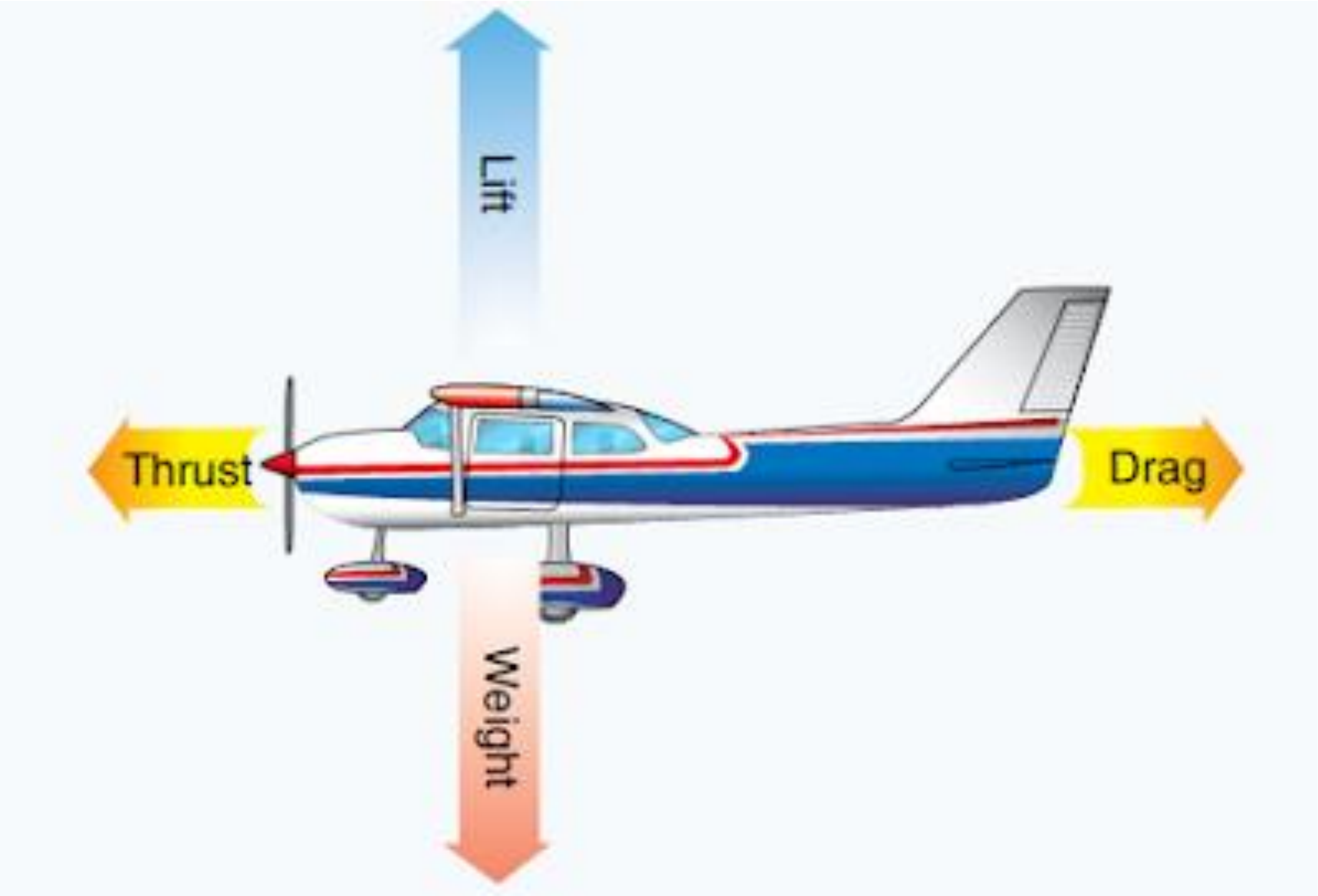


Aviation Department
First Grade- Spring Semester

Vector principle of force
(Lecture 2)

Lecturer: Ms. Jwan Khaleel M.

Lecture content:

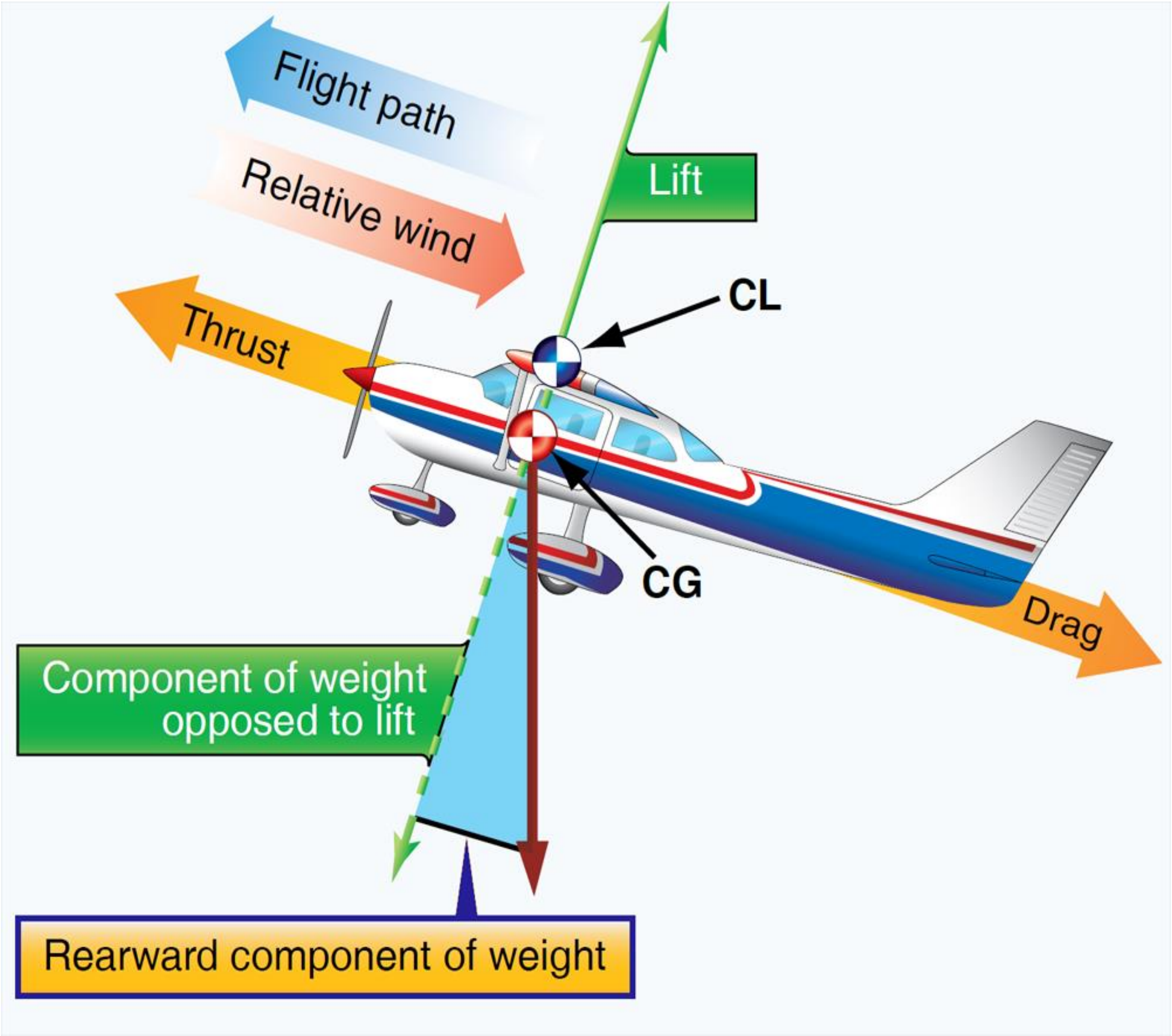
- Scalars and Vectors
- Vector Operations
- Vector Addition of Forces
- Addition of a System of Coplanar Forces
- Solving problems

Learning Outcomes:

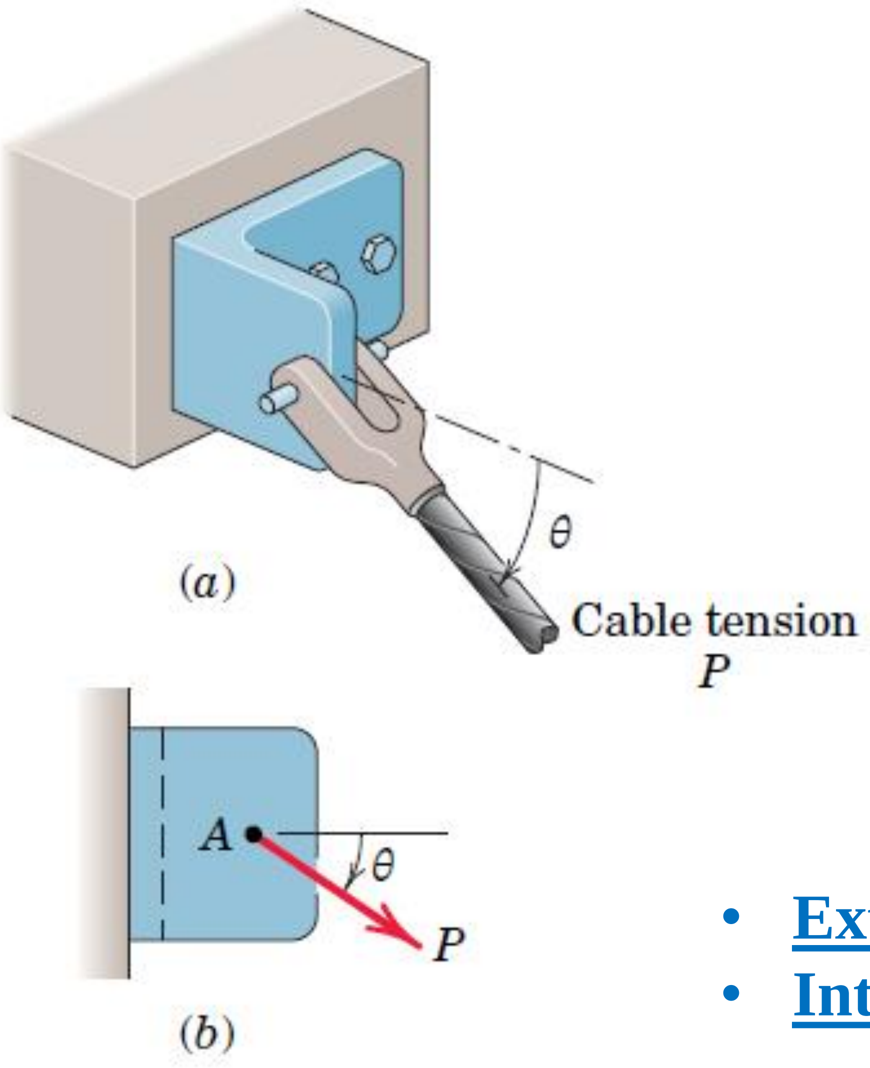
At the end of this lecture, you will be able to:

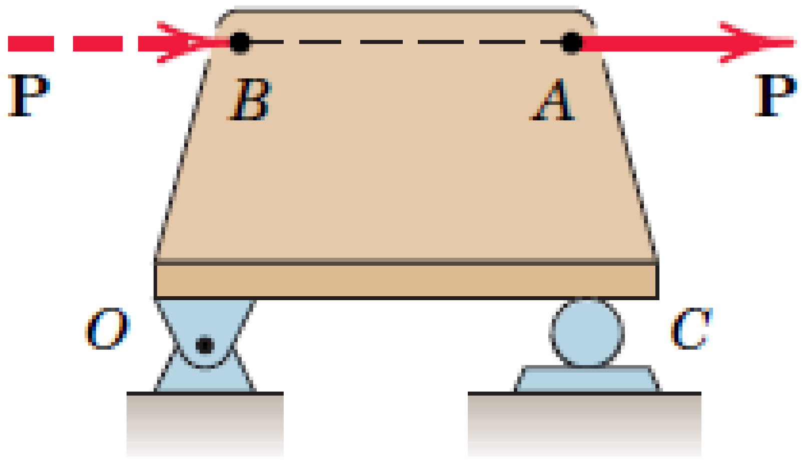
- Analyze the system of forces according to components and resultants.
- Express vectors in terms of unit vectors and perpendicular components
- Perform vector addition and subtraction.
- Compute Parallelogram law and trigonometric rules.
- Solve related problems.

