Aviation Department
First Grade- Spring Semester

Statics- 3D of force (Lecture 4)

Lecturer: Ms. Jwan Khaleel M.

Lecture Content:

- Force Vector Directed Along a Line
- Dot Product
- Solving problems

Learning Outcomes:

At the end of this lecture, you will be able to:

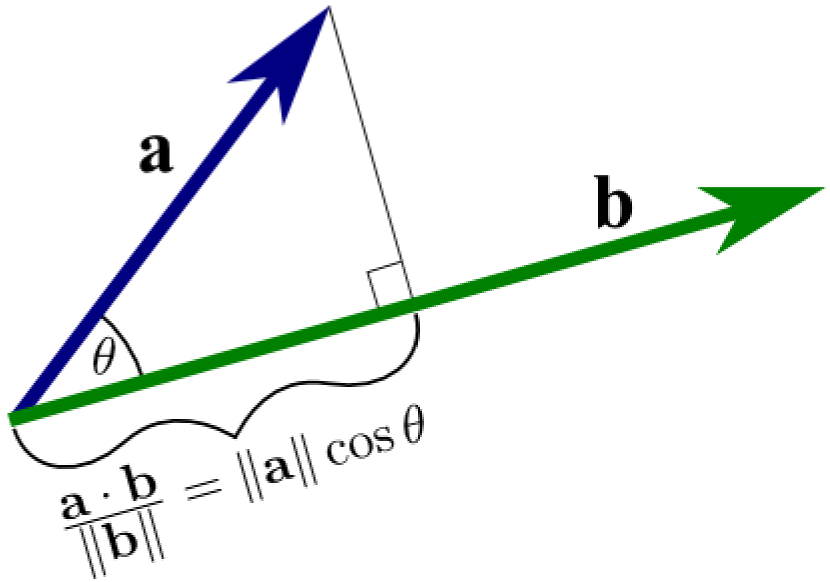
- Introduce the dot product in order to determine the angle between two vectors or the projection of one vector onto another.
- Solving problems using related equations

• Dot Product *(specify the forces by angle which oriented the line of action)*

- The dot product between two vectors **A** and **B** yields a scalar. If A and B are expressed in Cartesian vector form, then the dot product is the sum of the products of their $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ components.
- The dot product is used to determine the angle between two vectors or the projection of a vector in a specified direction.

$$\boxed{\mathbf{A} \cdot \mathbf{B} = AB\cos\theta}$$

where $0^\circ \leq \theta \leq 180^\circ$



Laws of Operation.

1. Commutative law: $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
2. Multiplication by a scalar: $a(\mathbf{A} \cdot \mathbf{B}) = (a\mathbf{A}) \cdot \mathbf{B} = \mathbf{A} \cdot (a\mathbf{B})$
3. Distributive law: $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$

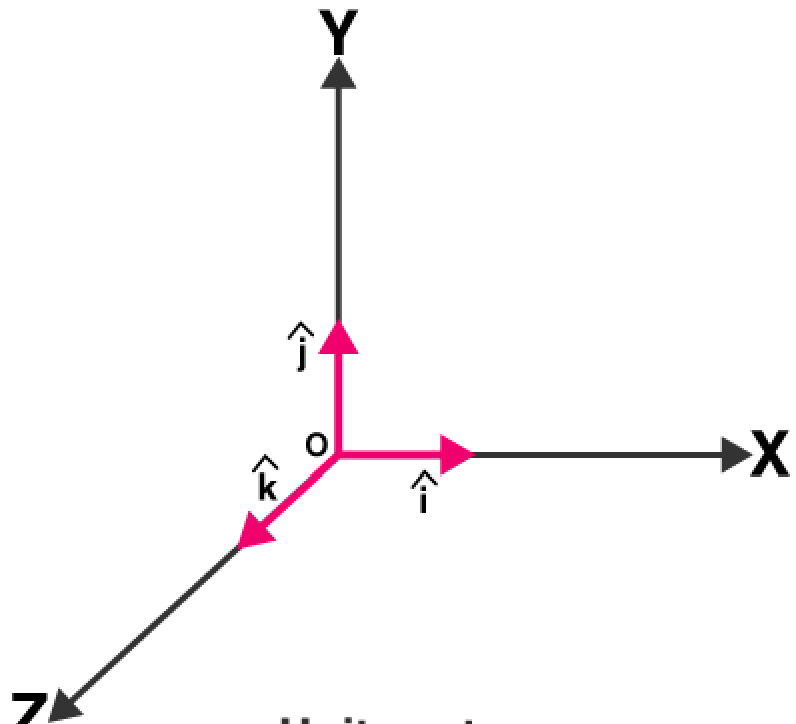
Cartesian Vector Formulation.

$$\mathbf{A} \cdot \mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \cdot (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$
$$= A_xB_x(\mathbf{i} \cdot \mathbf{i}) + A_xB_y(\mathbf{i} \cdot \mathbf{j}) + A_xB_z(\mathbf{i} \cdot \mathbf{k})$$
$$+ A_yB_x(\mathbf{j} \cdot \mathbf{i}) + (A_yB_y(\mathbf{j} \cdot \mathbf{j}) + A_yB_z(\mathbf{j} \cdot \mathbf{k})$$
$$+ A_zB_x(\mathbf{k} \cdot \mathbf{i}) + A_zB_y(\mathbf{k} \cdot \mathbf{j}) + A_zB_z(\mathbf{k} \cdot \mathbf{k})$$

Carrying out the dot-product operations, the final result becomes

$$\boxed{\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z}$$

$$\mathbf{i} \cdot \mathbf{i} = (1)(1) \cos 0^\circ = 1$$
$$\mathbf{i} \cdot \mathbf{j} = (1)(1) \cos 90^\circ = 0.$$



