Tishk International University
Mechatronics Engineering Department
Electrical Circuits Analysis II
Code: ME-114

AC Electrical Circuits Analysis:
Power Factor in AC circuits

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OUTLINE

- Apparent power
- The Relationship between P, Q, and S
- The RLC Circuits
  - The Power Factor
  - Power Triangular
  - The Impedance Triangle

Apparent power

- If the load contains both resistance and reactance, this product represents neither real power nor reactive power. Since *VI* appears to represent power, it is called **apparent power.**
- Apparent power is given the symbol ***S*** and has units of **volt-amperes (VA).** Thus,

$$S = VI \quad \text{(VA)}$$

where V and I are the magnitudes of the rms voltage and current, respectively. Since $V = IZ$ and $I = V/Z$, S can also be written as

$$S = I^2Z = V^2/Z \quad \text{(VA)}$$

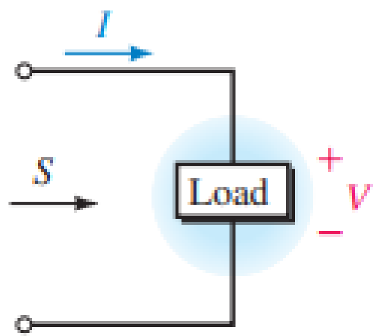


FIGURE 17–11 Apparent power $S = VI$.

For small equipment (such as found in electronics), VA is a convenient unit. However, for heavy power apparatus (Figure 17–12), it is too small and kVA (kilovolt-amps) is frequently used, where

$$S = \frac{VI}{1000} \quad \text{(kVA)}$$

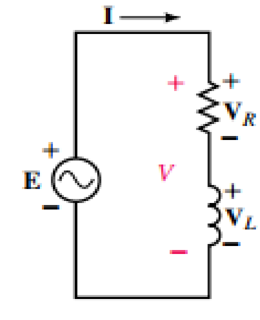
Apparent Power

In addition to its VA rating, it is common practice to rate electrical apparatus in terms of its operating voltage. Once you know these two, it is easy to determine rated current. For example, a piece of equipment rated 250 kVA, 4.16 kV has a rated current of $I = S/V = (250 \times 10^3\ \mathrm{VA})/(4.16 \times 10^3\ \mathrm{V}) = 60.1\ \mathrm{A}$.

FIGURE 17–12 Power apparatus is rated in apparent power. The transformer shown is a 167-kVA unit.

Power triangle

- The real, reactive, and apparent power separately are related by a very simple relationship through the Power Triangle.

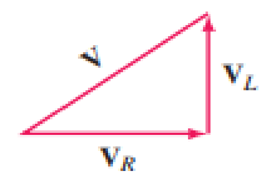


(a)

  - Consider the series circuit in figure (a)
  - Let the current through the circuit be $\mathbf{I} = I\angle 0°$ with phasor representation in (b)

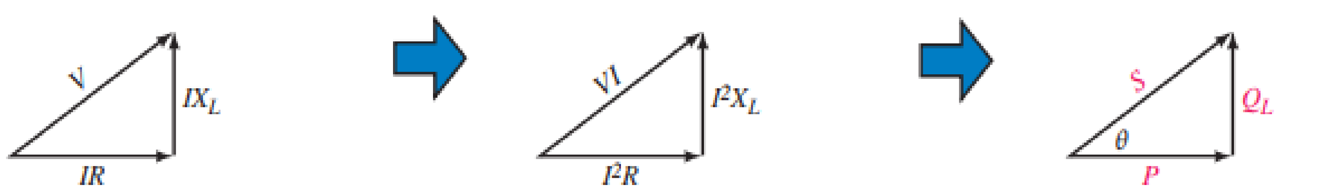
(b)

- VR is in phase with I, while VL leads it by 90°
- Kirchhof's voltage law applies for ac voltages in phasor form $\mathbf{V} = \mathbf{V}_R + \mathbf{V}_L$ as indicated in (c)



(c)

- The voltage triangle of (C) can be redrawn as



(a) Magnitudes only shown (b) Multiplied by I (c) Resultant power triangle

FIGURE 17–14 Steps in the development of the power triangle continued.

