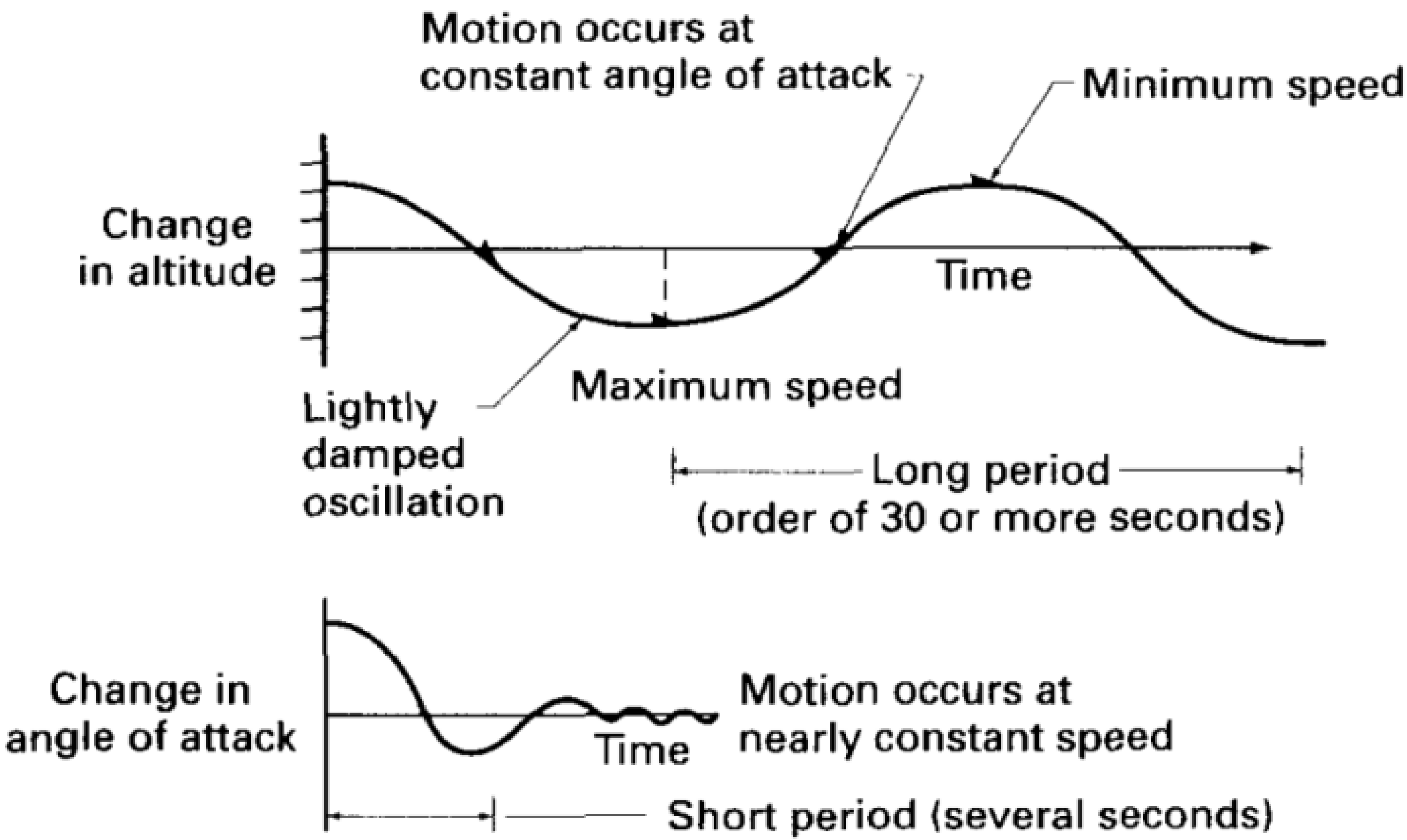
Lecture - 8
Application of Classical Control Theory to Aircraft Autopilot Design

- 1. AIRCRAFT TRANSFER FUNCTIONS
- The longitudinal and lateral equations of motion were described by a set of linear differential equations in Chapter 3.
- The transfer function gives the relationship between the output of and input to a system.
- In the case of aircraft dynamics it specifies the relationship between the motion variables and the control input
- In the following sections we develop the transfer function based on the longitudinal and lateral approximations developed in Chapters 4 and 5. We develop these simpler mathematical models so that we can examine the idea behind various autopilots without undue mathematical complexity.
- **The X-force, Z-force, and pitching moment equations forms the longitudinal motion. And the Y-force, rolling, and yawing moment equations form the lateral equations.**

- 2. Longitudinal T.F:
- The longitudinal motion of an airplane (controls fixed) disturbed from its equilibrium flight condition is characterized by two oscillatory modes of motion, Figure1 illustrates these basic modes.



- **Short-Period Dynamics** :
- In Chapter 4 the equations for the short-period motions were developed for the case where the control was held fixed. The equation with control input from the elevator in state space form can be written as

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha/u_0 & 1 \\ M_\alpha + M_{\dot{\alpha}} Z_\alpha/u_0 & M_q + M_{\dot{\alpha}} \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e}/u_0 \\ M_{\delta_e} + M_{\dot{\alpha}} Z_{\delta_e}/u_0 \end{bmatrix} [\Delta\delta_e] \quad (8.1)$$

Taking the Laplace transform of this equation yields

$$(s - Z_\alpha/u_0)\, \Delta\alpha(s) - \Delta q(s) = Z_{\delta_e}/u_0\, \Delta\delta_e(s)$$

$$-(M_\alpha + M_{\dot{\alpha}} Z_\alpha/u_0)\, \Delta\alpha(s) + [s - (M_q + M_{\dot{\alpha}})]\, \Delta q(s) = (M_{\delta_e} + M_{\dot{\alpha}} Z_{\delta_e}/u_0)\, \Delta\delta_e$$

If we divide these equations by $\Delta\delta_e(s)$

$$(s - Z_\alpha/u_0) \frac{\Delta\alpha(s)}{\Delta\delta_e(s)} - \frac{\Delta q(s)}{\Delta\delta_e(s)} = Z_{\delta_e}/u_0$$

$$-(M_\alpha + M_{\dot{\alpha}} Z_\alpha/u_0) \frac{\Delta\alpha(s)}{\Delta\delta_e(s)} + [s - (M_q + M_{\dot{\alpha}})] \frac{\Delta q(s)}{\Delta\delta_e(s)} = M_{\delta_e} + M_{\dot{\alpha}} \frac{Z_{\delta_e}}{u_0}$$

- **Short-Period Dynamics**

$$(s - Z_\alpha/u_0)\frac{\Delta\alpha(s)}{\Delta\delta_e(s)} - \frac{\Delta q(s)}{\Delta\delta_e(s)} = Z_{\delta_e}/u_0$$

$$-(M_\alpha + M_{\dot\alpha} Z_\alpha/u_0)\frac{\Delta\alpha(s)}{\Delta\delta_e(s)} + [s - (M_q + M_{\dot\alpha})]\frac{\Delta q(s)}{\Delta\delta_e(s)} = M_{\delta_e} + M_{\dot\alpha}\frac{Z_{\delta_e}}{u_0}$$