Unit vector

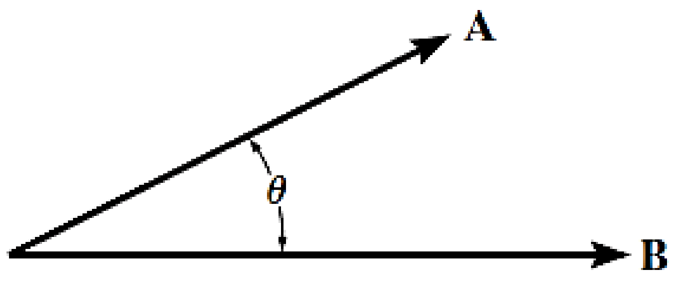
$$\vec{i} \cdot \vec{i} = 1$$
$$\vec{j} \cdot \vec{j} = 1$$
$$\vec{k} \cdot \vec{k} = 1$$

$$\vec{i} \cdot \vec{j} = 0$$
$$\vec{j} \cdot \vec{k} = 0$$
$$\vec{k} \cdot \vec{i} = 0$$

Applications. The dot product has two important applications in mechanics.

- ***The angle formed between two vectors or intersecting lines.***

$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) \quad 0^\circ \le \theta \le 180^\circ$$

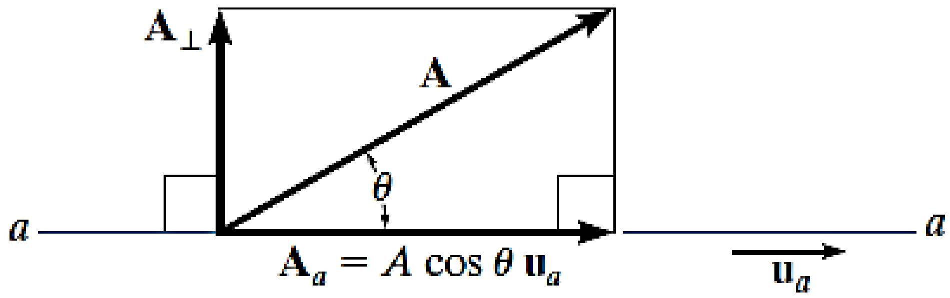


notice that if

$\mathbf{A}\cdot\mathbf{B} = 0, \theta = \cos^{-1} 0 = 90^\circ$ so that $\mathbf{A}$ will be *perpendicular* to $\mathbf{B}$.

- ***The components of a vector parallel and perpendicular to a line.***

$$A_a = A\cos\theta = \mathbf{A}\cdot\mathbf{u}_a$$



Since $\mathbf{A} = \mathbf{A}_a + \mathbf{A}_\perp$, then $\mathbf{A}_\perp = \mathbf{A} - \mathbf{A}_a$.

$$\theta = \cos^{-1}(\mathbf{A}\cdot\mathbf{u}_A / A)$$

$$A_\perp = A\sin\theta$$

Pythagorean's theorem

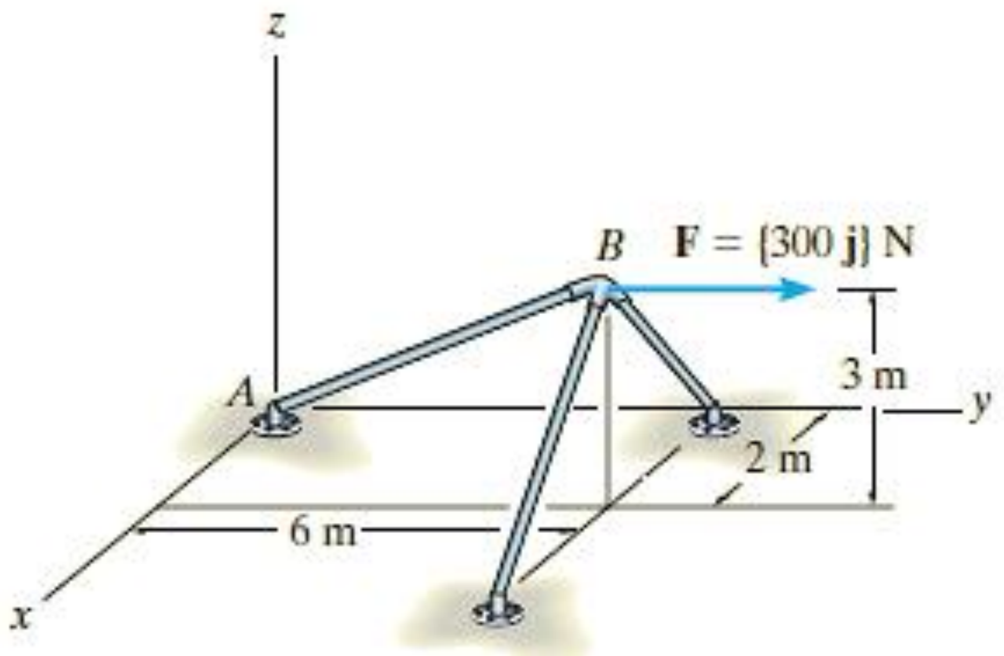
$$A_\perp = \sqrt{A^2 - A_a{}^2}$$

Important Points

- The dot product is used to determine the angle between two vectors or the projection of a vector in a specified direction.
- If vectors **A** and **B** are expressed in Cartesian vector form, the dot product is determined by multiplying the respective *x*, *y*, *z* scalar components and algebraically adding the results, i.e., $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$.
- From the definition of the dot product, the angle formed between the tails of vectors **A** and **B** is $\theta = \cos^{-1}(\mathbf{A} \cdot \mathbf{B}/AB)$.
- The magnitude of the projection of vector **A** along a line *aa* whose direction is specified by $\mathbf{u}_a$ is determined from the dot product $A_a = \mathbf{A} \cdot \mathbf{u}_a$.