• *What is Force?*



• External and Internal Effect of Force



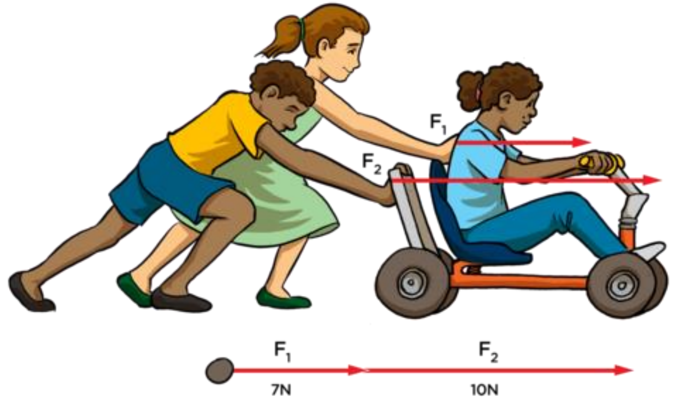


- **External force:** Applied or Reactive force
- **Internal force:** deformation through the material

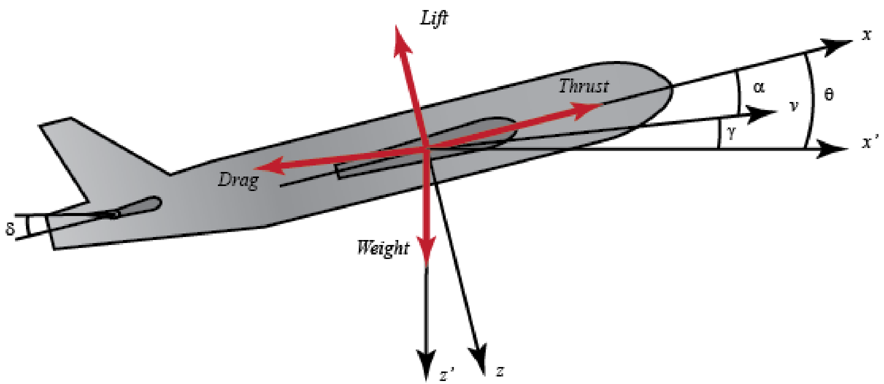
Lecturer: Jwan Khaleel M.

• *Force Classifications:*

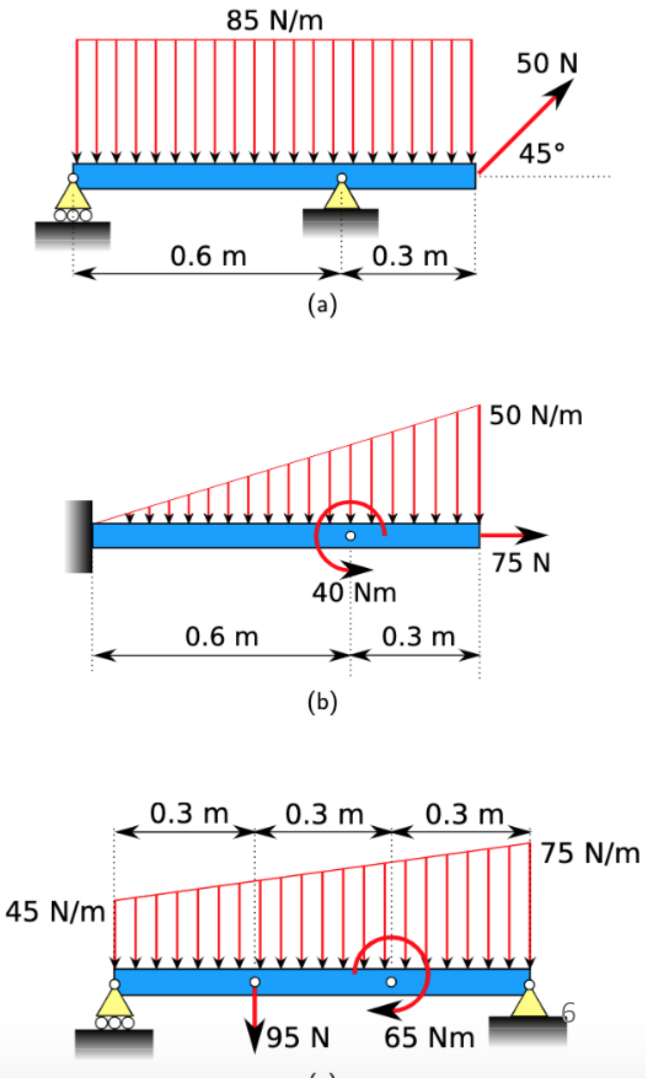
• Contact force



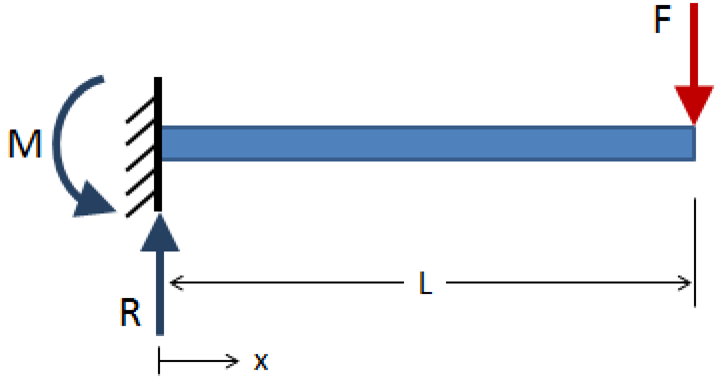
• Body force



• Distributed force



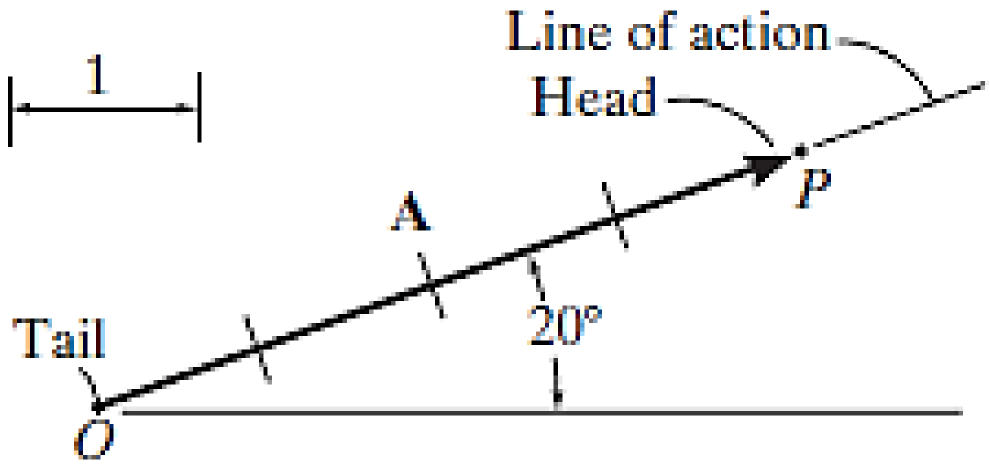
• Concentrated force



• *Scalars and Vectors*

- ***Scalar:*** A scalar is any positive or negative physical quantity that can be completely specified by its magnitude . Examples of scalar quantities include length, mass, and time.

- ***Vector:*** A vector is any physical quantity that requires both a magnitude and a direction for its complete description. Examples of vectors encountered in statics are force, position, and moment.



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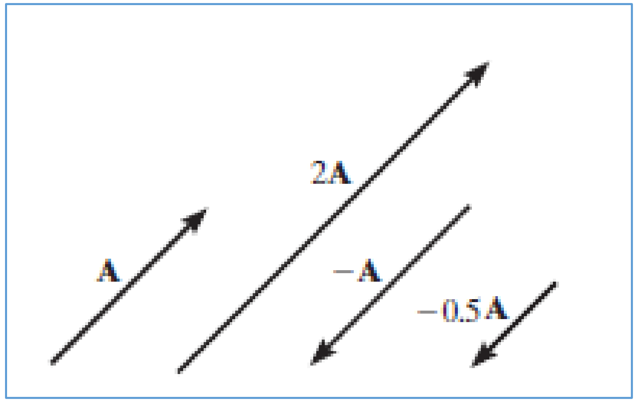
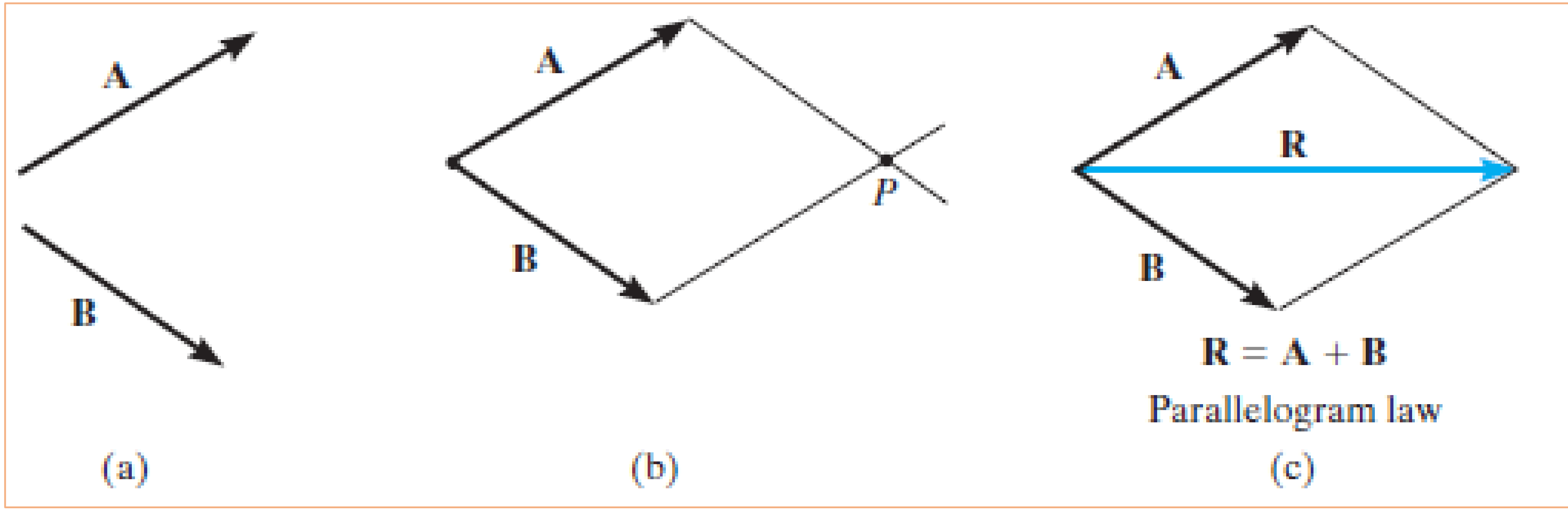
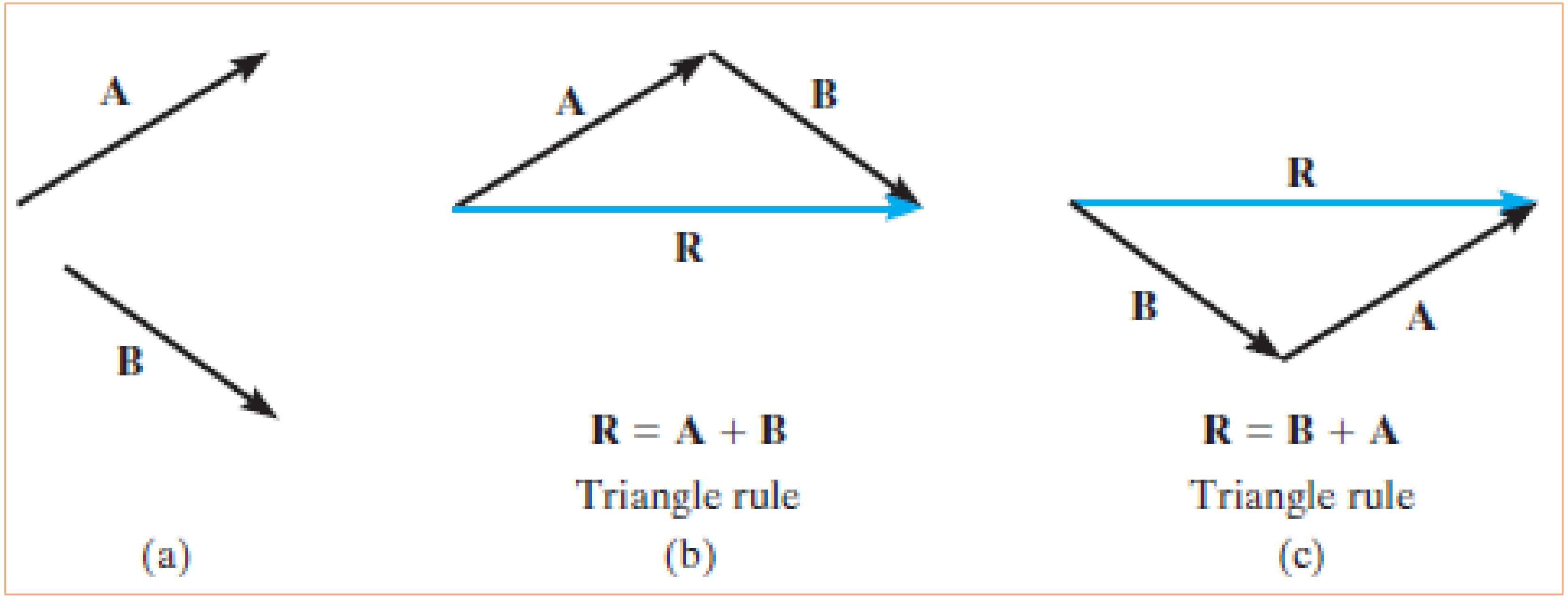
• Vector Operation