Solving for $\Delta\alpha(s)/\Delta\delta_e(s)$ and $\Delta q(s)/\Delta\delta_e(s)$ by Cramer's rule yields

$$\frac{\Delta\alpha(s)}{\Delta\delta_e(s)} = \frac{N^\alpha_{\delta_e}(s)}{\Delta_{sp}(s)} = \frac{\begin{vmatrix} Z_{\delta_e}/u_0 & -1 \\ M_{\delta_e} + M_{\dot\alpha}\frac{Z_{\delta_e}}{u_0} & s - (M_q + M_{\dot\alpha}) \end{vmatrix}}{\begin{vmatrix} s - Z_\alpha/u_0 & -1 \\ -(M_\alpha + M_{\dot\alpha} Z_\alpha/u_0) & s - (M_q + M_{\dot\alpha}) \end{vmatrix}}$$

$$\frac{\Delta\alpha(s)}{\Delta\delta_e(s)} = \frac{N^\alpha_{\delta_e}(s)}{\Delta_{sp}(s)} = \frac{A_\alpha s + B_\alpha}{As^2 + Bs + C}$$

Short-period transfer function approximations

	A, A_α, or A_q	B, B_α, or B_q	C
$\Delta_{sp}(s)$	1	$-(M_q + M_{\dot\alpha} + Z_\alpha/u_0)$	$Z_\alpha M_q/u_0 - M_\alpha$
$N^\alpha_{\delta_e}(s)$	Z_{δ_e}/u_0	$M_{\delta_e} - M_q Z_{\delta_e}/u_0$	
$N^q_{\delta_e}(s)$	$M_{\delta_e} + M_{\dot\alpha} Z_{\delta_e}/u_0$	$M_\alpha Z_{\delta_e}/u_0 - M_{\delta_e} Z_\alpha/u_0$	

- **Short-Period Dynamics** $(s - Z_\alpha/u_0)\dfrac{\Delta\alpha(s)}{\Delta\delta_e(s)} - \dfrac{\Delta q(s)}{\Delta\delta_e(s)} = Z_{\delta_e}/u_0$

$$-(M_\alpha + M_{\dot\alpha} Z_\alpha/u_0)\frac{\Delta\alpha(s)}{\Delta\delta_e(s)} + [s - (M_q + M_{\dot\alpha})]\frac{\Delta q(s)}{\Delta\delta_e(s)} = M_{\delta_e} + M_{\dot\alpha}\frac{Z_{\delta_e}}{u_0}$$

- The transfer function for the change in pitch rate to the change in elevator angle

$$\frac{\Delta q(s)}{\Delta\delta_e(s)} = \frac{N^q_{\delta_e}(s)}{\Delta_{sp}(s)} = \frac{\begin{vmatrix} s - Z_\alpha/u_0 & Z_{\delta_e}/u_0 \\ -(M_\alpha + M_{\dot\alpha} Z_\alpha/u_0) & M_{\delta_e} + M_{\dot\alpha}\dfrac{Z_{\delta_e}}{u_0} \end{vmatrix}}{\begin{vmatrix} s - Z_\alpha/u_0 & -1 \\ -(M_\alpha + M_{\dot\alpha} Z_\alpha/u_0) & s - (M_q + M_{\dot\alpha}) \end{vmatrix}}$$

$$\frac{\Delta q(s)}{\Delta\delta_e(s)} = \frac{N^q_{\delta_e}(s)}{\Delta_{sp}(s)} = \frac{A_q s + B_q}{As^2 + Bs + C}$$

Short-period transfer function approximations

	A, A_α, or A_q	B, B_α, or B_q	C
$\Delta_{sp}(s)$	1	$-(M_q + M_{\dot\alpha} + Z_\alpha/u_0)$	$Z_\alpha M_q/u_0 - M_\alpha$
$N^\alpha_{\delta_e}(s)$	Z_{δ_e}/u_0	$M_{\delta_e} - M_q Z_{\delta_e}/u_0$	
$N^q_{\delta_e}(s)$	$M_{\delta_e} + M_{\dot\alpha} Z_{\delta_e}/u_0$	$M_\alpha Z_{\delta_e}/u_0 - M_{\delta_e} Z_\alpha/u_0$	

- **Long-Period or Phugoid Dynamics**
- The state-space equation for the long period or phugoid approximation are as follows:

$$\begin{bmatrix}\Delta\dot{u}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix} X_u & -g \\ -\dfrac{Z_u}{u_0} & 0 \end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta\theta\end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_T} \\ -\dfrac{Z_{\delta_e}}{u_0} & -\dfrac{Z_{\delta_T}}{u_0}\end{bmatrix}\begin{bmatrix}\Delta\delta_e\\ \Delta\delta_T\end{bmatrix}$$

The Laplace transformation of the approximate equations for the long period are

$$(s - X_u)\,\Delta u(s) + g\,\Delta\theta(s) = X_{\delta_e}\,\Delta\delta_e(s) + X_{\delta_T}\,\Delta\delta_T(s)$$

$$\frac{Z_u}{u_0}\,\Delta u(s) + s\,\Delta\theta(s) = -\frac{Z_{\delta_e}}{u_0}\,\Delta\delta_e(s) - \frac{Z_{\delta_T}}{u_0}\,\Delta\delta_T(s)$$

The transfer function $\Delta u(s)/\Delta\delta_e(s)$ and $\Delta\theta(s)/\Delta\delta_e(s)$ can be found by setting $\Delta\delta_T(s)$ to 0 and solving for the appropriate transfer function as follows:

$$(s - X_u)\,\frac{\Delta u(s)}{\Delta\delta_e(s)} + g\,\frac{\Delta\theta(s)}{\Delta\theta_e(s)} = X_{\delta_e}$$

$$\frac{Z_u}{u_0}\,\frac{\Delta u(s)}{\Delta\delta_e(s)} + s\,\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = -\frac{Z_{\delta_e}}{u_0}$$

$$\frac{\Delta u(s)}{\Delta\delta_e(s)} = \frac{\begin{vmatrix} X_{\delta_e} & g \\ \dfrac{-Z_{\delta_e}}{u_0} & s \end{vmatrix}}{\begin{vmatrix} s - X_u & g \\ \dfrac{Z_u}{u_0} & s \end{vmatrix}}$$

$$\frac{\Delta u(s)}{\Delta\delta_e(s)} = \frac{X_{\delta_e}s + gZ_{\delta_e}/u_0}{s^2 + X_u s - \dfrac{Z_u g}{u_0}}$$

- **Long Period Dynamics**
- The transfer functions can be written in
- a symbolic form in the following manner:

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{\begin{vmatrix} s - X_u & X_{\delta_e} \\ \frac{Z_u}{u_0} & \frac{-Z_{\delta_e}}{u_0} \end{vmatrix}}{\begin{vmatrix} s - X_u & g \\ \frac{Z_u}{u_0} & s \end{vmatrix}}$$

$$\frac{\Delta u(s)}{\Delta\delta_e(s)} = \frac{N^u_{\delta_e}(s)}{\Delta_p(s)} = \frac{A_u s + B_u}{As^2 + Bs + C}$$

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{N^\theta_{\delta_e}}{\Delta_p(s)} = \frac{A_\theta s + B_\theta}{As^2 + Bs + C}$$

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{-\frac{Z_{\delta e}}{u_0}s + \left(\frac{X_u Z_{\delta e}}{u_0} - \frac{Z_u X_{\delta e}}{u_0}\right)}{s^2 - X_u s - \frac{Z_u g}{u_0}}$$

Long-period transfer function approximati...