Example 5:

The frame shown in Figure is subjected to a horizontal force $F = \{300\ j\}$. Determine the magnitude of the components of this force parallel and perpendicular to member AB .



Solution:

The magnitude of the component of **F** along AB is equal to the dot product of **F** and the unit vector $\mathbf{u}_B$, which defines the direction of AB,

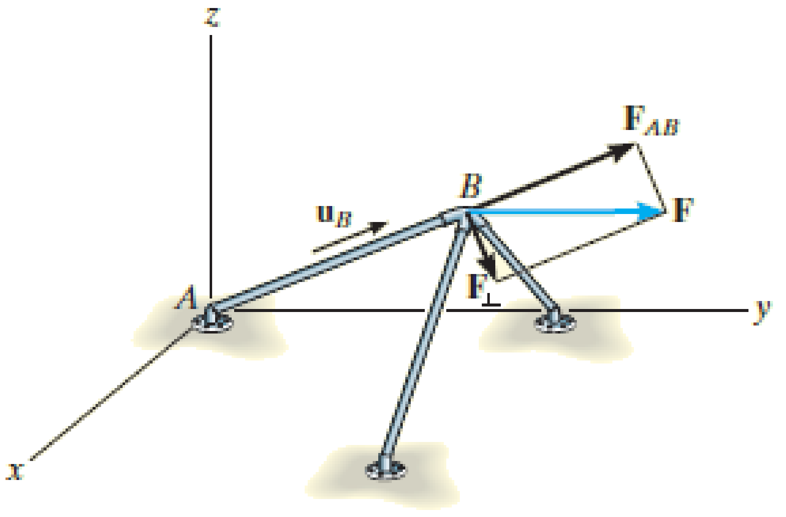
$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = 0.286\,\mathbf{i} + 0.857\,\mathbf{j} + 0.429\,\mathbf{k}$$

then

$$\begin{aligned} F_{AB} = F\cos\theta = \mathbf{F}\cdot\mathbf{u}_B &= (300\mathbf{j})\cdot(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k}) \\ &= (0)(0.286) + (300)(0.857) + (0)(0.429) \\ &= 257.1\text{ N} \end{aligned}$$

Ans.

Since the result is a positive scalar, $\mathbf{F}_{AB}$ has the same sense of direction as $\mathbf{u}_B$, Fig



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Solution cont.:

Expressing $\mathbf{F}_{AB}$ in Cartesian vector form, we have

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_B = (257.1\text{ N})(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k})$$
$$= \{73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$

The perpendicular component, Fig. is therefore

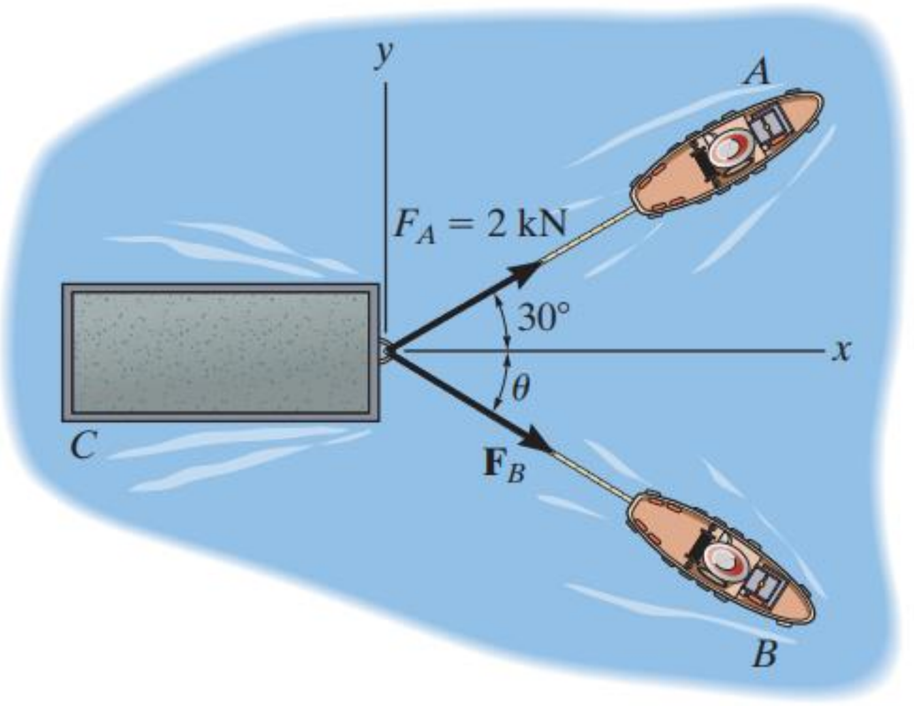
$$\mathbf{F}_\perp = \mathbf{F} - \mathbf{F}_{AB} = 300\mathbf{j} - (73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k})$$
$$= \{-73.5\mathbf{i} + 79.6\mathbf{j} - 110\mathbf{k}\}\text{ N}$$

Its magnitude can be determined either from this vector or by using the Pythagorean theorem, Fig.

$$F_\perp = \sqrt{F^2 - F_{AB}^2} = \sqrt{(300\text{ N})^2 - (257.1\text{ N})^2}$$
$$= 155\text{ N} \qquad \textit{Ans.}$$