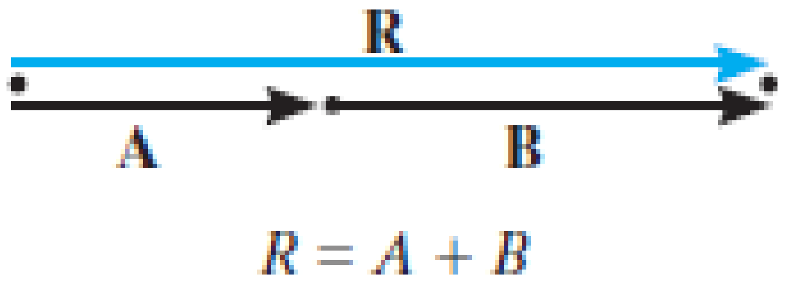
Multiplication and Division of a Vector by a Scalar



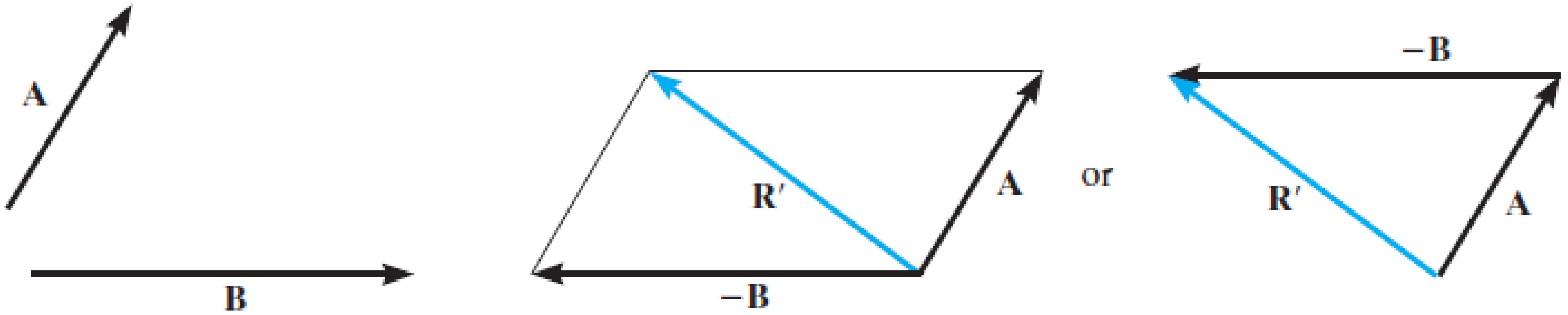
Vector Addition



- ***Vector Operation***



Addition of collinear vectors



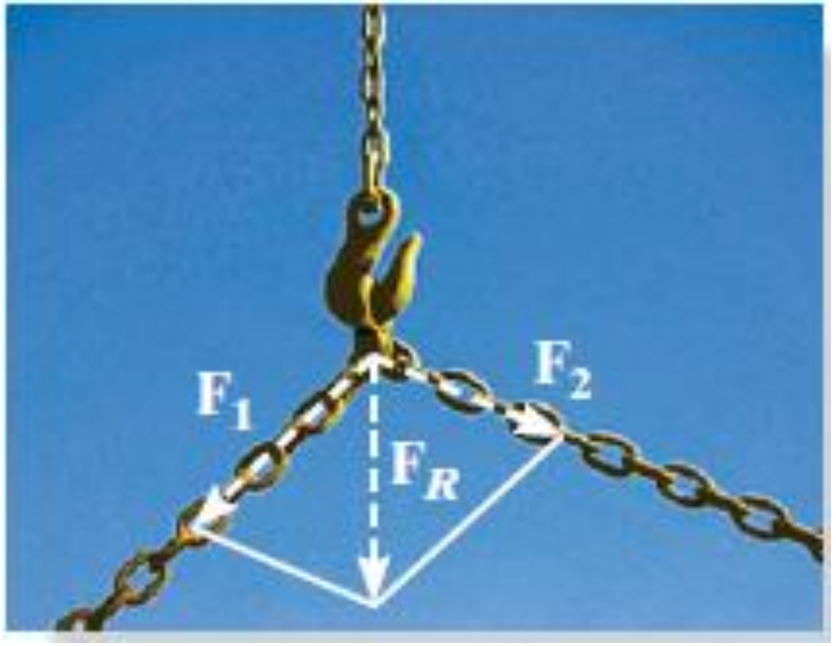
Parallelogram law

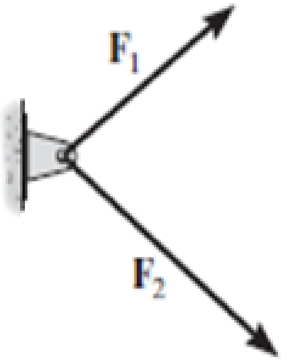
Triangle construction

Vector subtraction

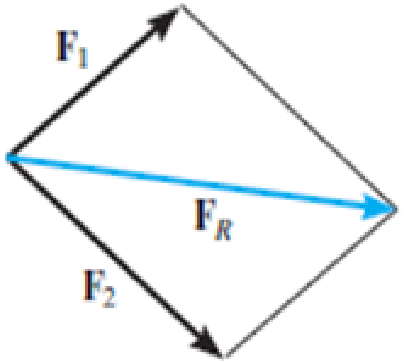
• *Vector Addition of Forces*

Concurrent forces: the line of action of two or more forces intersect at that point.

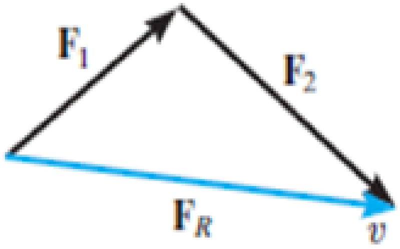




(a) Concurrent Forces



(b) Parallelogram Law

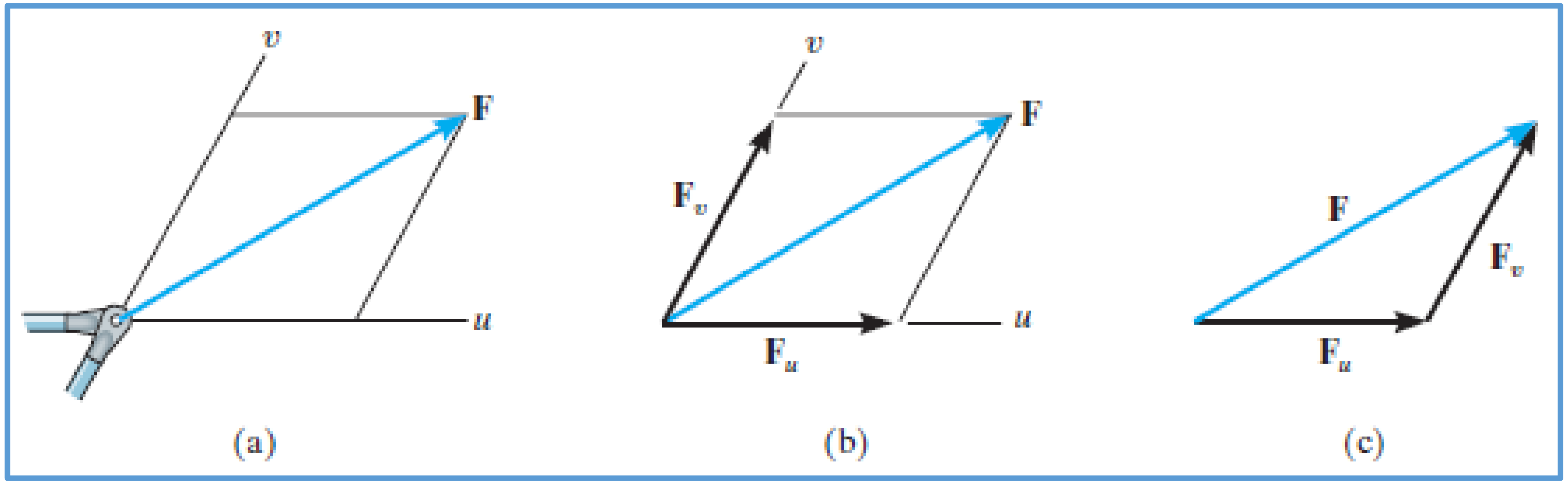


$F_R = F_1 + F_2$

(c) Triangle Rule

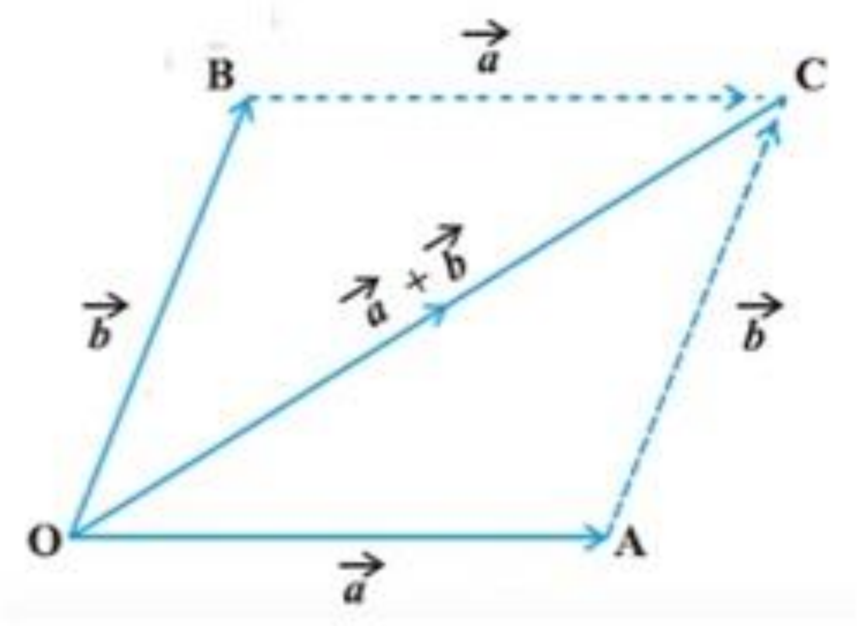
$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$$


F
v
u
F_u
F_v


v
F
u
(a)
v
F
F_v
F_u
u
(b)
F
F_v
F_u
(c)

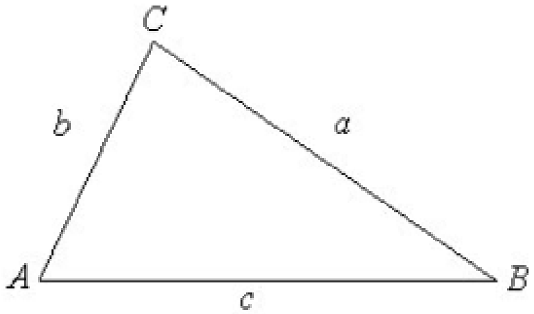
Parallelogram law

The law of parallelogram of forces states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.



