N}) \times p(\text{N})$$

$$p(\text{N} | \text{Hello, Friend}) = p(\text{Hello} | \text{N}) \times p(\text{Friend} | \text{N}) \times p(\text{N})$$

$$p(\text{N} | \text{Hello, Friend}) = 0.47 \times 0.29 \times 0.67 \approx 0.09$$



# Example



New Message: **“Hello Friend”**

## Spam Class

- $p(\text{Hello}|\text{S})=0.29$
- $p(\text{Friend}|\text{S})=0.14$
- $p(\text{Dinner}|\text{S})=0.0$
- $p(\text{Discount}|\text{S})=0.57$
- $p(\text{S}) = 0.33$

$$p(\text{S} | \text{Hello, Friend}) = p(\text{Hello, Friend} | \text{S}) \times p(\text{S})$$

$$p(\text{S} | \text{Hello, Friend}) = p(\text{Hello} | \text{S}) \times p(\text{Friend} | \text{S}) \times p(\text{S})$$

$$p(\text{S} | \text{Hello, Friend}) = 0.29 \times 0.14 \times 0.33 \approx 0.01$$

# Example



$$p(N | \text{Hello, Friend}) = 0.47 \times 0.29 \times 0.67 \approx 0.09$$

$$p(S | \text{Hello, Friend}) = 0.29 \times 0.14 \times 0.33 \approx 0.01$$

Thus, according to the Bayes classifier, the message "Hello Friend" is more likely to belong to class N (Normal) since the posterior probability for N is higher than that for S.



New Message: **“Friend Discount Discount Discount”**

## Normal & Spam Class

**Hint:**

$$p(N | \text{Hello, Friend}) = p(\text{Hello, Friend} | N) \times p(N)$$

$$p(N | \text{Hello, Friend}) = p(\text{Hello} | N) \times p(\text{Friend} | N) \times p(N)$$

$$p(S | \text{Hello, Friend}) = p(\text{Hello, Friend} | S) \times p(S)$$

$$p(S | \text{Hello, Friend}) = p(\text{Hello} | S) \times p(\text{Friend} | S) \times p(S)$$

- $p(\text{Hello}|N)=0.47$
- $p(\text{Friend}|N)=0.29$
- $p(\text{Dinner}|N)=0.18$
- $p(\text{Discount}|N)=0.06$
- $p(N) = 0.67$
  
- $p(\text{Hello}|S)=0.29$
- $p(\text{Friend}|S)=0.14$
- $p(\text{Dinner}|S)=0.0$
- $p(\text{Discount}|S)=0.57$
- $p(S) = 0.33$

New Message: “Friend Discount Discount Discount”

## Normal Class

- $p(\text{Hello}|N)=0.47$
- $p(\text{Friend}|N)=0.29$
- $p(\text{Dinner}|N)=0.18$
- $p(\text{Discount}|N)=0.06$
- $p(N) = 0.67$

$$\begin{aligned} p(N | \text{Friend, Discount, Discount, Discount}) &= p(\text{Friend, Discount, Discount, Discount} | N) \times p(N) \\ &= p(\text{Friend} | N) \times p(\text{Discount} | N) \times p(\text{Discount} | N) \times p(\text{Discount} | N) \times p(N) \\ &= 0.29 \times 0.06 \times 0.06 \times 0.06 \times 0.67 \approx 0.000042 \end{aligned}$$

New Message: **“Friend Discount Discount Discount”**

## Spam Class

- $p(\text{Hello}|\text{S})=0.29$
- $p(\text{Friend}|\text{S})=0.14$
- $p(\text{Dinner}|\text{S})=0.0$
- $p(\text{Discount}|\text{S})=0.57$
- $p(\text{S}) = 0.33$

$$\begin{aligned} p(\text{S} | \text{Friend, Discount, Discount, Discount}) &= p(\text{Friend, Discount, Discount, Discount} | \text{S}) \times p(\text{S}) \\ &= p(\text{Friend} | \text{S}) \times p(\text{Discount} | \text{S}) \times p(\text{Discount} | \text{S}) \times p(\text{Discount} | \text{S}) \times p(\text{S}) \\ &= 0.14 \times 0.57 \times 0.57 \times 0.57 \times 0.33 \approx 0.0086 \end{aligned}$$



# Exercise



$$p(N | \text{Friend, Discount, Discount, Discount}) = 0.29 \times 0.06 \times 0.06 \times 0.06 \times 0.67 \approx 0.000042$$

$$p(S | \text{Friend, Discount, Discount, Discount}) = 0.14 \times 0.57 \times 0.57 \times 0.57 \times 0.33 \approx 0.0086$$

Thus, according to the Bayes classifier, the message "Friend Discount Discount Discount" is more likely to belong to class S (Spam) since the posterior probability for S is higher than that for N (Normal).

Thank You