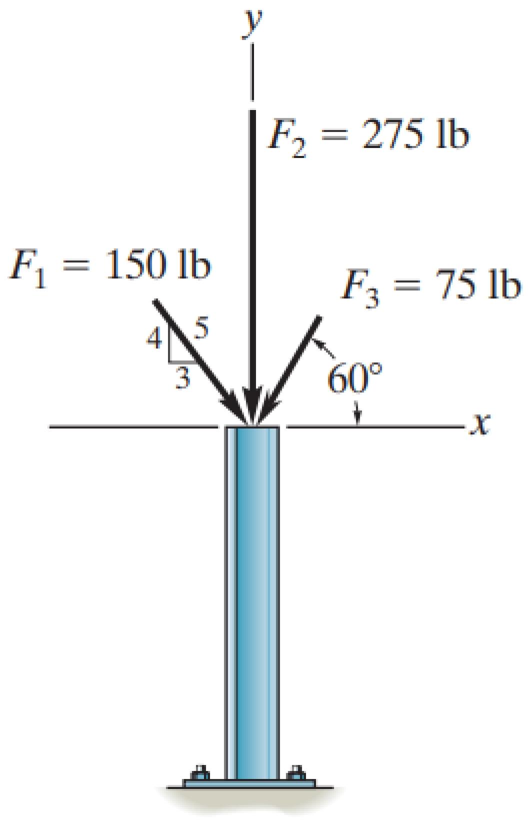
CLASS ACTIVITIES

1. If $F_B = 3$ kN and $\theta = 45°$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise from the positive x axis.



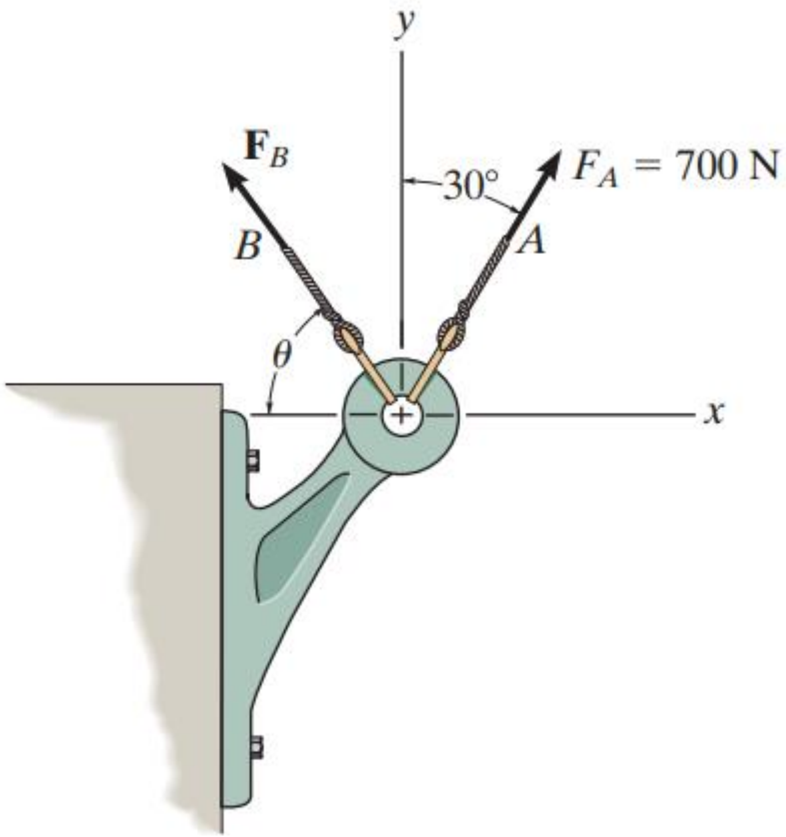
CLASS ACTIVITIES

2. Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.



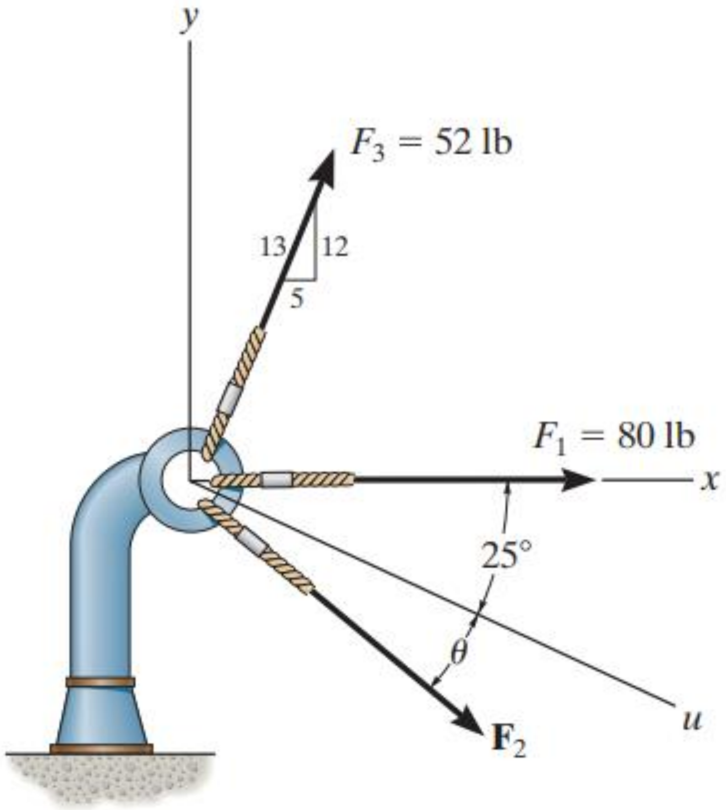
CLASS ACTIVITIES

3. Determine the magnitude and orientation, measured counterclockwise from the positive y axis, of the resultant force acting on the bracket, if $F_B = 600$ N and $\theta = 20°$.