FIGURE 17–13 Steps in the development of the power triangle—continued in Figure 17–14.

The Relationship between P, Q, and S

Power Triangle of an AC Circuit

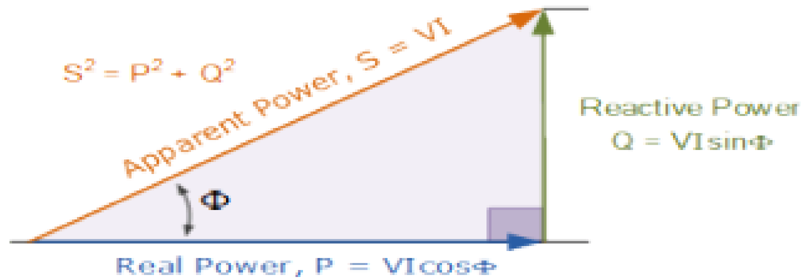
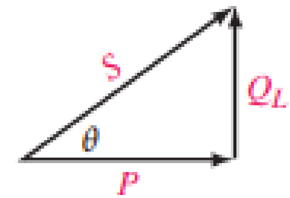


Figure power triangle

- Where:
- **P** is the I^2*R or Real power that performs work measured in watts, W
- **Q** is the I^2*X or Reactive power measured in volt-amperes reactive, VAr
- **S** is the I^2*Z or Apparent power measured in volt-amperes, VA
- **Φ** is the phase angle in degrees. The larger the phase angle, the greater the reactive power
- $\text{Cos}(\Phi) = P/S = W/VA$ = power factor, p.f.
- $\text{Sin}(\Phi) = Q/S = VAr/VA$
- $\text{Tan}(\Phi) = Q/P = VAr/W$

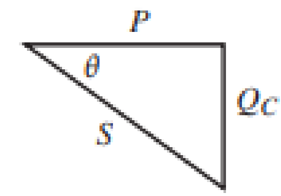
The power factor is calculated as the ratio of the real power to the apparent power because this ratio equals cos(Φ).

From the geometry of the resultant triangle,



- Apparent Power Magnitude: $S = \sqrt{P^2 + Q_L^2}$
- Complex power representation: $\mathbf{S} = P + jQ_L$

$\mathbf{S} = S\angle\theta$



- Incase the circuit has CL instead of VL: $\mathbf{S} = P - jQ_C$
- Real and Reactive Power equations represent as:

$$P = VI\cos\theta = S\cos\theta \quad \text{(W)}$$

$$Q = VI\sin\theta = S\sin\theta \quad \text{(VAR)}$$

  - V and I are the magnitude of the rms values
  - **θ** is the angle between the rms values V,I
  - **Θ = θ*v* – θ*i***
  - P is always positive
  - Q is positive for Inductive circuits and negative for Capacitive circuits.

Some useful power triangle relationships are listed following:

$S = v_{RMS} \times i_{RMS}$

$S = \sqrt{P^2 + Q^2}$

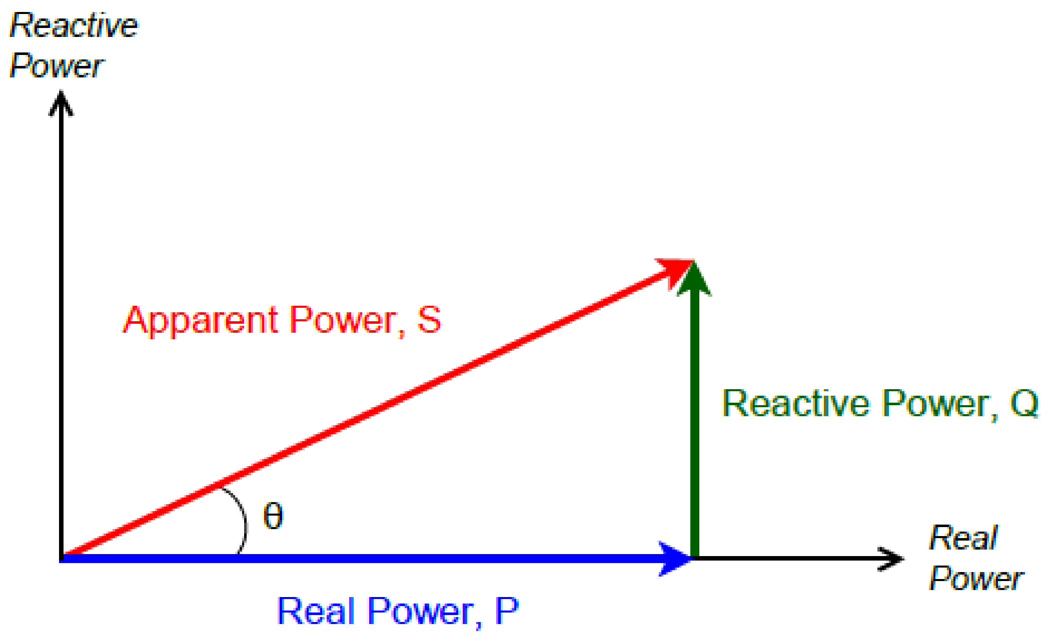
$\theta = tan^{-1}\ (Q/P)$ (i.e., the impedance angle)

$P = S \cos \theta$

$P = \sqrt{S^2 - Q^2}$

$Q = S \sin\theta$

$Q = \sqrt{S^2 - P^2}$



Example,

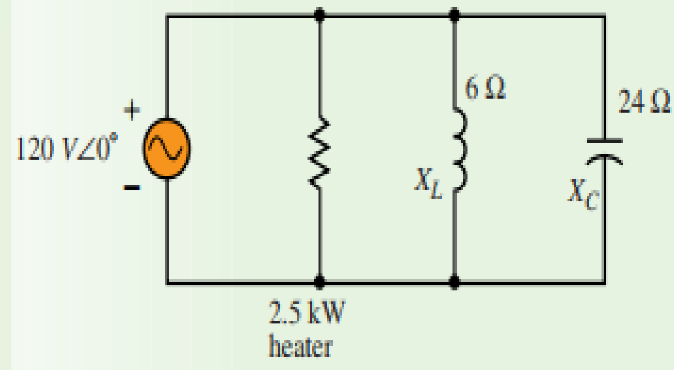
A generator supplies power to an electric heater, an inductive element, and a capacitor as in Figure 17–17(a).

a. Find P and Q for each load.

b. Find total active and reactive power supplied by the generator.

c. Draw the power triangle for the combined loads and determine total apparent power.

d. Find the current supplied by the generator.



(a)

Solution

a. The components of power are as follows:

Heater: $P_H = 2.5\text{ kW}$ $Q_H = 0\text{ VAR}$

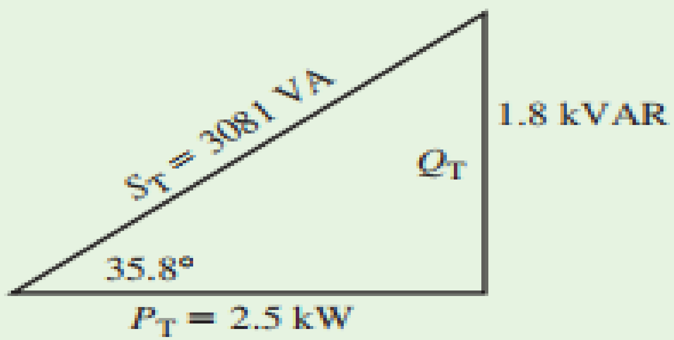
Inductor: $P_L = 0\text{ W}$ $Q_L = \dfrac{V^2}{X_L} = \dfrac{(120\text{ V})^2}{6\ \Omega} = 2.4\text{ kVAR (ind.)}$

Capacitor: $P_C = 0\text{ W}$ $Q_C = \dfrac{V^2}{X_C} = \dfrac{(120\text{ V})^2}{24\ \Omega} = 600\text{ VAR (cap.)}$

b. $P_T = 2.5\text{ kW} + 0\text{ W} + 0\text{ W} = 2.5\text{ kW}$

$Q_T = 0\text{ VAR} + 2.4\text{ kVAR} - 600\text{ VAR} = 1.8\text{ kVAR (ind.)}$

c. The power triangle is sketched as Figure 17–7(b). Both the hypotenuse and the angle can be obtained easily using rectangular to polar conversion. $\mathbf{S}_T = P_T + jQ_T = 2500 + j1800 = 3081\angle 35.8°$. Thus, apparent power is $S_T = 3081\text{ VA}$.