	A, A_u, or A_θ	B, B_u, or B_θ	C
$\Delta_p(s)$	1	$-X_u$	$-Z_u g/u_0$
$N^u_{\delta_e}(s)$	X_{δ_e}	gZ_{δ_e}/u_0	
$N^\theta_{\delta_e}(s)$	$-Z_{\delta_e}/u_0$	$X_u Z_{\delta_e}/u_0 - Z_u X_{\delta_e}/u_0$	

- **Roll Dynamics**

The equation of motion for a pure rolling motion, developed in Chapter 5, is

$$\Delta\dot{p} - L_p\,\Delta p = L_{\delta_a}\,\Delta\delta_a$$

The transfer function $\Delta p(s)/\delta_a(s)$ and $\Delta\phi(s)/\Delta\delta_a(s)$ can be obtained by taking the Laplace transform of the roll equation:

$$(s - L_p)\,\Delta p(s) = L_{\delta_a}\,\Delta\delta_a(s) \tag{8.21}$$

or

$$\frac{\Delta p(s)}{\Delta\delta_a(s)} = \frac{L_{\delta a}}{s - L_p} \tag{8.22}$$

But the roll rate Δp is defined as $\Delta\dot{\phi}$; therefore,

$$\Delta p(s) = s\,\Delta\phi(s) \tag{8.23}$$

or

$$\frac{\Delta\phi(s)}{\Delta\delta_a(s)} = \frac{L_{\delta_a}}{s(s - L_p)} \tag{8.24}$$

- **Dutch Roll Approximation**
- The approximate equations can be shown to be

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} Y_\beta/u_0 & -(1 - Y_r/u_0) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} Y_{\delta_r}/u_0 & 0 \\ N_{\delta_r} & N_{\delta_a} \end{bmatrix} \begin{bmatrix} \Delta\delta_r \\ \Delta\delta_a \end{bmatrix}$$

Taking the Laplace transform and rearranging yields

$$(s - Y_\beta/u_0)\,\Delta\beta(s) + (1 - Y_r/u_0)\,\Delta r(s) = Y_{\delta_r}/u_0\,\Delta\delta_r(s)$$

$$-N_\beta\,\Delta\beta(s) + (s - N_r)\,\Delta r(s) = N_{\delta_a}\,\Delta\delta_a(s) + N_{\delta_r}\,\Delta\delta_r(s)$$

The transfer functions $\Delta\beta(s)/\Delta\delta_r(s)$ and $\Delta r(s)/\Delta\delta_r(s)$ are obtained as follows:

$$(s - Y_\beta/u_0)\frac{\Delta\beta(s)}{\Delta\delta_r(s)} + (1 - Y_r/u_0)\frac{\Delta r(s)}{\Delta\delta_r(s)} = Y_{\delta_r}/u_0$$

$$-N_\beta\frac{\Delta\beta(s)}{\Delta\delta_r(s)} + (s - N_r)\frac{\Delta r(s)}{\Delta\delta_r(s)} = N_{\delta_r}$$

$$\frac{\Delta\beta(s)}{\Delta\delta_r(s)} = \frac{\begin{vmatrix} Y_{\delta_r}/u_0 & 1 - Y_r/u_0 \\ N_{\delta_r} & s - N_r \end{vmatrix}}{\begin{vmatrix} s - Y_\beta/u_0 & 1 - Y_r/u_0 \\ -N_\beta & s - N_r \end{vmatrix}}$$

$$\frac{\Delta r(s)}{\Delta\delta_r(s)} = \frac{\begin{vmatrix} s - Y_\beta/u_0 & Y_{\delta_r}/u_0 \\ -N_\beta & N_{\delta_r} \end{vmatrix}}{\begin{vmatrix} s - Y_\beta/u_0 & 1 - Y_r/u_0 \\ -N_\beta & s - N_r \end{vmatrix}}$$

$$\frac{\Delta\beta(s)}{\Delta\delta_r(s)} = \frac{N^\beta_{\delta_r}(s)}{\Delta_{DR}(s)} = \frac{A_\beta s + B_\beta}{As^2 + Bs + C}$$

$$\frac{\Delta r(s)}{\Delta\delta_r(s)} = \frac{N^\beta_{\delta_r}(s)}{\Delta_{DR}(s)} = \frac{A_r s + B_r}{As^2 + Bs + C}$$

Dutch Roll Approximation

$$\frac{\Delta\beta(s)}{\Delta\delta_r(s)} = \frac{N^{\beta}_{\delta_r}(s)}{\Delta_{\mathrm{DR}}(s)} = \frac{A_\beta s + B_\beta}{As^2 + Bs + C}$$

$$\frac{\Delta r(s)}{\Delta\delta_r(s)} = \frac{N^{\beta}_{\delta_r}(s)}{\Delta_{\mathrm{DR}}(s)} = \frac{A_r s + B_r}{As^2 + Bs + C}$$

In a similar manner the aileron transfer function can be shown to be

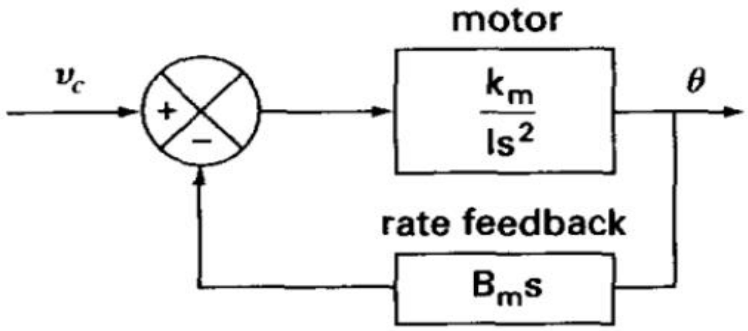
$$\frac{\Delta\beta(s)}{\Delta\delta_a(s)} = \frac{N^{\beta}_{\delta_a}(s)}{\Delta_{\mathrm{DR}}(s)} = \frac{A_\beta s + B_\beta}{As^2 + Bs + C}$$

$$\frac{\Delta r(s)}{\Delta\delta_a(s)} = \frac{N^{r}_{\delta_a}(s)}{\Delta_{\mathrm{DR}}(s)} = \frac{A_r s + B_r}{As^2 + Bs + C}$$

Dutch roll transfer function approximations

	A, A_β, or A_r	B, B_β, or B_r	C
$\Delta_{\mathrm{DR}}(s)$	1	$-(Y_\beta + u_0 N_r)/u_0$	$(Y_\beta N_r - N_\beta Y_r + N_\beta u_0)/u_0$
$N^{\beta}_{\delta_r}(s)$	Y_r/u_0	$(Y_r N_{\delta_r} - Y_{\delta_r} N_r - N_{\delta_r} u_0)/u_0$	
$N^{r}_{\delta_r}(s)$	N_{δ_r}	$(N_\beta Y_{\delta_r} - Y_\beta N_{\delta_r})/u_0$	
$N^{\beta}_{\delta_a}(s)$	0	$(Y_r N_{\delta_a} - u_0 N_{\delta_a})/u_0$	
$N^{r}_{\delta_a}(s)$	N_{δ_a}	$-Y_\beta N_{\delta_a}/u_0$	

- **CONTROL SURFACE ACTUATOR:**
- Control surface servo actuators can be either electrical, hydraulic, pneumatic, or some combination of the three. The transfer function is similar for each type.
- servo actuator transfer function for a servo based on an electric motor.