CLASS ACTIVITIES

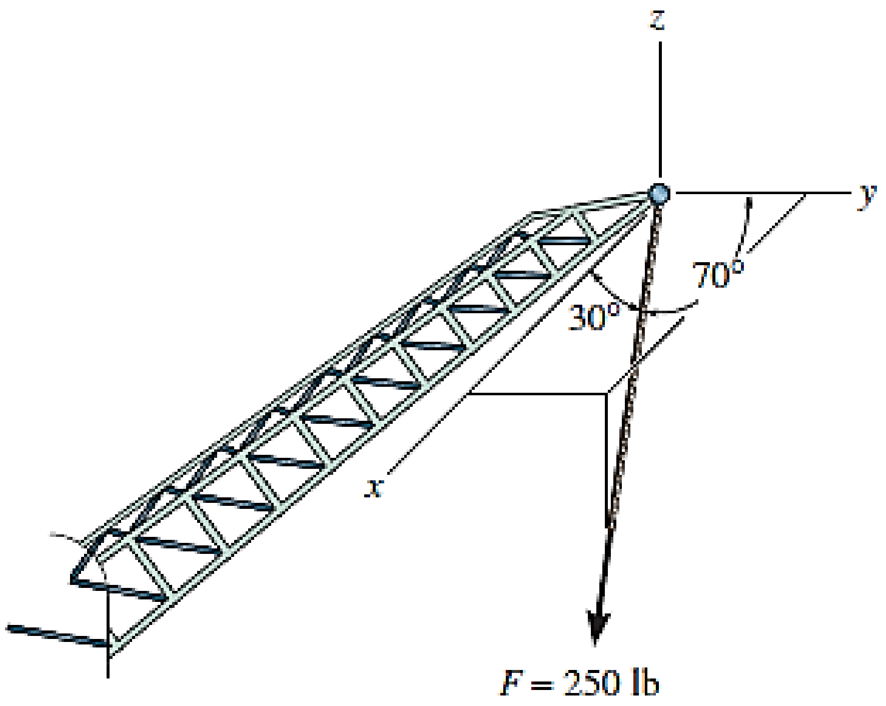
4. If $F_2 = 150$ lb and $\theta = 55°$, determine the magnitude and direction, measured clockwise from the positive x axis, of the resultant force of the three forces acting on the bracket.



CLASS ACTIVITIES

5.

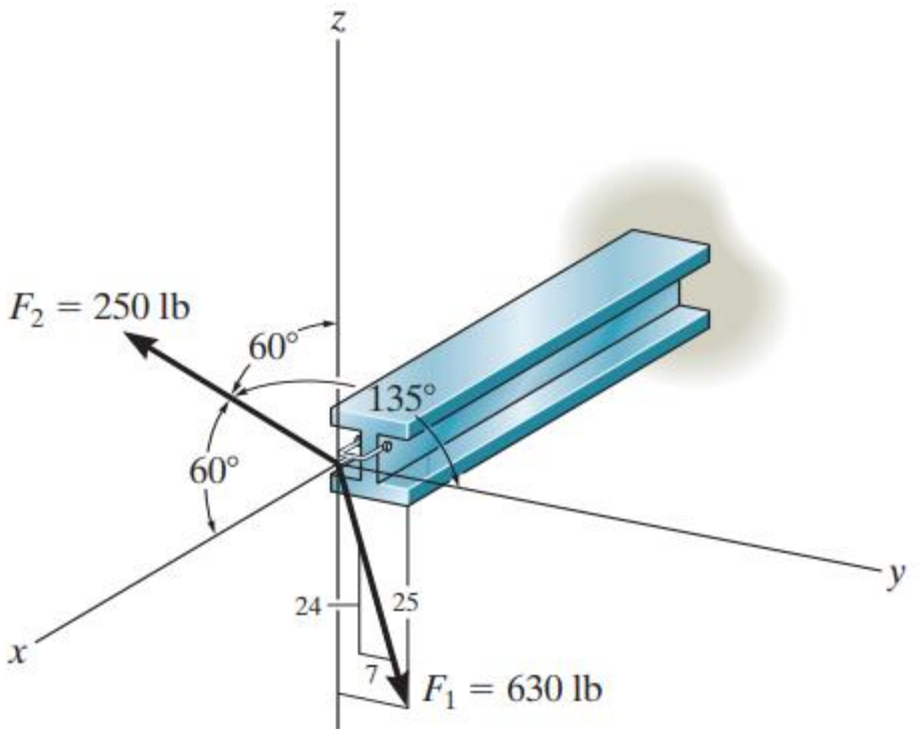
The cable at the end of the crane boom exerts a force of 250 lb on the boom as shown. Express **F** as a Cartesian vector.