Sine Formula

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$



Cosine Formula

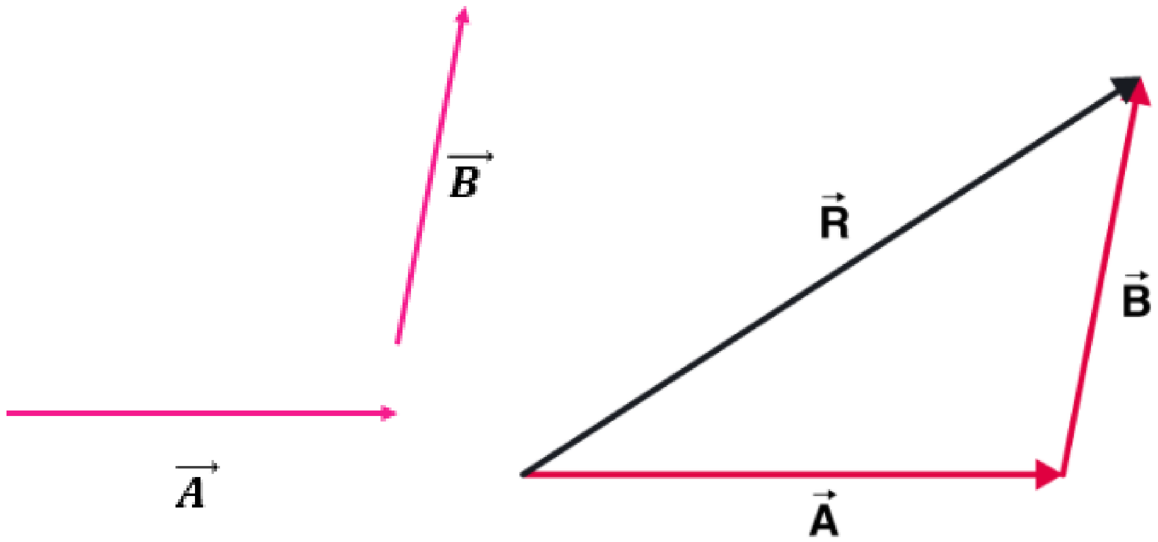
$$a^2=b^2+c^2-2bc\cos A \longrightarrow \cos A=\frac{b^2+c^2-a^2}{2bc}$$

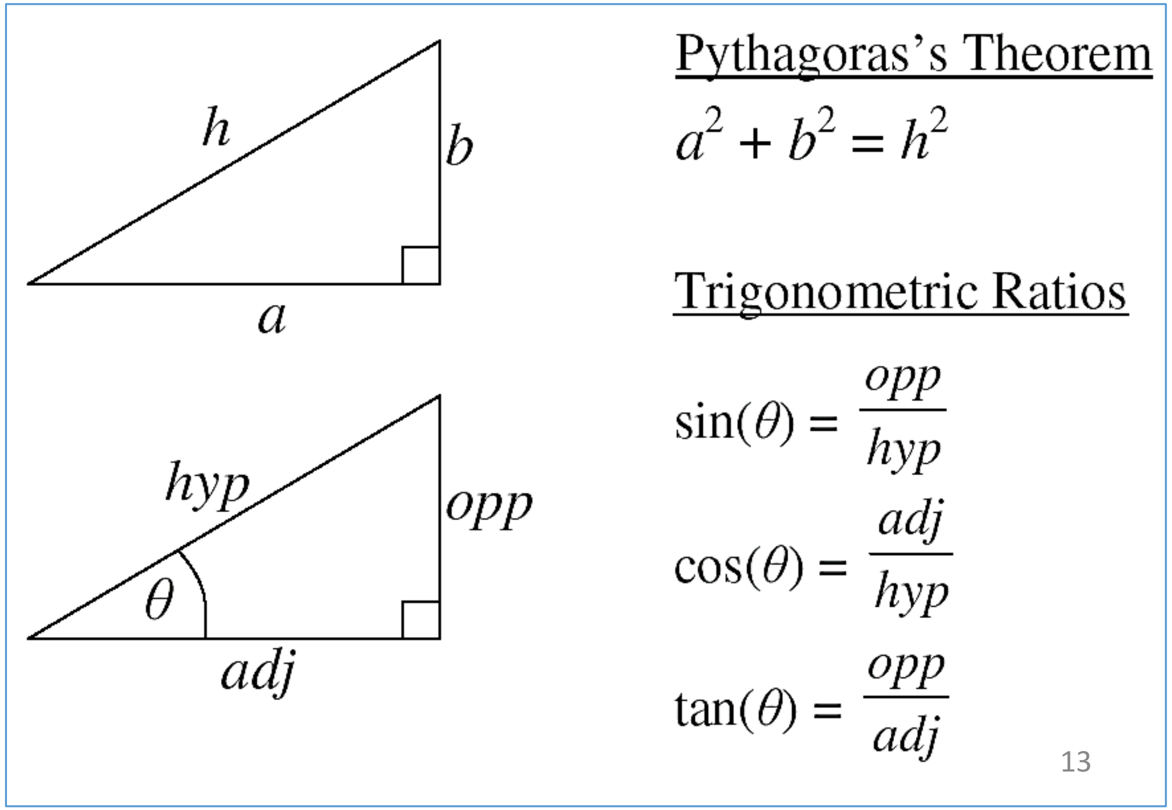
$$b^2=a^2+c^2-2ac\cos B \longrightarrow \cos B=\frac{a^2+c^2-b^2}{2ac}$$

$$c^2=a^2+b^2-2ab\cos C \longrightarrow \cos C=\frac{a^2+b^2-c^2}{2ab}$$

Triangle law (Trigonometry)

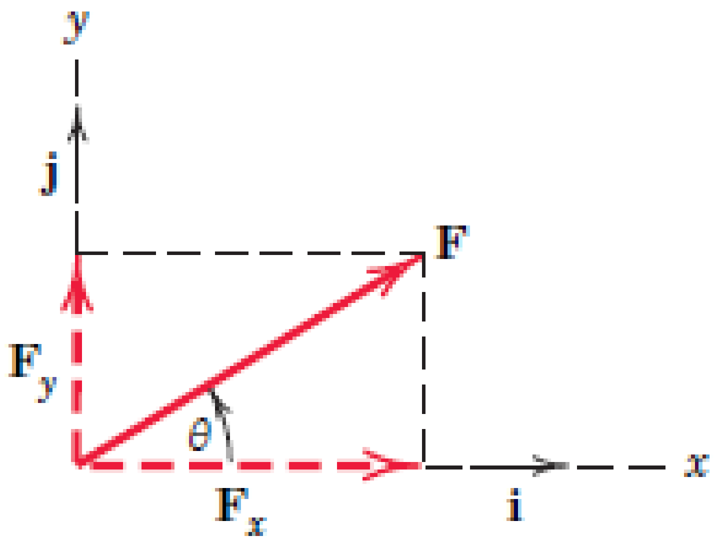
Triangle law of vector addition states that when two vectors are represented as two sides of the triangle with the order of magnitude and direction, then the third side of the triangle represents the magnitude and direction of the resultant vector.

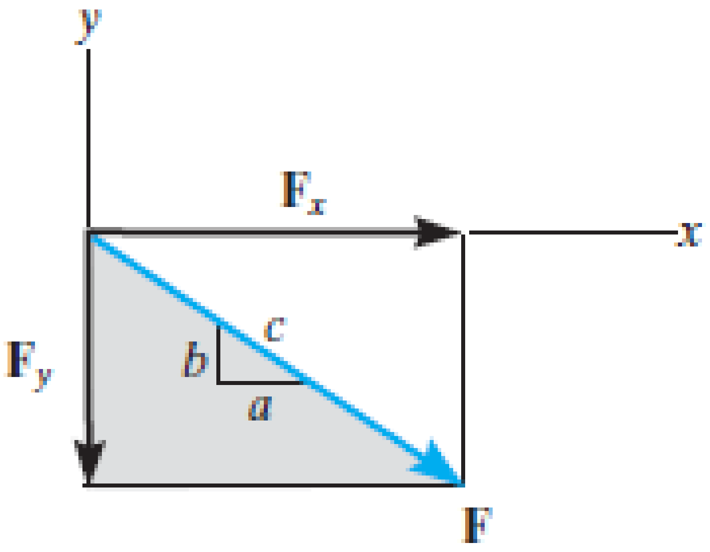




• Finding the Components of a Force

➢ Scalar notation:





$$F_x = F\cos\theta \qquad F = \sqrt{F_x^{\ 2} + F_y^{\ 2}}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

$$\frac{F_x}{F} = \frac{a}{c}$$

or

$$F_x = F\left(\frac{a}{c}\right)$$

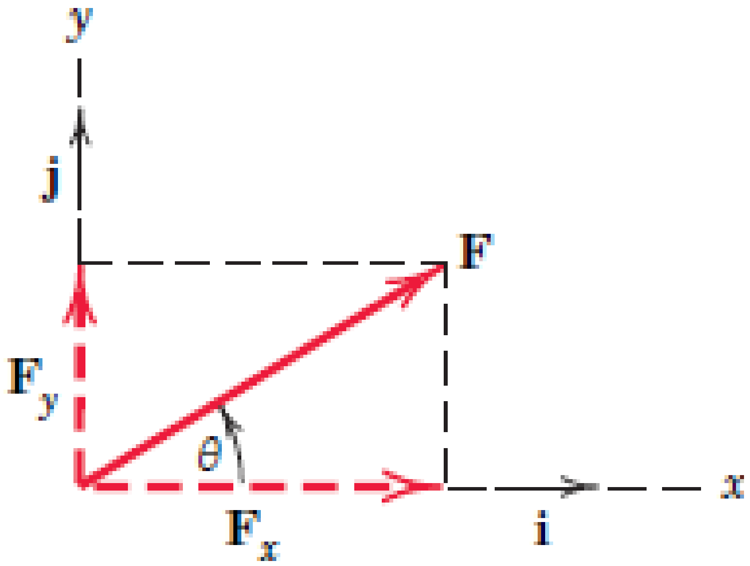
and

$$\frac{F_y}{F} = \frac{b}{c}$$

or

$$F_y = -F\left(\frac{b}{c}\right)$$

➢*Cartesian Vector notation:*

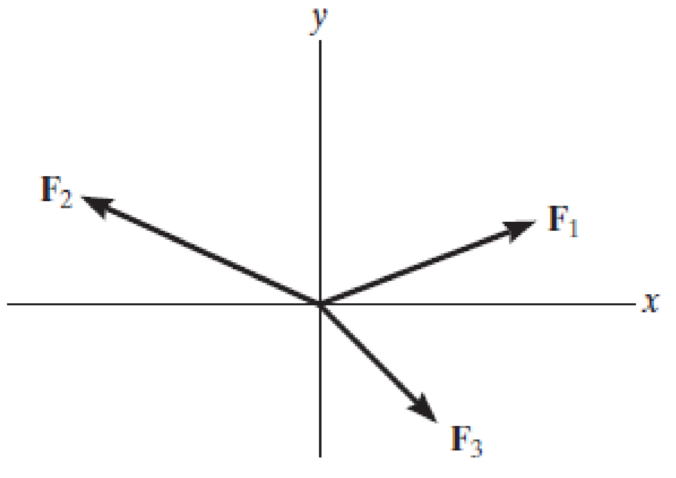


Where $\boldsymbol{F_x}$ and $\boldsymbol{F_y}$ are ***vector components*** of **F** in the *x*- and *y*-directions.