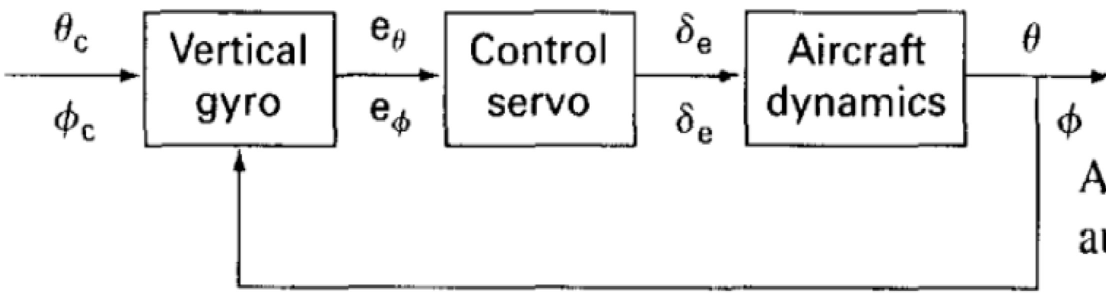
Motor with rate feedback.

$$T_m = k_m v_c \qquad I\ddot{\theta} = T_m \qquad \frac{\theta}{v_c} = \frac{k_m}{Is^2}$$

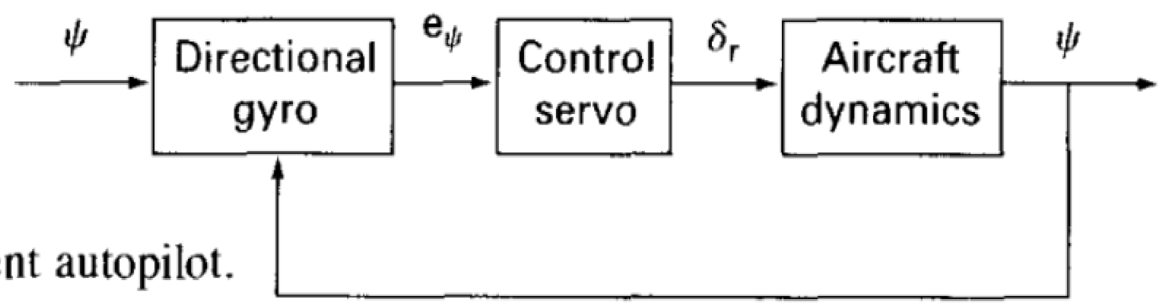
$$\frac{\theta}{v_c} = \frac{k}{s(\tau_m s + 1)} \quad \text{where} \quad \tau_m = \frac{I}{k_m B_m} \quad \text{and} \quad k = \frac{1}{B_m}$$

- If τm (time constant), is small, the motor responds rapidly and the transfer function of the motor with rate feedback can be approximated as:

- **DISPLACEMENT AUTOPILOT**
- In practice, the displacement autopilot is engaged once the airplane has been trimmed in straight and level flight.



A roll or pitch displacement autopilot.



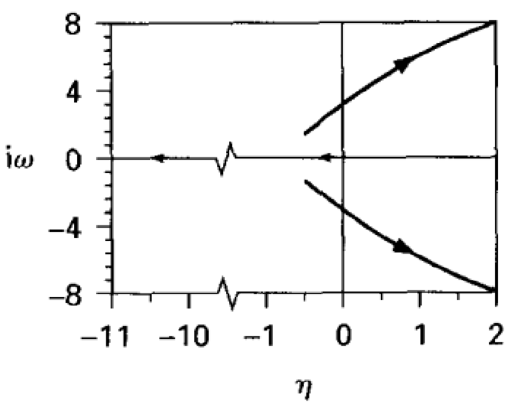
A heading displacement autopilot.

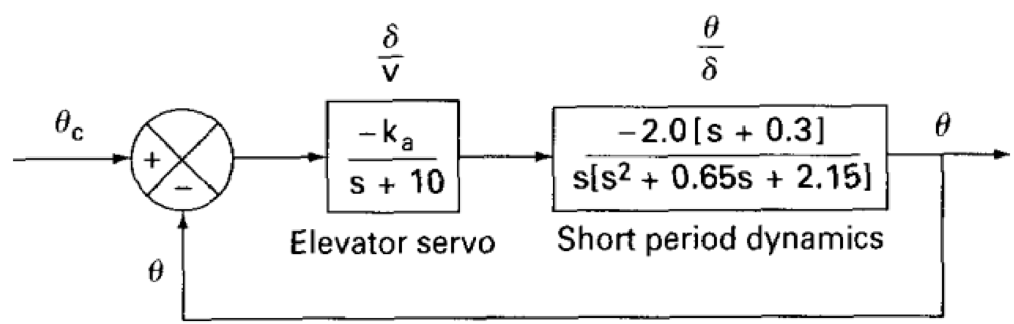
- **Pitch Displacement Autopilot**
- The elevator servo transfer function can be represented as a first-order system:

$$\frac{\delta_e}{v} = \frac{k_a}{\tau s + 1}$$

- where δ_e, v, k_a, - are the elevator deflection angle, input voltage, elevator servo gain, and **τ is time constant** of servomotor .
- Time constants for typical servomotors fall in a range 0.05-0.25 s
- For Example:
- The short-period transfer function for the business jet in (Appendix B) can be shown to be