CLASS ACTIVITIES

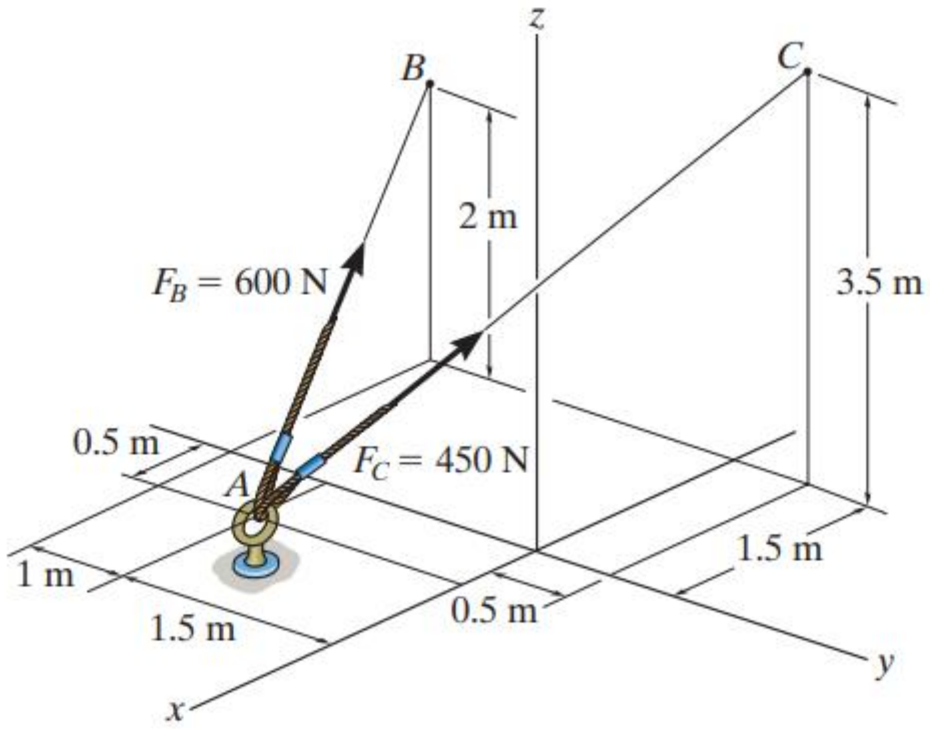
6.

The beam is subjected to the two forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



CLASS ACTIVITIES

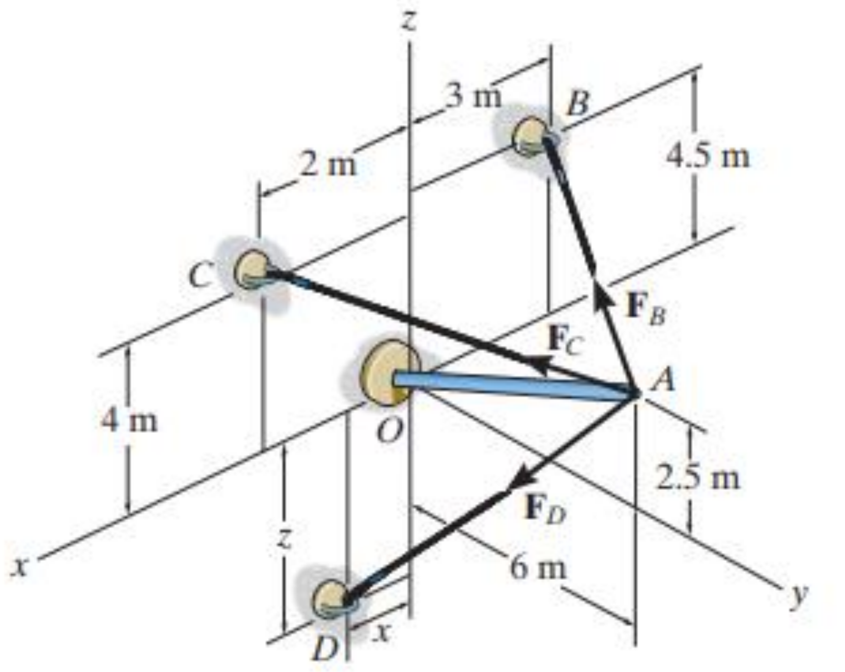
7. Express $\mathbf{F}_B$ and $\mathbf{F}_C$ in Cartesian vector form.



CLASS ACTIVITIES

8.

If the magnitude of the resultant force is and acts along the axis of the strut, directed from point towards , determine the magnitudes of the three forces acting on the strut (FB, FC, FD).



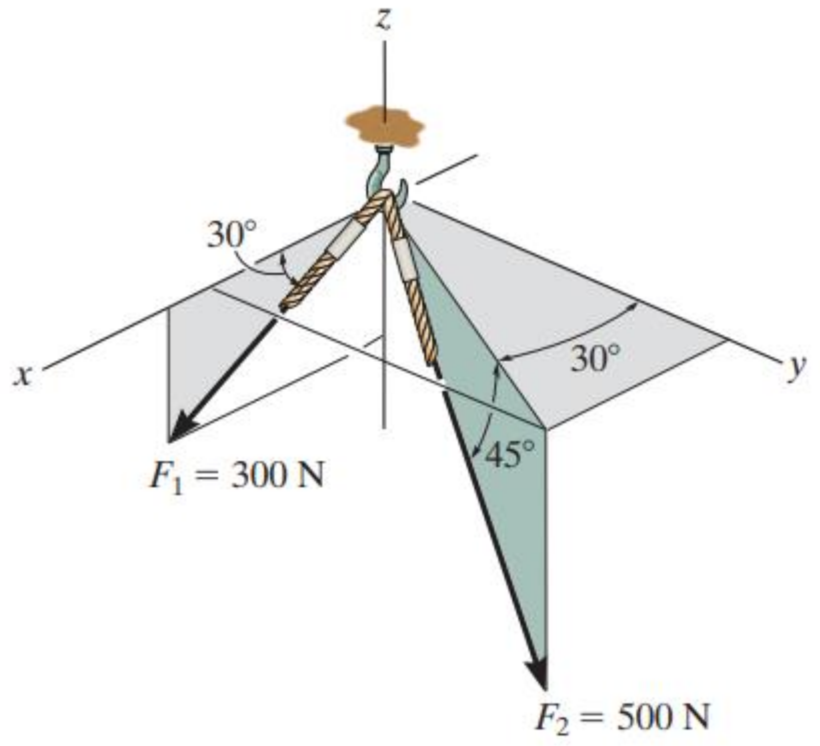
ASSIGNMENTS

Solve the following assignments and submit your answer on google classroom.