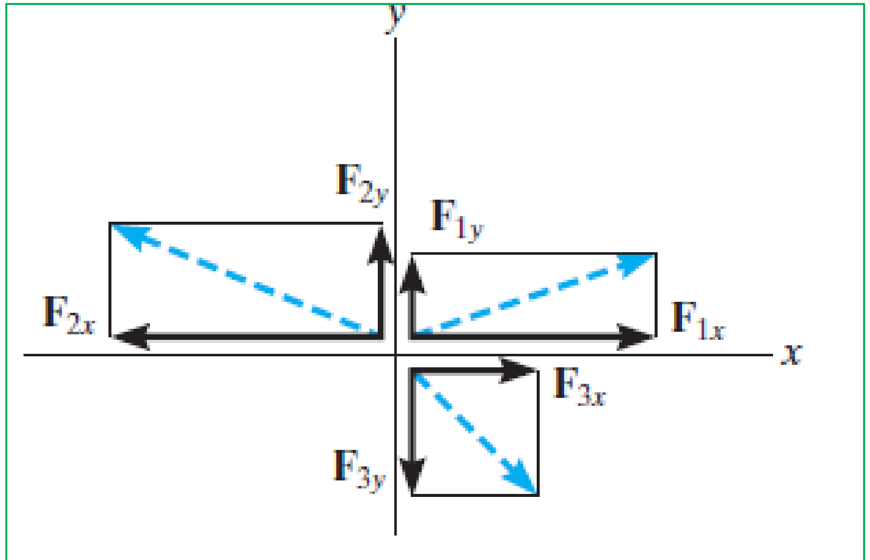
$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

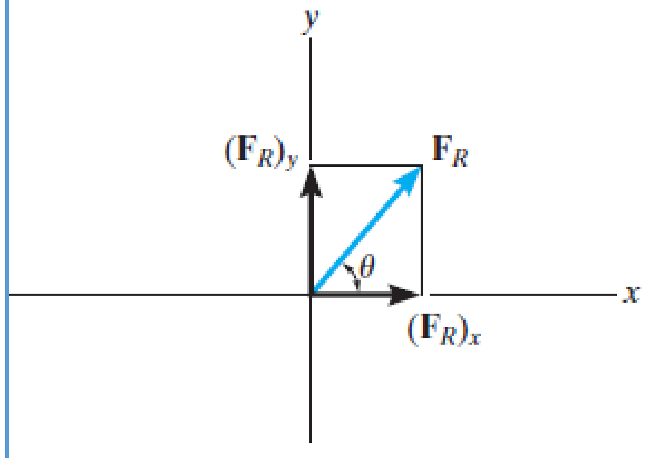
➢Coplanar Force Resultants:



(a)



(b)



(c)

$$\mathbf{F}_1 = F_{1x}\mathbf{i} + F_{1y}\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\mathbf{i} + F_{2y}\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$$

The vector resultant is therefore

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j} \\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

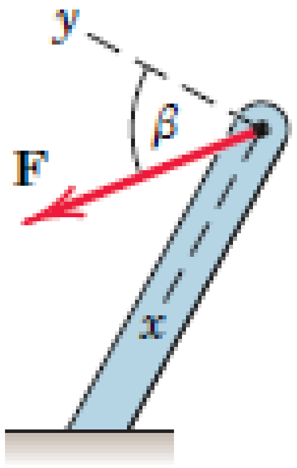
If *scalar notation* is used, then from Figure, we have

$$(\xrightarrow{+}) \qquad (F_R)_x = F_{1x} - F_{2x} + F_{3x}$$
$$(+\uparrow) \qquad (F_R)_y = F_{1y} + F_{2y} - F_{3y}$$

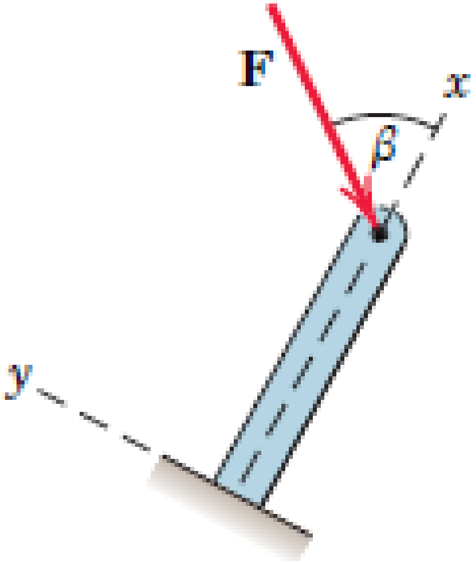
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

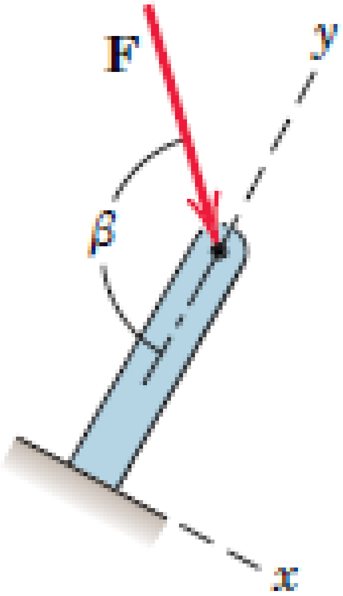
➢*Angles with their axis:*



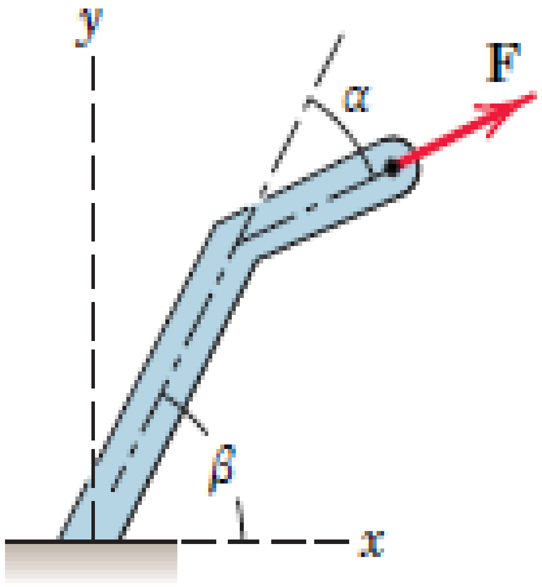
$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$



$$F_x = F \sin (\pi - \beta)$$
$$F_y = -F \cos (\pi - \beta)$$

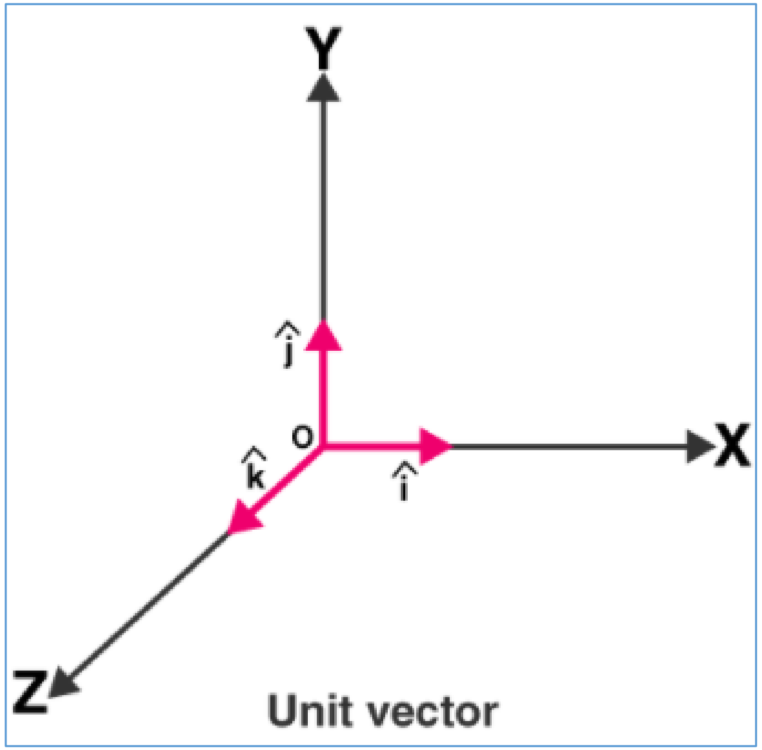


$$F_x = F \cos (\beta - \alpha)$$
$$F_y = F \sin (\beta - \alpha)$$

Unit Vector:

A vector is a quantity that has both magnitude, as well as direction. A vector that has a magnitude of **1** is a unit vector. It is also known as Direction Vector.

$$\text{Unit Vector} = \frac{\text{Vector}}{\text{Magnitude of Vector}}$$

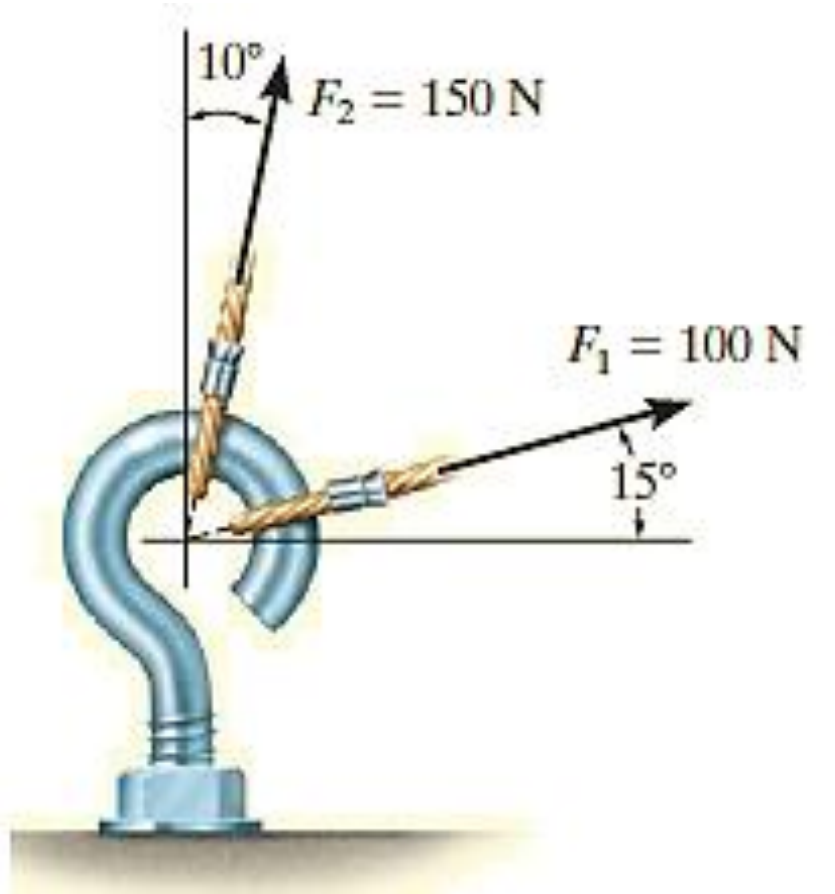


EXAMPLE// Find the unit vector $\vec{q}$ for the given vector, $-2\hat{\imath}+4\hat{\jmath}-4\hat{k}$. Show Unit vector component **q**.

$$q = \frac{-2\hat{\imath}+4\hat{\jmath}-4\hat{k}}{\sqrt{(-2)^2+(4)^2+(-4)^2}} = \frac{-2\hat{\imath}+4\hat{\jmath}-4\hat{k}}{\sqrt{36}} = \frac{-2\hat{\imath}+4\hat{\jmath}-4\hat{k}}{6} = \frac{-2\hat{\imath}}{6} + \frac{4\hat{\jmath}}{6} - \frac{4\hat{k}}{6} = \frac{-1}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} - \frac{2}{3}\hat{k}$$

Example 1//

The screw eye in Figure shown *a* is subjected to two forces, F_1 and F_2. Determine the magnitude and direction of the resultant force.



Solution//

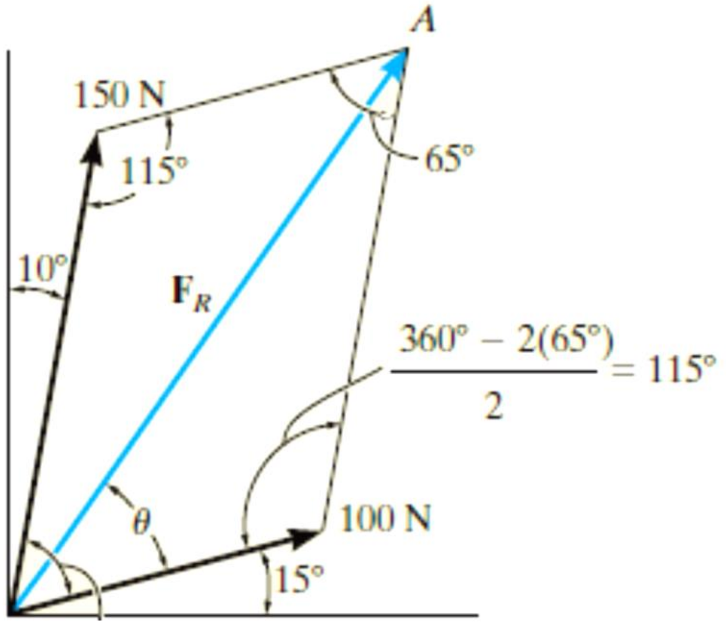
$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115°}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N}$$ *Ans.*

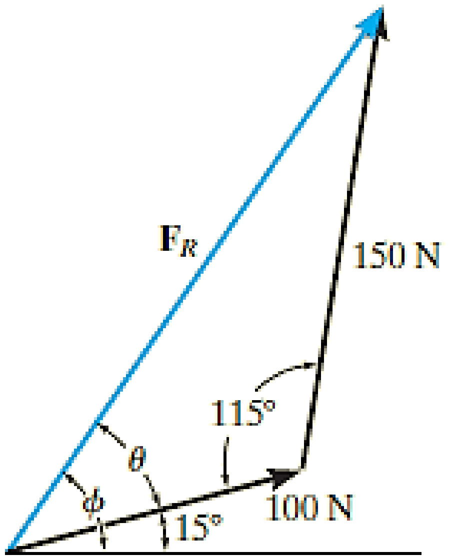
Applying the law of sines to determine θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115°} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115°)$$
$$\theta = 39.8°$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