(b)

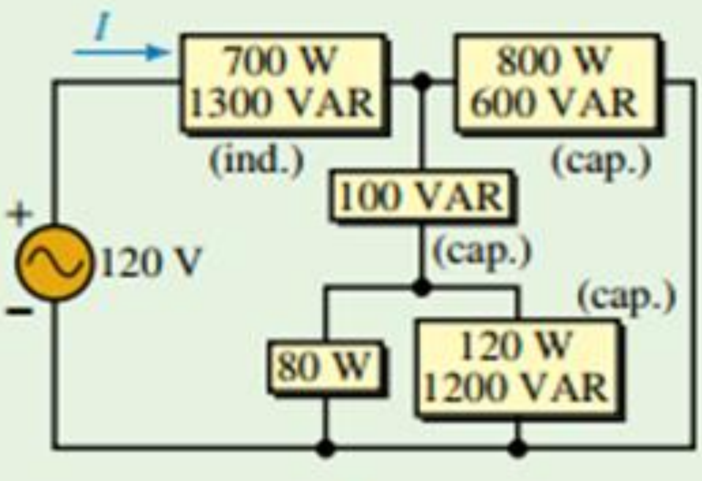
d. Generator current is $I = \dfrac{S_T}{E} = \dfrac{3081\text{ VA}}{120\text{ V}} = 25.7\text{ A}$

Class Activity,

The P and Q values for a circuit are shown in Figure 17–16(a).

a. Determine the power triangle.

b. Determine the magnitude of the current supplied by the source.



(a)

Solution

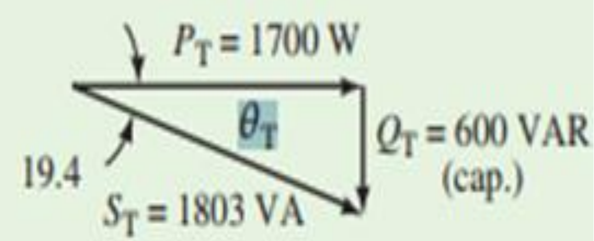
a. $P_T = 700 + 800 + 80 + 120 = 1700\text{ W}$

$Q_T = 1300 - 600 - 100 - 1200 = -600\text{ VAR} = 600\text{ VAR (cap.)}$

$S_T = P_T + jQ_T = 1700 - j600 = 1803\angle -19.4°\text{ VA}$

The power triangle is as shown. The load is net capacitive.

b. $I = S_T/E = 1803\text{ VA}/120\text{ V} = 15.0\text{ A}.$

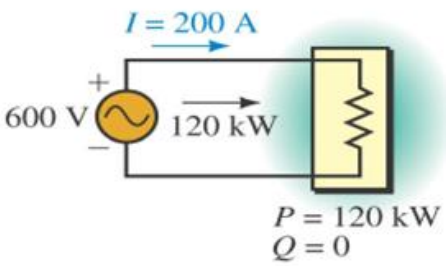


(b)

Why equipment are rated in VA instead of Watts ?