Assignment 1:

(solve this problems then submit your answer)

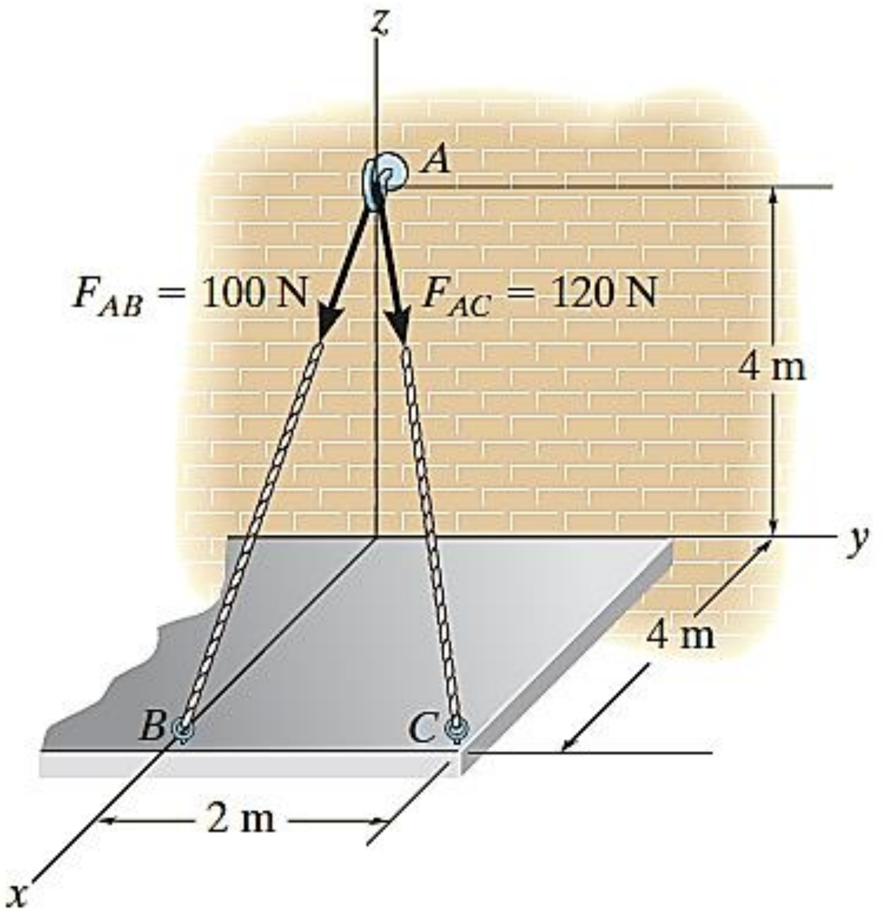
Determine the magnitude and coordinate direction angles of the resultant force acting on the hook.



Assignment 2:

(solve this problems then submit your answer)

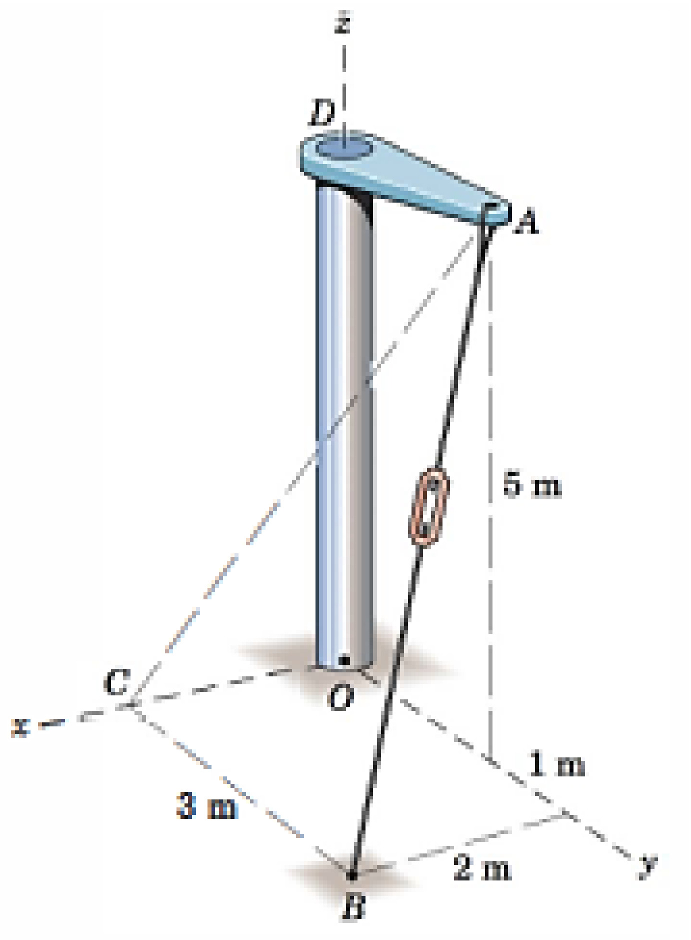
The roof is supported by cables as shown in the photo. If the cables exert forces $F_{AB} = 100\ N$ and $F_{AC} = 120\ N$ on the wall hook at A as shown in Fig., determine the resultant force acting at A . Express the result as a Cartesian vector.