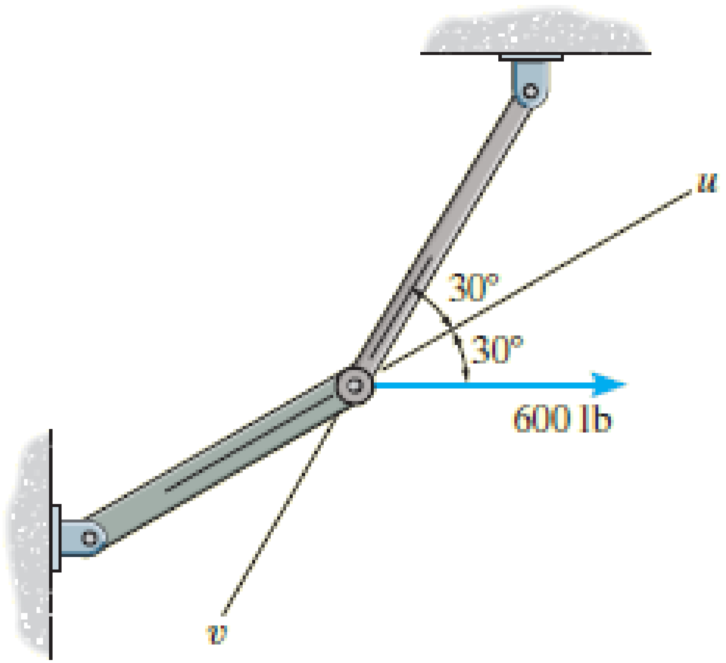
$$\phi = 39.8° + 15.0° = 54.8°$$ *Ans.*





Example 2//

Resolve the horizontal 600-lb force in Figure shown into components acting along the u and v axes and determine the magnitudes of these components.



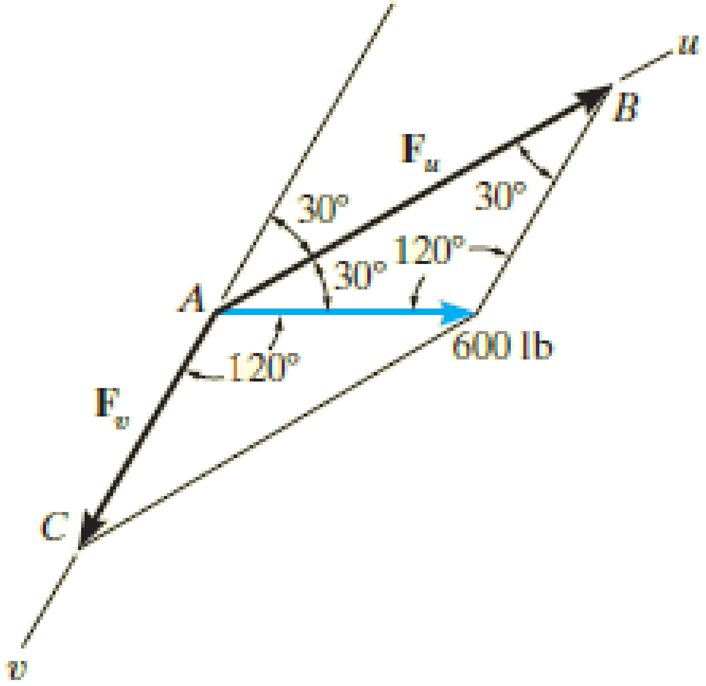
Solution//

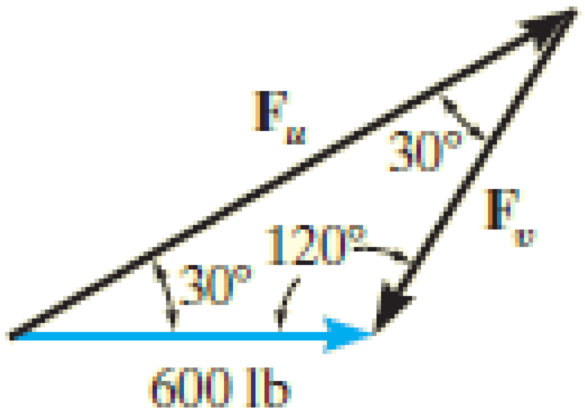
$$\frac{F_u}{\sin 120^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039 \text{ lb}$$

$$\frac{F_v}{\sin 30^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

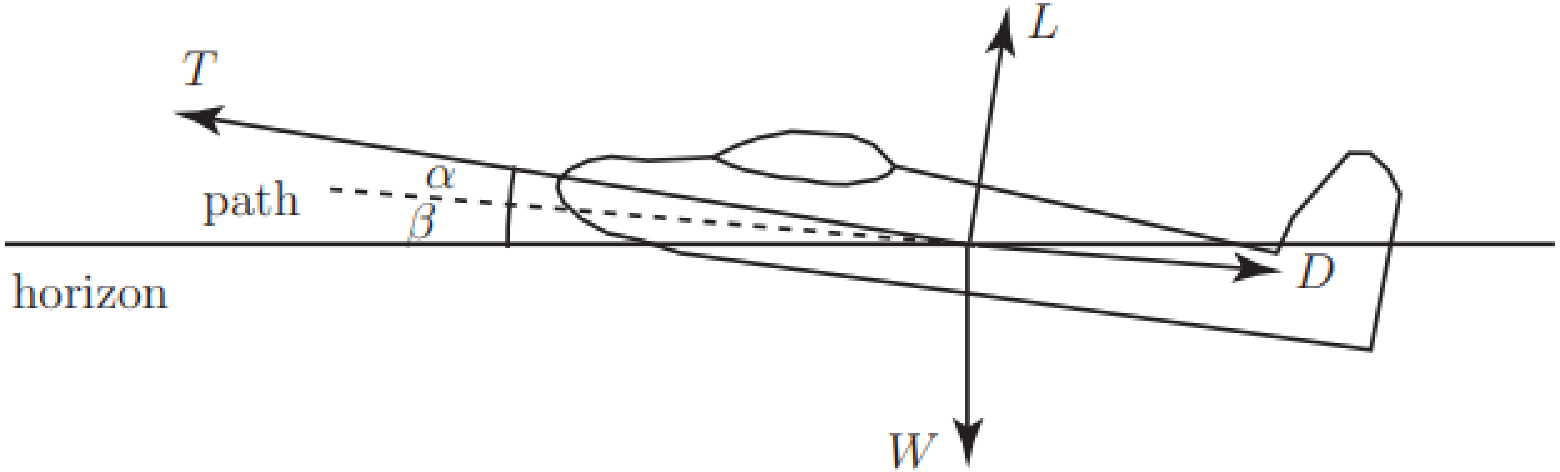
$$F_v = 600 \text{ lb}$$





Example 3//

The forces acting on an aeroplane are shown in Figure. If the plane has mass 72 000 tones, the drag is 130 kN, the lift is 690 kN and β = 6o find the magnitude of the thrust and the value of α to maintain steady flight.



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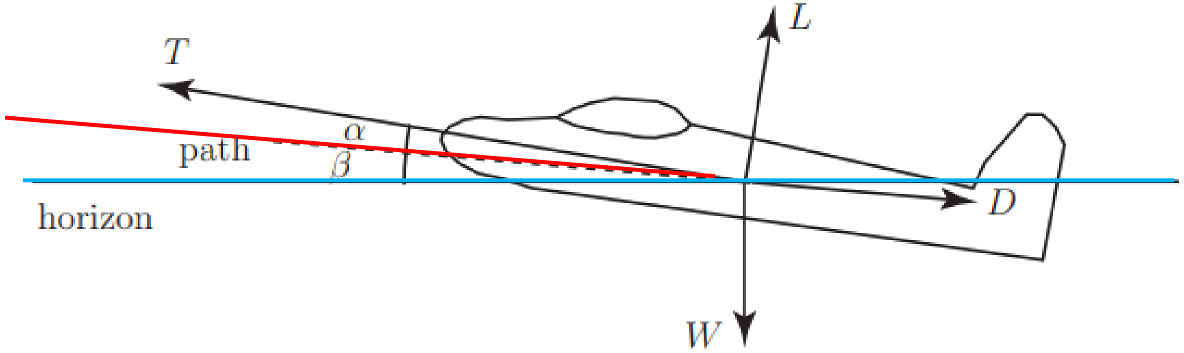
Solution//

The **magnitude** (strength) of the forces are indicated by

T: the thrust provided by the engines, W: the weight,

D: the drag (acting against the direction of flight) and

L: the lift (taken perpendicular to the path.)



As the plane is in *steady* flight the sum of the forces in any direction is zero. (If this were not the case, then, by Newton's second law, the non-zero resultant force would cause the aeroplane to accelerate.)

So, resolving forces in the direction of the path: $T\cos\alpha - D - W\sin\beta = 0$

Then, resolving forces perpendicular to the path: $T\sin\alpha + L - W\cos\beta = 0$

$$T\cos\alpha = D + W\sin\beta = 130000 + (72000)(9.81)\sin 6^o = 203830.54$$

and $T\sin\alpha = W\cos\beta - L = (72000)(9.81)\cos 6^o - 690000 = 12450.71$

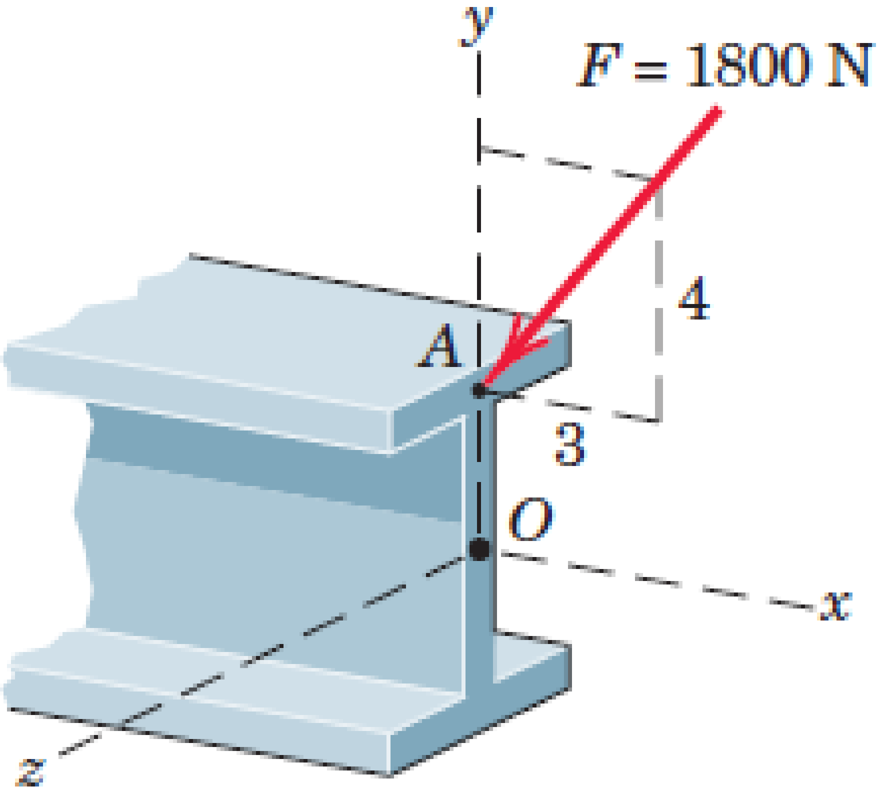
hence $\tan\alpha = \dfrac{T\sin\alpha}{T\cos\alpha} = \dfrac{12450.71}{203830.54} = 0.061084 \quad \rightarrow \quad \alpha = 3.50^0$

and consequently, for the thrust:

$$T = 204210 \text{ N.}$$

Example 4//

The 1800-N force F is applied to the end of the I-beam. Express ***F*** as a vector notation i and j.