Assume that the generator is rated at 600V, 120kVA:

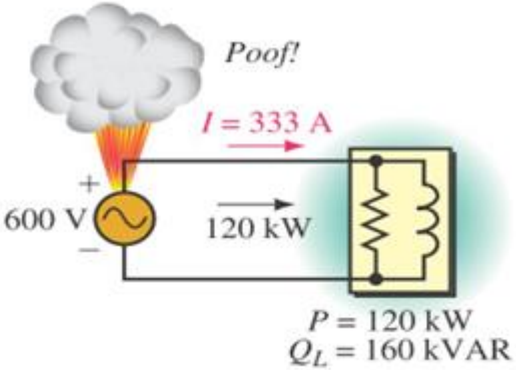
Current capability, **I = 120kVA/600V = 200A**



(a) S = 120 kVA

- In Figure (a), the generator is supplying a purely resistive load with 120 kW.
- Since S = P for a purely resistive load, S = 120 kVA and the generator is supplying its rated current.

- **In Figure (b), the generator is supplying a load with P= 120 kW as before, but Q=160 kVAR.**
- **Its apparent power is therefore S = 200 kVA,**



(b) $S = \sqrt{(120)^2 + (160)^2} = 200$ kVA
The generator is overloaded

Supplied current: **I = 200 kVA / 600 V** = 333.3 A

So, the current-carrying capability can be greatly exceeded (even though its power rating is not) which means overloading and possible damage.

Thank you for being patient and a good listener,

Take a rest please!

The Power Factor

Power factor is the ratio of the average power to the apparent power.

- The angle θ = θv – θi is called the **power factor angle**, which is equal to the angle of the ***load impedance***

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle \theta_v - \theta_i = \frac{V_{\text{rms}}}{I_{\text{rms}}} \angle \theta_v - \theta_i$$

$$\boxed{\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)}$$

- the power factor cannot exceed 1.0 (or 100% if expressed in percent).

- **A load's power factor shows how much it's apparent power is actually real power:**

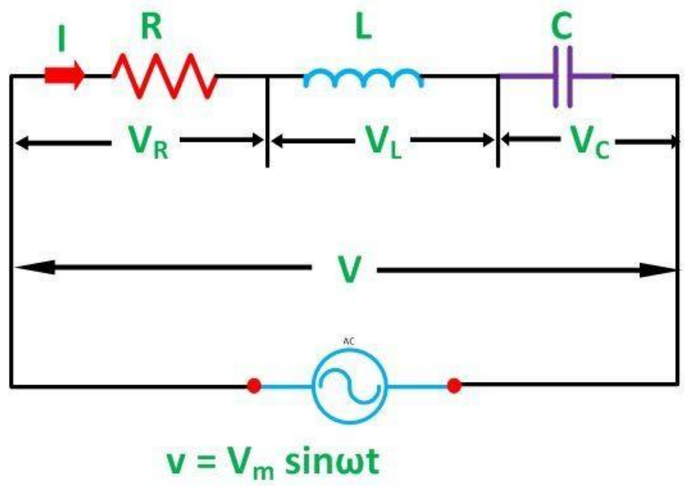
✓ **For a purely resistive circuit: θ = 0, pf = cos(0) = 1**
  - All the load's apparent power is real power
  - This case is referred to as unity power factor.

✓ **For a Purely reactive circuit: θv – θi = ± 90◦, pf = 0.**
  - In this case the average power is zero.

✓ **In between these two extreme cases, pf is said to be leading or lagging.**
  - Leading power factor means that current leads voltage, which implies a capacitive load.
  - Lagging power factor means that current lags voltage, implying an inductive load.

The RLC Circuits

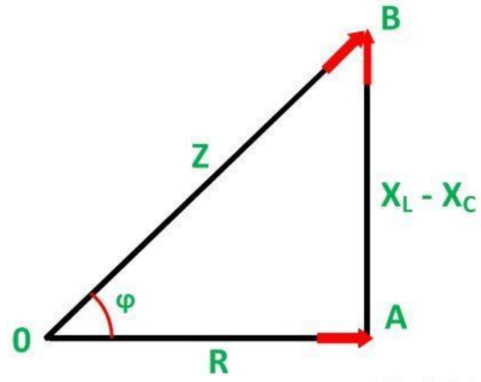
- In the RLC Series circuit, XL = 2πfL and XC = 1/2πfC



- When the AC voltage is applied through the RLC Series circuit the resulting current I flows through the circuit, and thus the voltage across each element will be:
  - VR = IR that is the voltage across the resistance R and is in phase with the current I.
  - VL = IXL that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.
  - VC = IXC that is the voltage across capacitor C and it lags the current I by an angle of 90 degrees.
- The three cases of RLC Series Circuit
  - When XL > XC, the phase angle φ is positive. The circuit behaves as RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.
  - When XL < XC, the phase angle φ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.
  - When XL = XC, the phase angle φ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of the power factor is unity.