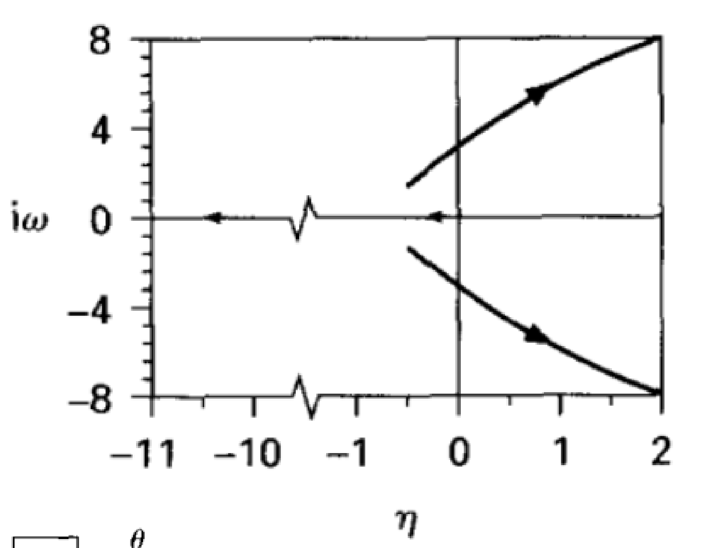
$$\frac{\Delta\theta}{\Delta\delta_e} = \frac{-2.0(s + 0.3)}{s(s^2 + 0.65s + 2.15)}$$

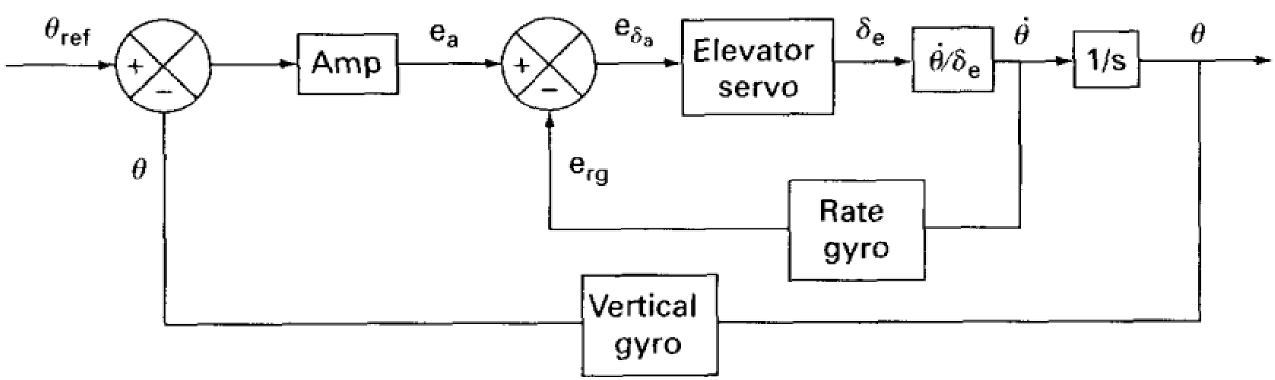




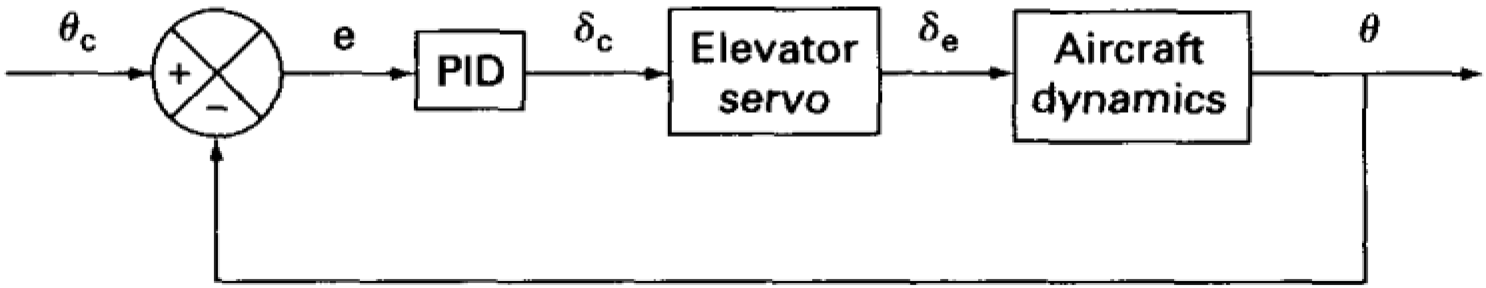
- **Pitch Displacement Autopilot**
- The problem now is one of determining the gain ka so that the control system will have the desired performance.
- Selection of the gain k, can be determined using a root locus plot of the loop transfer function. Figure is the root locus plot for the business jet pitch autopilot. As the gain is increased from 0, the system damping decreases rapidly and the system becomes unstable

--- To improve the design we could increase the damping of the short-period mode by adding an inner feedback loop.





- **EXAMPLE.** Use the PID controller for a pitch attitude autopilot as illustrated in Figure. The transfer functions for each component are given in Table



Data for Example Problem 8.1

Control element	Parameters	Transfer function
PID	$k_p = ?$ $k_i = ?$ $k_d = ?$	$\frac{\delta_c}{e} = k_p + \frac{k_i}{s} + k_d s$
Elevator servo	$A = -0.1$ $\tau = 0.1$	$\frac{\delta_e}{\delta_c} = \frac{A}{\tau s + 1}$
Aircraft dynamics	$M_{\delta e} = -3\ s^{-2}$ $M_q = -2\ s^{-1}$ $M_\alpha = -5\ s^{-2}$	$\frac{\theta}{\delta_e} = \frac{M_{\delta e}}{s^2 - M_q s - M_\alpha}$

- **EXAMPLE**
- Using the Ziegler and Nichols method discussed in Section 7.8, the PID gains can be estimated from the ultimate gain $\mathbf{k_{pu}}$, which is the gain for which the system is marginally stable when only the proportional control is being used. Following is the root locus sketch of the transfer function:

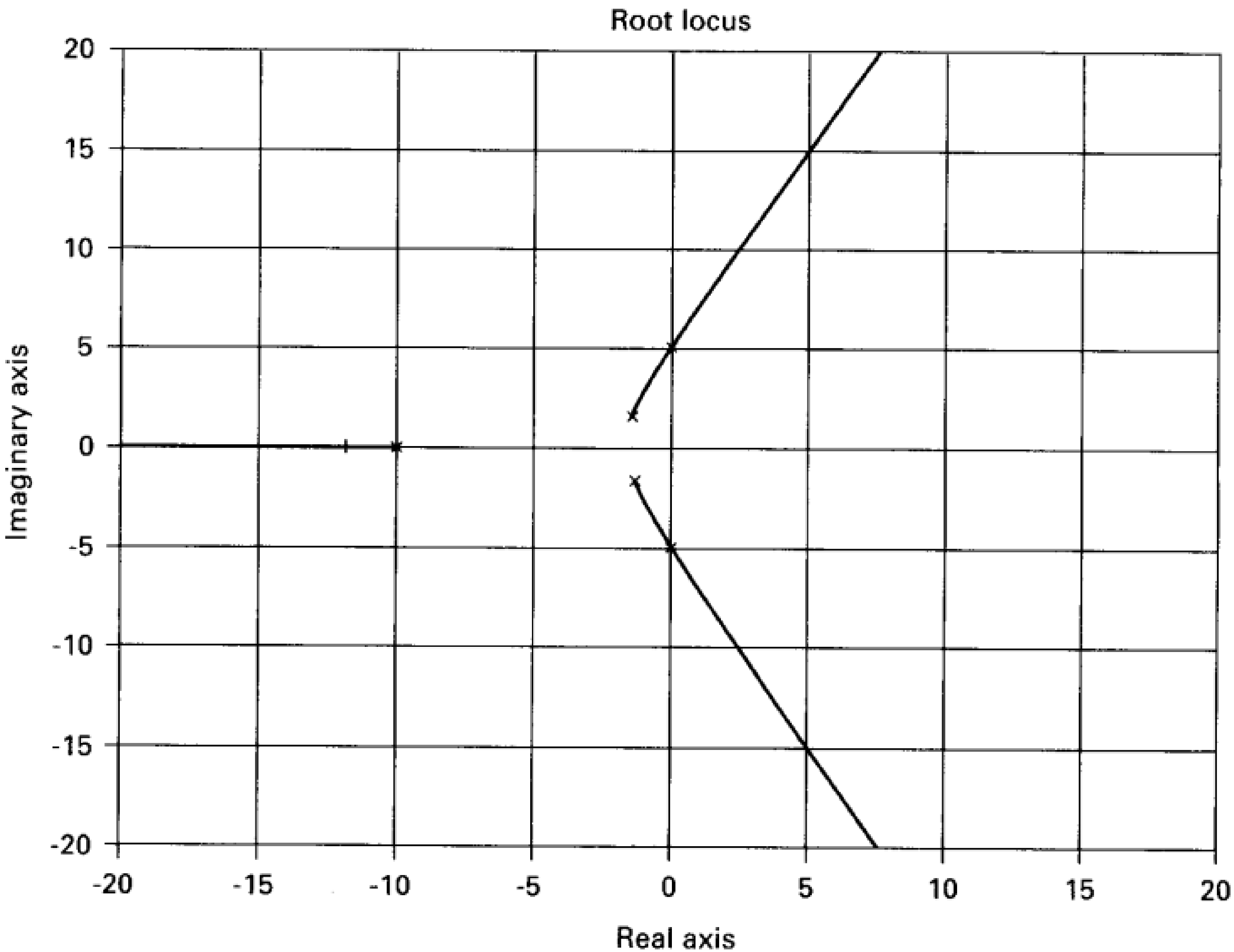
$$G(s)H(s) = \frac{3.0k_p}{(s + 10)(s^2 + 2s + 5)}$$

- The root locus crosses the imaginary axis at s = 5.13i. The gain of the system can be found from the magnitude criteria to be $\mathbf{k_{pu}}$ = 88.7.
- The period, Tu = 2pi/w = 1.22

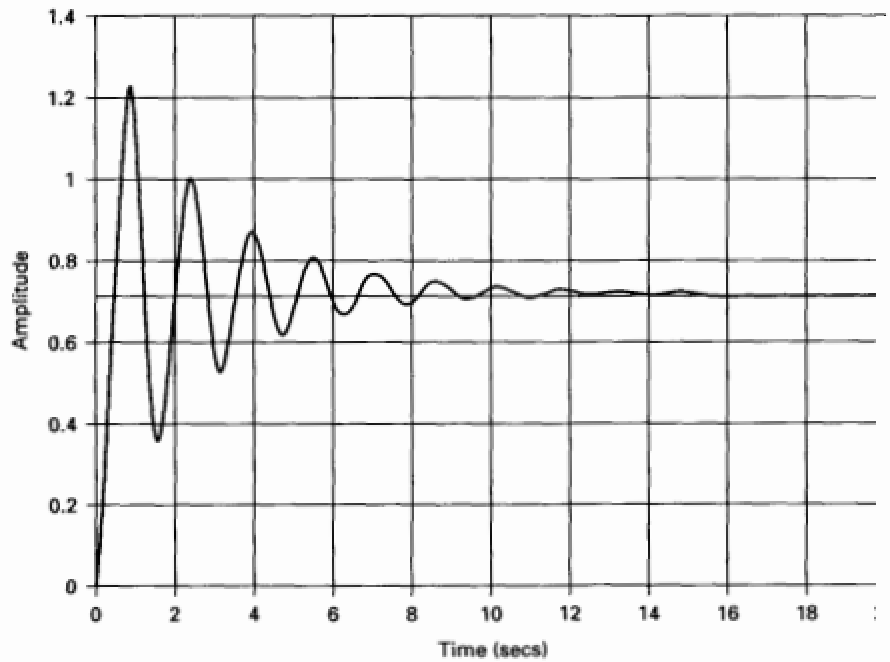
Gains for P, PI, and PID controllers

P control	$k_p = 0.5k_{pu} = 44.35$
PI control	$k_p = 0.45k_{pu} = 39.92$ $k_i = 0.45k_{pu}/(0.83T_u) = 39.42$
PID control	$k_p = 0.6k_{pu} = 53.22$ $k_i = 0.6k_{pu}/(0.5T_u) = 87.24$ $k_d = 0.6k_{pu}(0.125T_u) = 8.12$

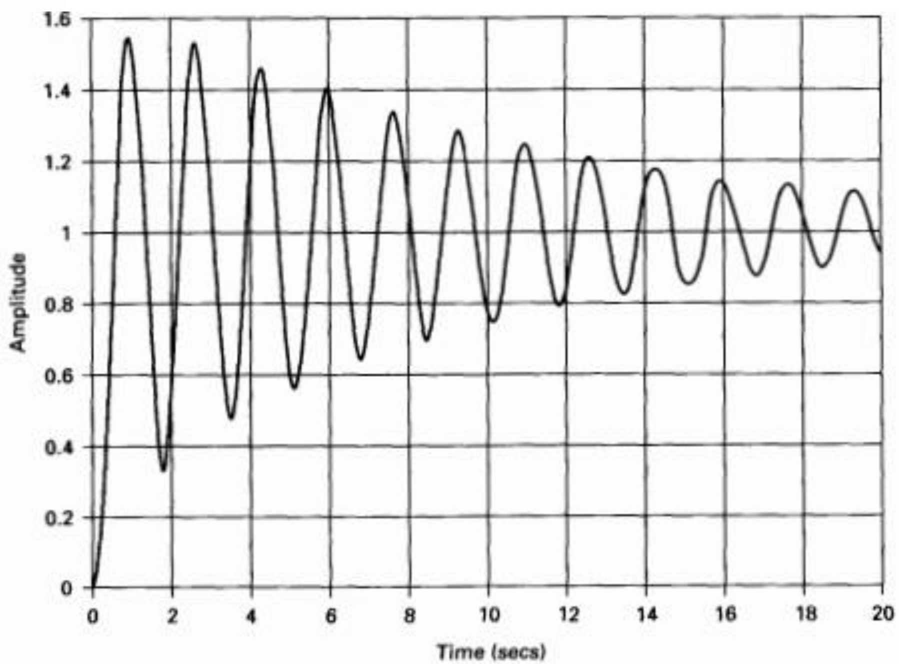
- **EXAMPLE**


Root locus
20
15
10
5
0
-5
-10
-15
-20
Imaginary axis
-20
-15
-10
-5
0
5
10
15
20
Real axis