Solution:

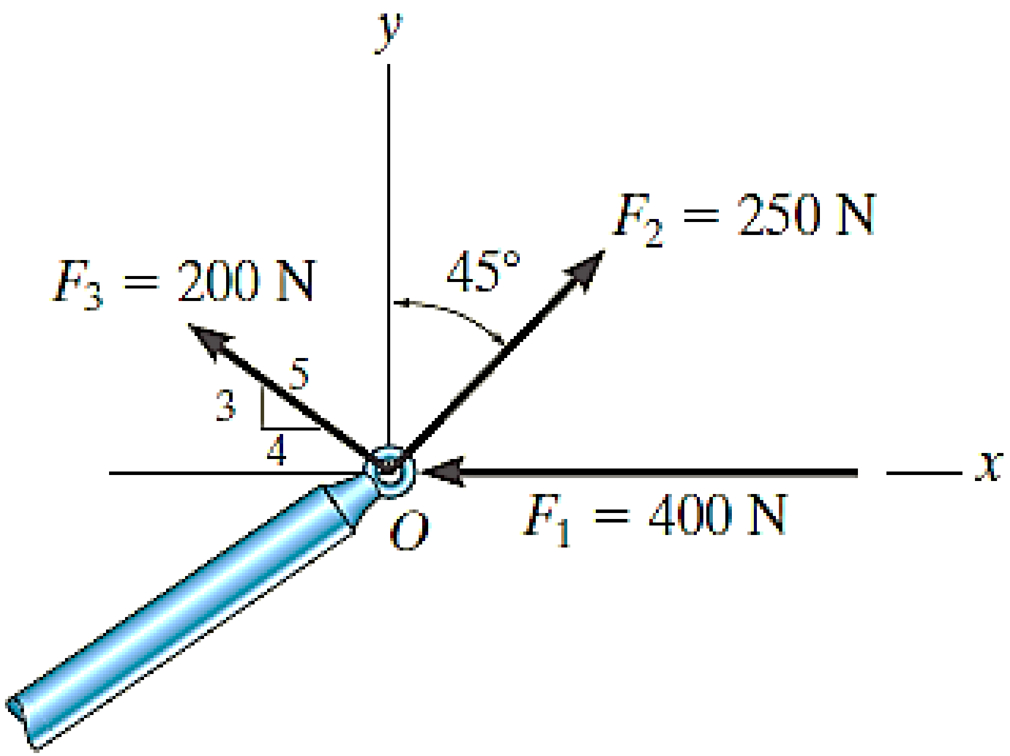
$$F = -1800\frac{3}{5}i - 1800\frac{4}{5}j \quad = (-1080i - 1440j)\text{N}$$

Lecturer: Jwan Khaleel M.

Example 6//

"Class activity (you have 10 minutes to think/solve it by group)"

The end of the boom *O* in Fig. *a* is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.

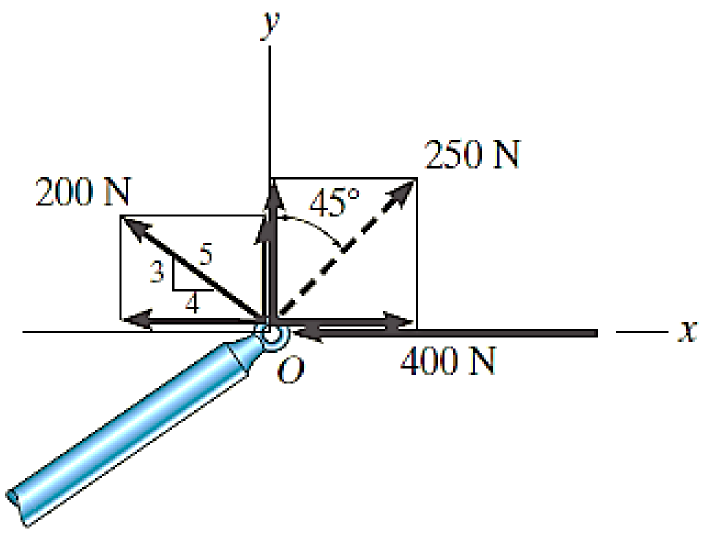


(a)

Solution:

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -400\text{ N} + 250 \sin 45^\circ\text{ N} - 200\left(\tfrac{4}{5}\right)\text{ N}$$

$$= -383.2\text{ N} = 383.2\text{ N} \leftarrow$$



(b)

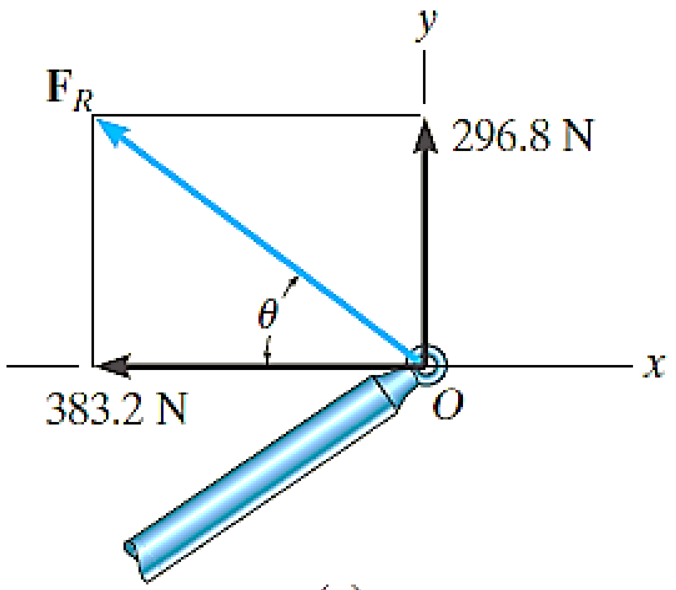
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 250 \cos 45^\circ\text{ N} + 200\left(\tfrac{3}{5}\right)\text{ N}$$

$$= 296.8\text{ N}\uparrow$$

$$F_R = \sqrt{(-383.2\text{ N})^2 + (296.8\text{ N})^2}$$

$$= 485\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^\circ$$



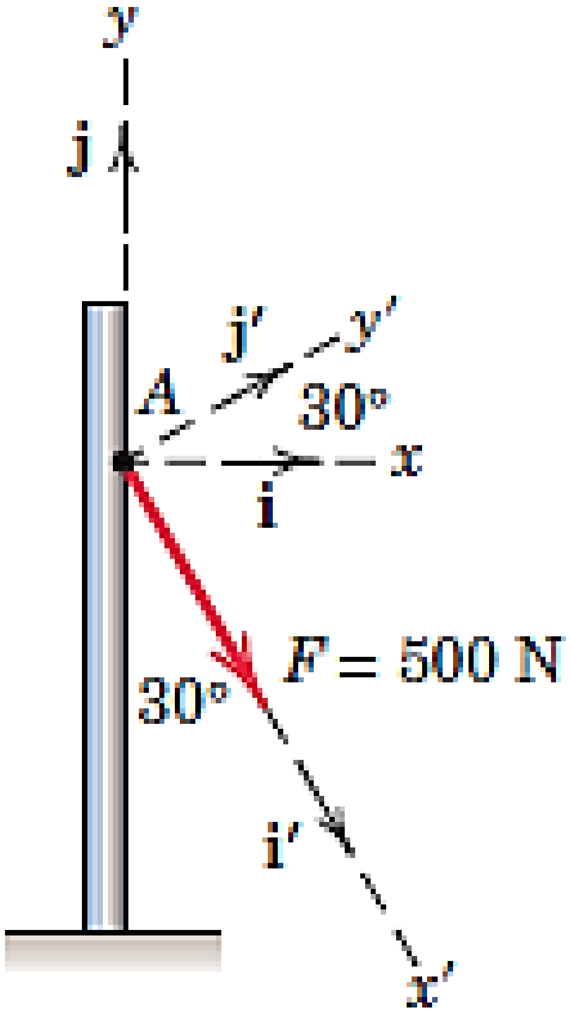
(c)

Example 7//

"Class activity (you have 10 minutes to think/solve it by group)"

The 500-N force ***F*** is applied to the vertical pole as shown:

1) Write ***F*** in terms of the unit vectors ***i*** and ***j*** and identify both its vector and scalar components.
2) Determine the scalar components of the force vector ***F*** along the x'- and y'-axes.
3) Determine the scalar components of ***F*** along the x- and y'-axes.



Lecturer: Jwan Khaleel M.

Solution ***Part (1).*** From Fig. *a* we may write **F** as

$$\mathbf{F} = (F\cos\theta)\mathbf{i} - (F\sin\theta)\mathbf{j}$$
$$= (500\cos 60^\circ)\mathbf{i} - (500\sin 60^\circ)\mathbf{j}$$
$$= (250\mathbf{i} - 433\mathbf{j})\ \text{N} \qquad \textit{Ans.}$$

The scalar components are $F_x = 250$ N and $F_y = -433$ N. The vector components are $\mathbf{F}_x = 250\mathbf{i}$ N and $\mathbf{F}_y = -433\mathbf{j}$ N.

Part (2). From Fig. *b* we may write **F** as $\mathbf{F} = 500\mathbf{i}'$ N, so that the required scalar components are

$$F_{x'} = 500\ \text{N} \qquad F_{y'} = 0 \qquad \textit{Ans.}$$

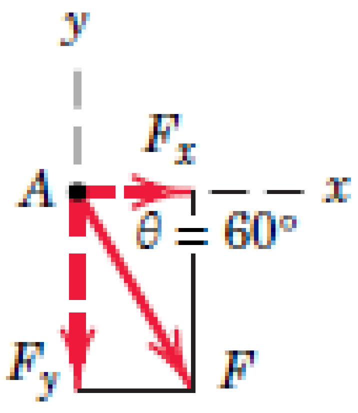
Part (3).

$$\frac{|F_x|}{\sin 90^\circ} = \frac{500}{\sin 30^\circ} \qquad |F_x| = 1000\ \text{N} \quad ①$$

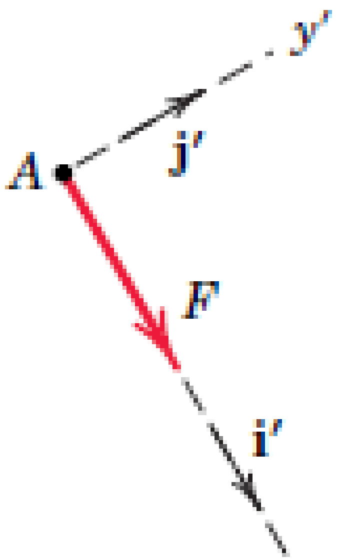
$$\frac{|F_{y'}|}{\sin 60^\circ} = \frac{500}{\sin 30^\circ} \qquad |F_{y'}| = 866\ \text{N}$$

The required scalar components are then

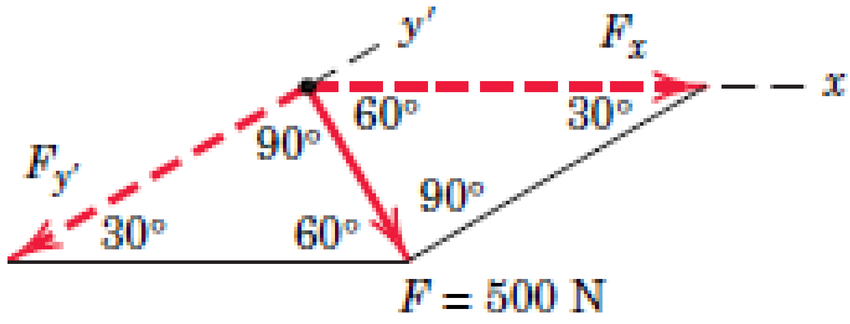
$$F_x = 1000\ \text{N} \qquad F_{y'} = -866\ \text{N} \qquad \textit{Ans.}$$



(a)



(b)

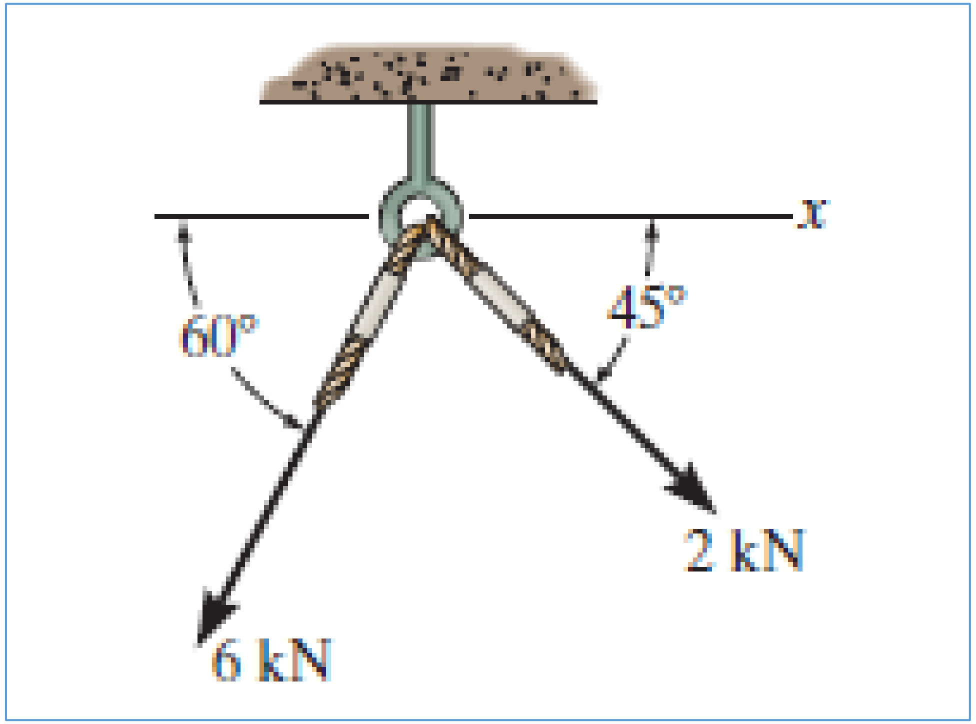


(c)