Assignment 3:

(solve this problems then submit your answer)

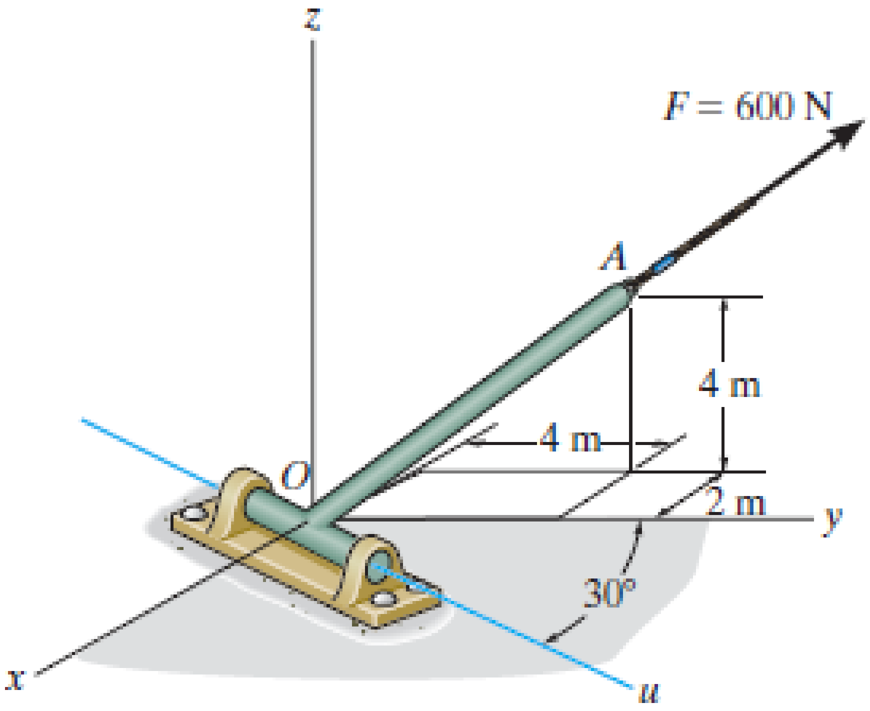
The turnbuckle is tightened until the tension in the cable AB equals $2.4\ kN$. Determine the vector expression for the tension T as a force acting on member AD. Also find the magnitude of the projection of T along the line AC



Assignment 4:

(solve this problems then submit your answer)

Determine the magnitude of the projection of force F = 600 N along the u axis.

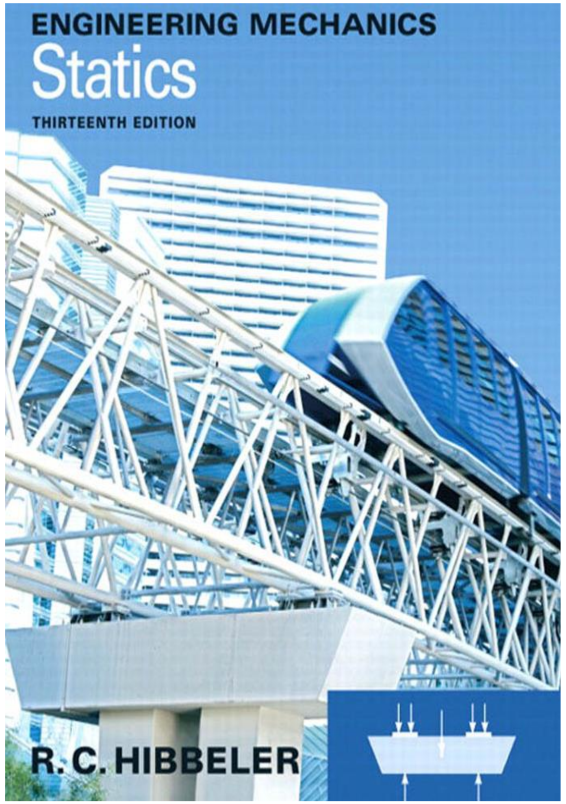


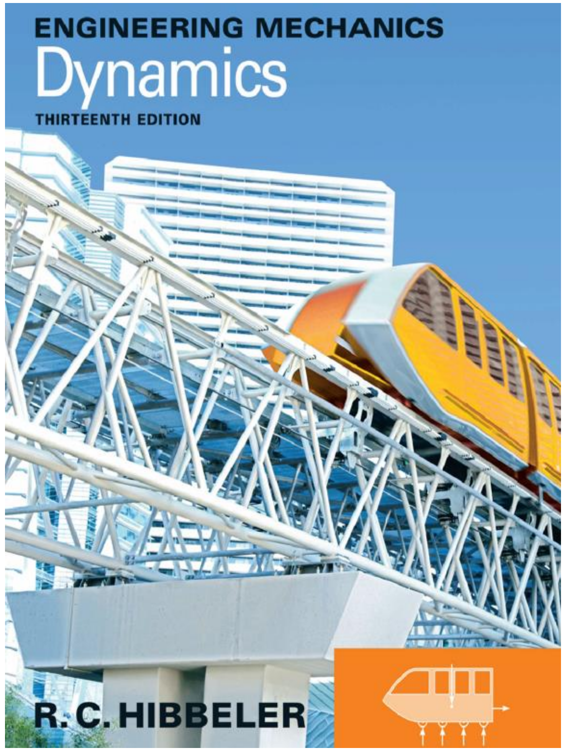
Next Lecture:

- Moment of a Force - Scalar Formation
- Cross Product
- Moment of a Force -Vector Formulation
- Solving Related examples

References:

Engineering Mechanics R.C. Hibbeler 13th edition (Statics and Dynamics).





The end of the lecture
Enjoy your time