- **EXAMPLE**
- The response of the pitch attitude autopilot for the three different controllers to a step input is shown.
- Notice that the proportional controller has a steady-state error; that is, it does not go to 1 but converges to a value of approximately 0.7.
- The magnitude of the steady-state error can be predicted using the steady-state error constants in Chapter 7:

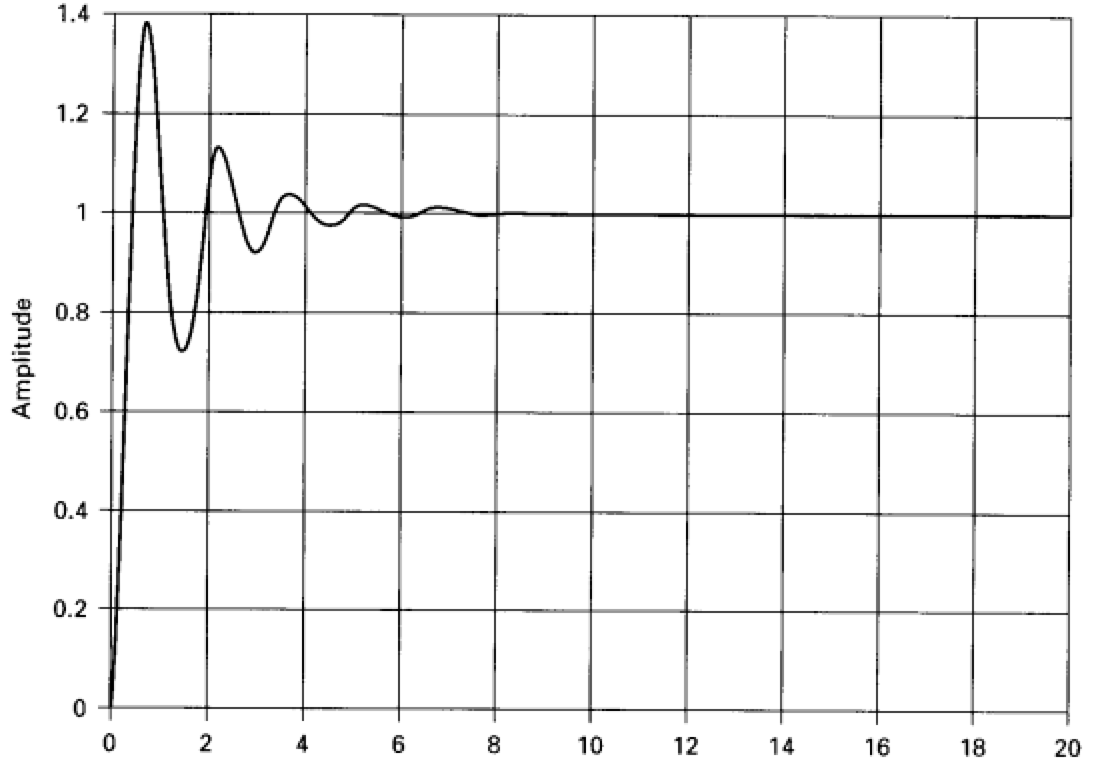


(a) Proportional, integral, and derivative control



(b) Proportional plus integral control

- **EXAMPLE**
- The magnitude of the steady-
- state error can be predicted
- using the steady-state error
- constants in Chapter 7:
-



(c) Proportional, integral, and derivative control

$$e_{ss} = \frac{1}{1 + K_p}$$

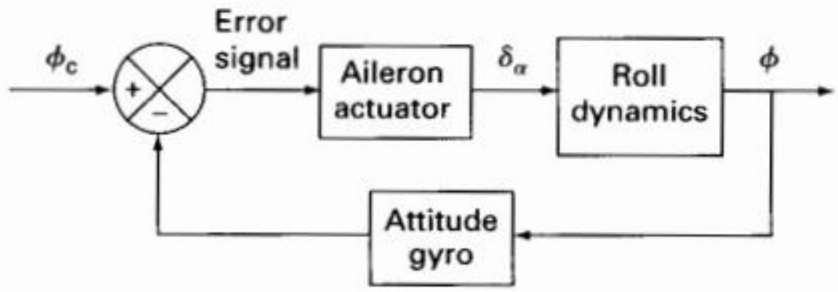
$$K_p = \underset{s \to 0}{\text{Limit}}\, G(s)H(s) = \underset{s \to 0}{\text{Limit}} \frac{3.0k_p}{s^3 + 12s^2 + 25s + 50}$$

for a proportional gain $k_p = 44.35$ **is** $K_p = 2.66$

The steady-state error e_{ss} can then be calculated: $e_{ss} = \frac{1}{1 + K_p} = \frac{1}{3.66} = 0.27$

- Therefore tk e response will go to 0.73 instead of 1 due to the steady-state error

- **2 Roll Attitude Autopilot**
- The roll attitude of an airplane can be controlled by a simple bank angle autopilot as illustrated in Figure



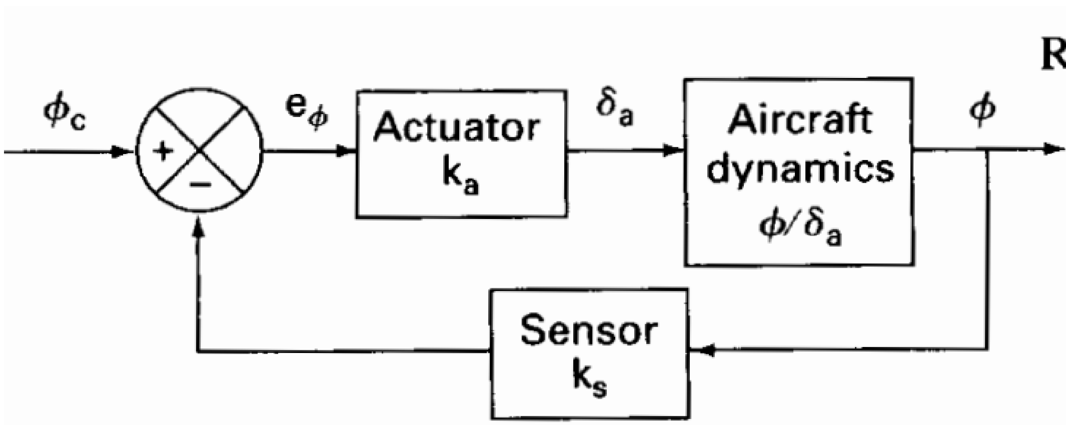
Simple roll attitude control system.