Assignment 1//

(solve this problems then submit your answer)

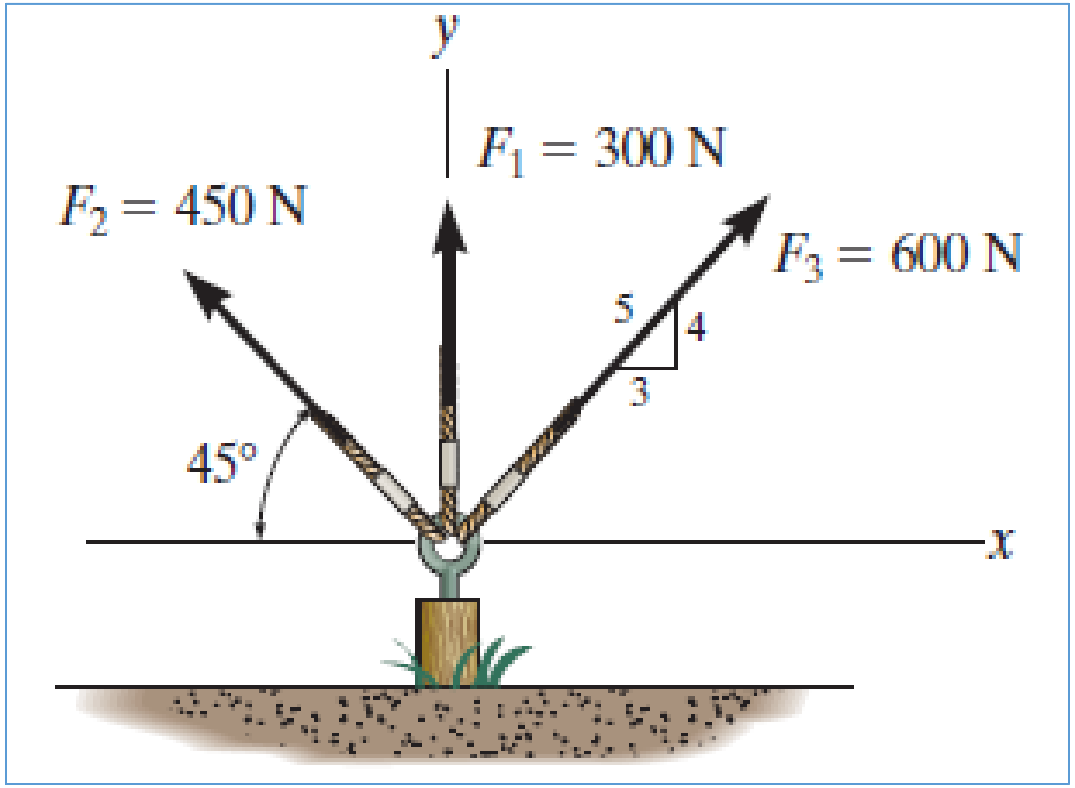
Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x-axis.



Assignment 2//

(solve this problems then submit your answer)

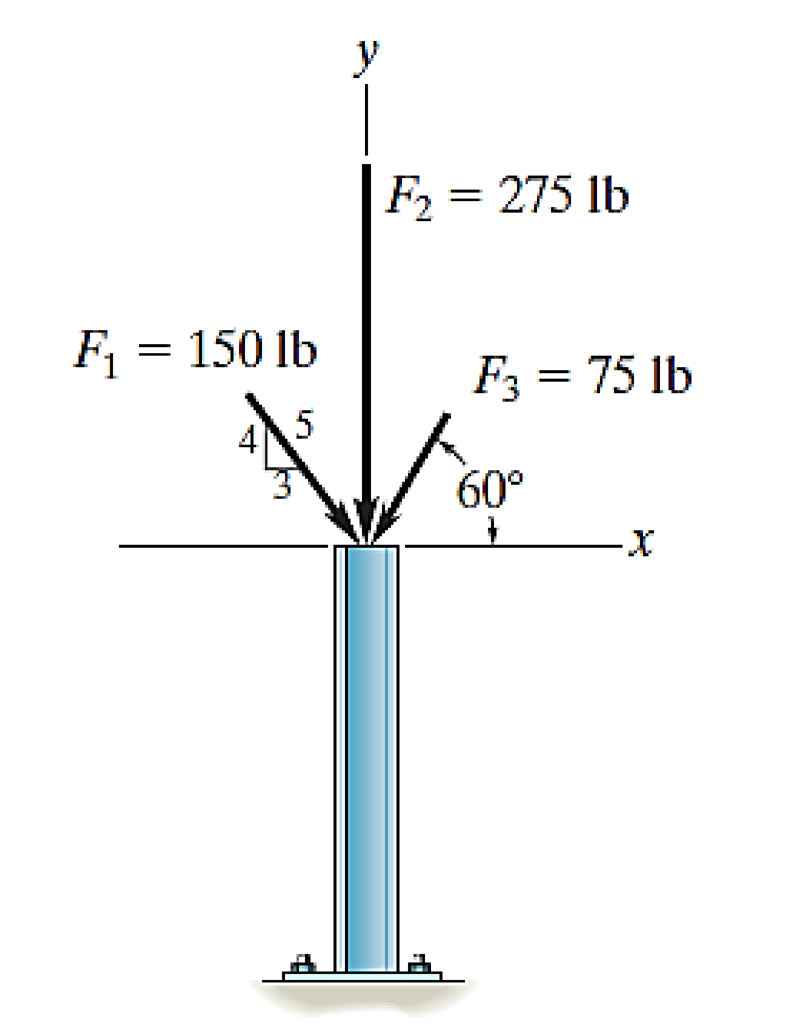
Resolve each force acting on the post into its x and y components.



Assignment 3

(solve this problems then submit your answer)

Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.



Assignment 4

(solve this problems then submit your answer)

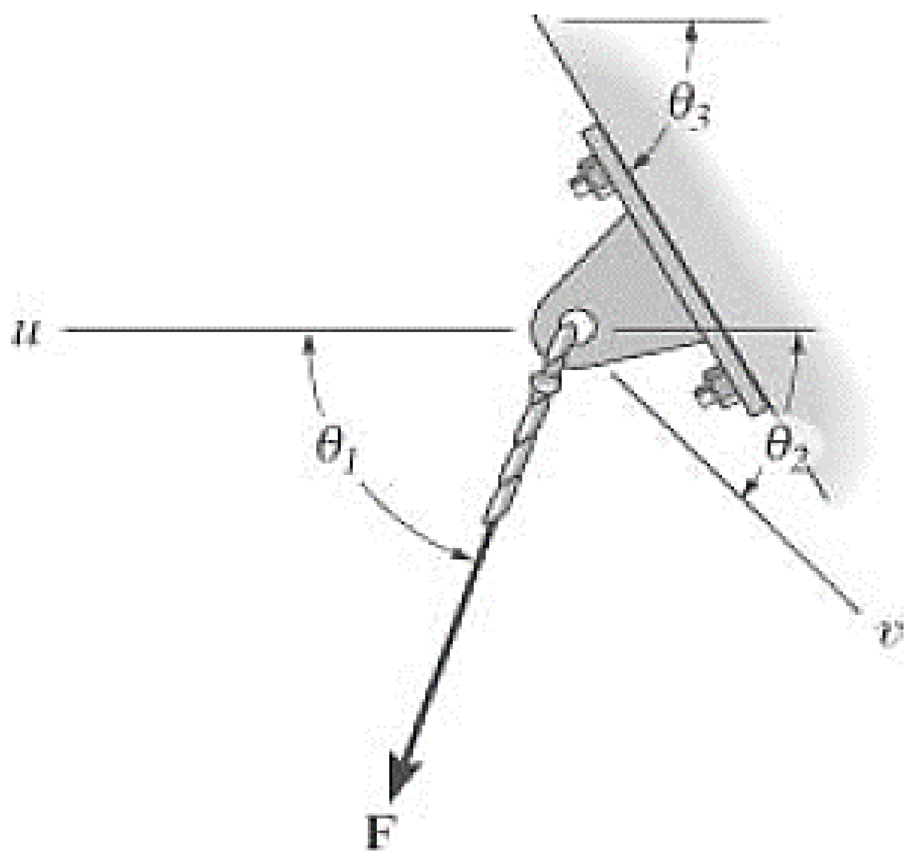
Determine the components of the **F** force acting along the *u* and *v* axes. If you know:

$\theta_1 = 70°$

$\theta_2 = 45°$

$\theta_3 = 60°$

$F = 250$ N

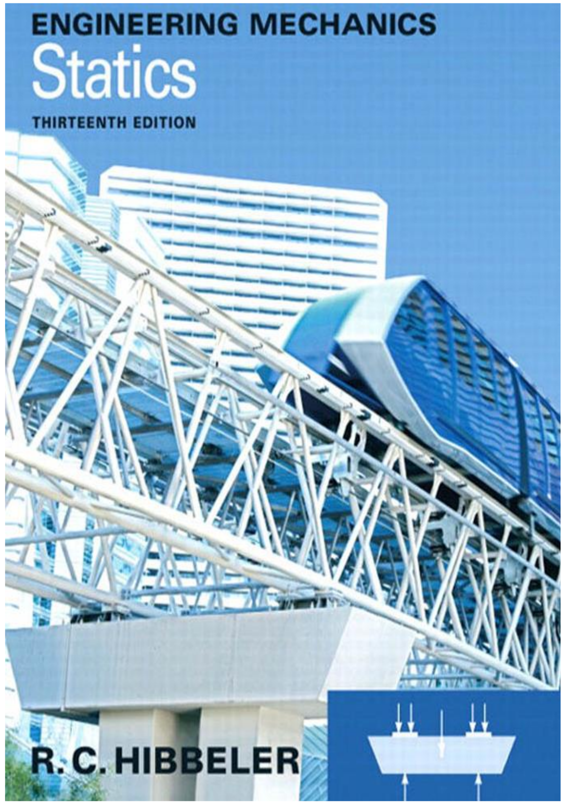


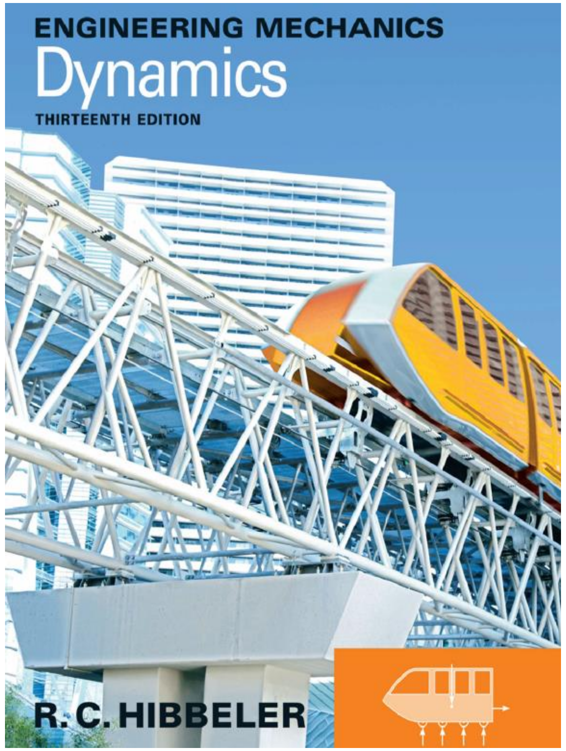
Next lecture:

- Cartesian Vectors
- Addition of Cartesian Vectors
- Position Vectors
- Force Vector Directed Along a Line
- Dot Product

References:

Engineering Mechanics R.C. Hibbeler 13th edition (Statics and Dynamics).





The end of the lecture
Enjoy your time