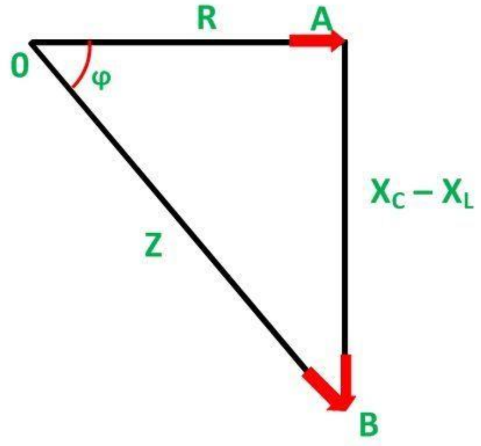
➢ Impedance Triangle of RLC Series Circuit

- The impedance triangle of the RL series circuit, when (XL > XC) is shown below:



  – If the inductive reactance is greater than the capacitive reactance than the circuit reactance is inductive giving a lagging phase angle.

- Impedance triangle is shown below when the circuit acts as an RC series circuit (XL< XC):



  – When the capacitive reactance is greater than the inductive reactance the overall circuit reactance acts as capacitive and the phase angle will be leading.

- **The complex power maybe expressed in term of the load impedance,**

Since:

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \angle \theta_v - \theta_i$$

$$\mathbf{V}_{\text{rms}} = \mathbf{Z}\mathbf{I}_{\text{rms}}$$

Therefore:

$$\boxed{\mathbf{S} = I_{\text{rms}}^2 \mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*}}$$

Since $\mathbf{Z} = R + jX$

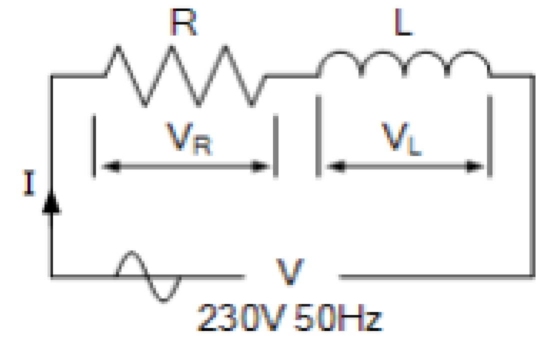
$$\mathbf{S} = I_{\text{rms}}^2 (R + jX) = P + jQ$$

$$P = \text{Re}(\mathbf{S}) = I_{\text{rms}}^2 R$$

$$Q = \text{Im}(\mathbf{S}) = I_{\text{rms}}^2 X$$

1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

Example, A solenoid coil with a resistance of 30 ohms and an inductance of 200mH is connected to a 230VAC, 50Hz supply. Calculate: (a) the solenoids impedance, (b) the current consumed by the solenoid, (c) the phase angle between the current and the applied voltage, and (d) the average power consumed by the solenoid.



Data given: $R = 30\Omega$, $L = 200mH$, $V = 230V$ and $f = 50Hz$.

(a) Impedance (Z) of the solenoid coil:

$$R = 30\Omega$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 200 \times 10^{-3} = 62.8\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 62.8^2} = 69.6\Omega$$