- In practice we would typically design the autopilot to maintain a wings level attitude or $\phi = 0$
- The autopilot is composed of a comparator, aileron actuator, aircraft equation of motion (i.e., transfer function), and an attitude gyro to measure the airplane's roll angle

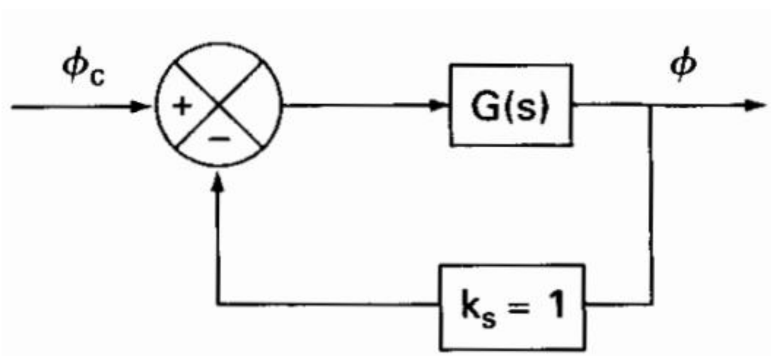
- **E X A M P L E 2.** Design a roll attitude control system to maintain a wings
- level attitude for a vehicle having the following characteristics:

$$L_{\delta_a} = 2.0/s^2 \qquad L_p = -0.5/s$$

- The system performance is to have a damping ratio, 5 = 0.707, and an undamped natural frequency, w,, = 10 rad/s. A potential concept of a roll attitude control system is shown in the block diagram in Figure



Roll attitude control concept.



Simplified roll control system.

- **E X A M P L E 2.**
- The roll angle to aileron input transfer function for an airplane can be shown to be

$$\frac{\Delta\phi(s)}{\Delta\delta_a(s)} = \frac{L_{\delta_a}}{s(s - L_p)} \qquad G(s) = \frac{\Delta\delta_a(s)}{e(s)}\frac{\Delta\phi(s)}{\Delta\delta_a(s)} = k_a\frac{L_{\delta_a}}{s(s - L_p)}$$

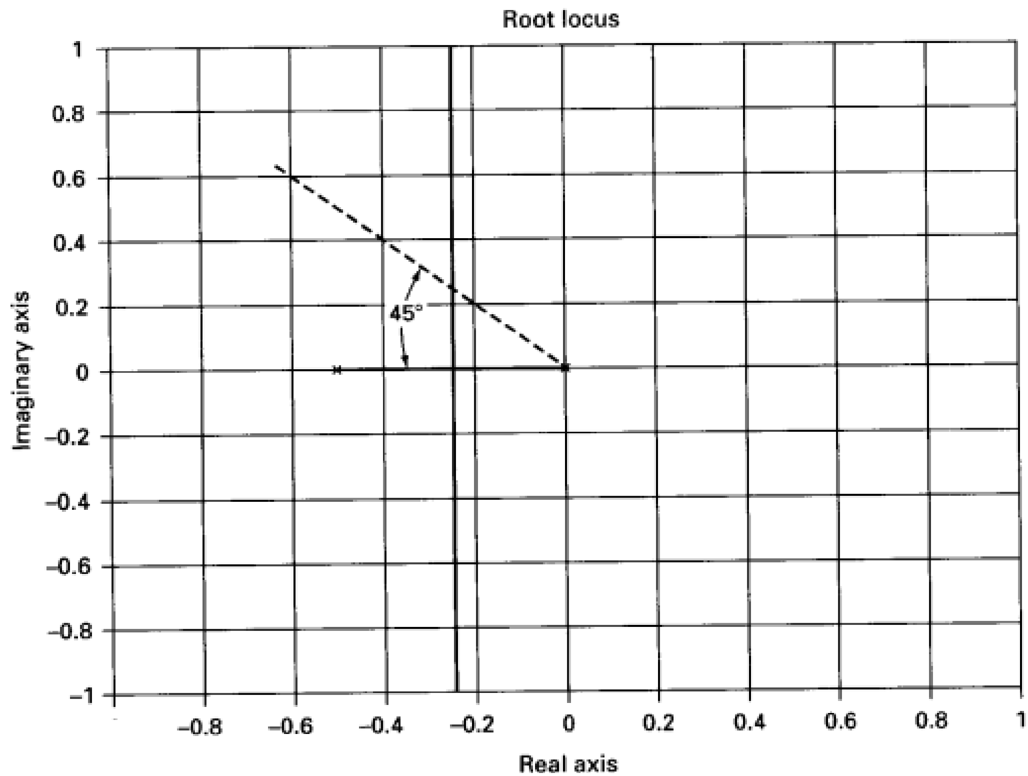
$$G(s)H(s) = \frac{k}{s(s - L_p)} \qquad k = k_a L_{\delta_a} \qquad G(s)H(s) = \frac{k}{s(s + 0.5)}$$

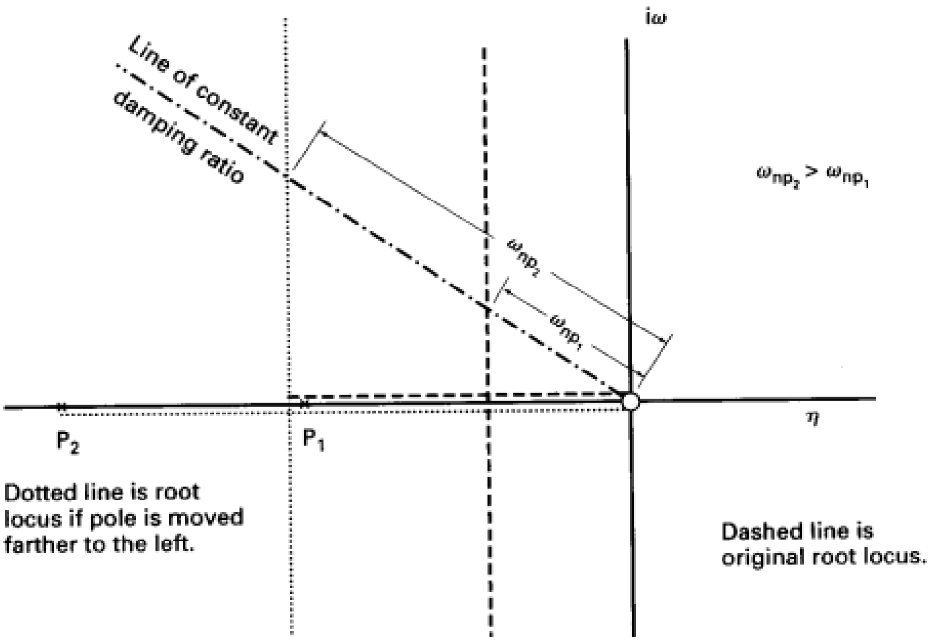
- The desired damping ratio of 5 = 0.707 can be achieved with the present control system. The gain for the system is determined by drawing a line from the origin at 45' as indicated in the root locus plot. Recall that the damping ratio was shown to be equal to the following expression: $\zeta = \cos\theta$

$$\frac{|k|}{|s|\;|s + 0.5|} = 1 \qquad \text{where } s = -0.25 + 0.25\text{i}. \qquad k = 0.0139$$

- , the undamped natural frequency is much lower than specified $\omega_n = 0.35$ rad/s

- **Example 2**
- L_p the roll damping root, was
- shown to be a function of the
- wing span; therefore, we could
- make L, more negative by
- increasing the wing span
- of the vehicle.





--This may be impractical and so we need to look at providing increased damping by means of a stability augmentation system