(b) Current (I) consumed by the solenoid coil:

$$V = I \times Z$$

$$\therefore I = \frac{V}{Z} = \frac{230}{69.6} = 3.3\,A_{(rms)}$$

(c) The phase angle, θ:

$$\cos\theta = \frac{R}{Z}, \quad \text{or} \quad \sin\theta = \frac{X_L}{Z}, \quad \text{or} \quad \tan\theta = \frac{X_L}{R}$$

$$\therefore \cos\theta = \frac{R}{Z} = \frac{30}{69.6} = 0.431$$

$$\cos^{-1}(0.431) = 64.5^{\circ} \text{ lagging}$$

(d) Average AC power consumed by the solenoid coil:

$$P = V \times I \times \cos\theta$$

$$P = 230 \times 3.3 \times \cos\left(64.5^{\circ}\right)$$

$$\therefore P = 327 \text{ watts}$$

Class Activity, Three coils of resistances 20, 30 and 40 Ohm and inductance 0.5, 0.3 and 0.2 *H*, respectively are connected in series across a 230 V, 50 Hz supply. Calculate the total current, power factor and the power consumed in the circuit.

$$R = 20 + 30 + 40 = 90 \text{ Ohm}$$

$$L = 0.5 + 0.3 + 0.2 = 1.0 \text{ Ohm}$$

$$X_L = 2\pi f L = 2\pi * 50 * 1.0 = 314\ \Omega$$

$$\text{Impedance } Z = \sqrt{R^2 + X_L^2}$$

$$= 90^2 + 314^2 = 327\ \Omega$$

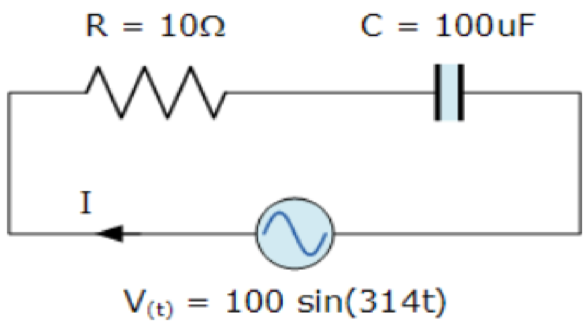
$$\text{Current } I = \frac{E}{Z} = \frac{230}{327} = 0.704 \text{ A}$$

$$power\ factor\ \cos\varphi = \frac{R}{Z} = \frac{90}{327} = 0.275 \text{ lagging}$$

$$\text{Power consumed} = EI\cos\varphi.$$

$$= 230 * 0.704 * 0.275 = 44.5 \text{ watts.}$$

- Example, A capacitor which has an internal resistance of 10Ω's and a capacitance value of 100uF is connected to a supply voltage given as $V_{(t)} = 100 \sin(314t)$. Calculate the current flowing into the capacitor. Also construct a voltage triangle showing the individual voltage drops.



The capacitive reactance and circuit impedance is calculated as:

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 100uF} = 31.85\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{10^2 + 31.85^2} = 33.4\Omega$$

Then the current flowing into the capacitor and the circuit is given as:

$$I = \frac{V_C}{Z} = \frac{100}{33.4} = 3\text{Amps}$$

The phase angle between the current and voltage is calculated from the impedance triangle above as:

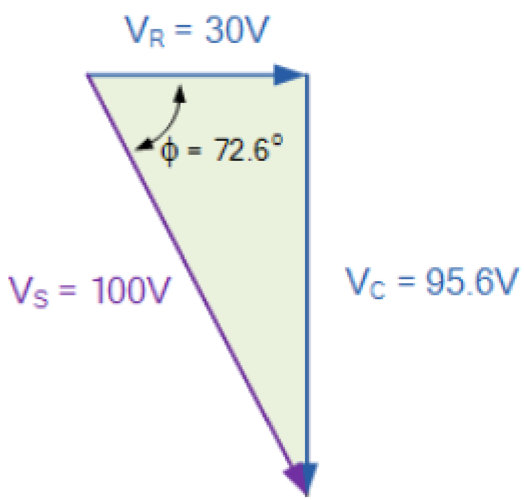
$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \frac{31.85}{10} = 72.6^\circ \text{ leading}$$

Then the individual voltage drops around the circuit are calculated as:

$$V_R = I \times R = 3 \times 10 = 30V$$

$$V_C = I \times X_C = 3 \times 31.85 = 95.6V$$

$$V_S = \sqrt{V_R^2 + V_C^2} = \sqrt{30^2 + 95.6^2} = 100V$$



Then the resultant voltage triangle will be.