Aviation Department

First Grade- Spring Semester

Moment of Force (Lecture 5)

Lecturer: Ms. Jwan Khaleel M.

Lecture Content:

- Moment of a Force - Scalar Formation
- Cross Product
- Moment of a Force -Vector Formulation
- Solving Related examples

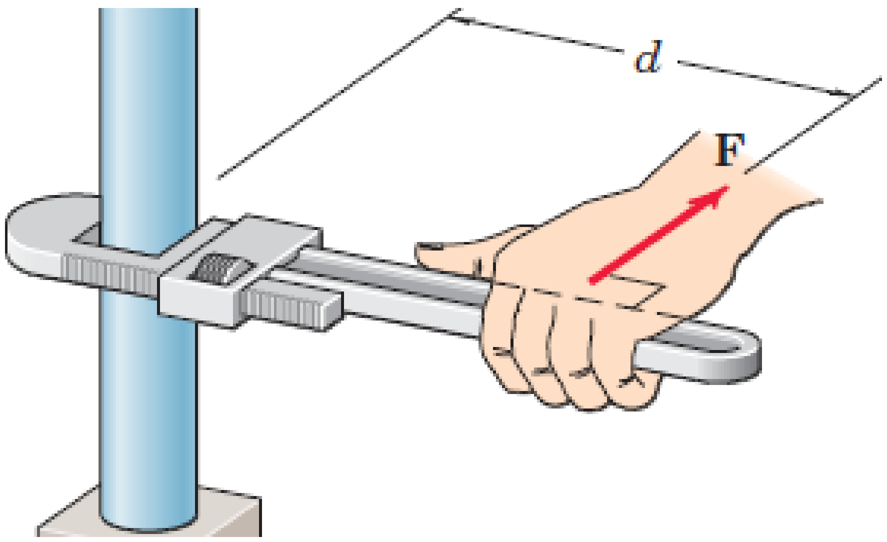
Learning Outcomes:

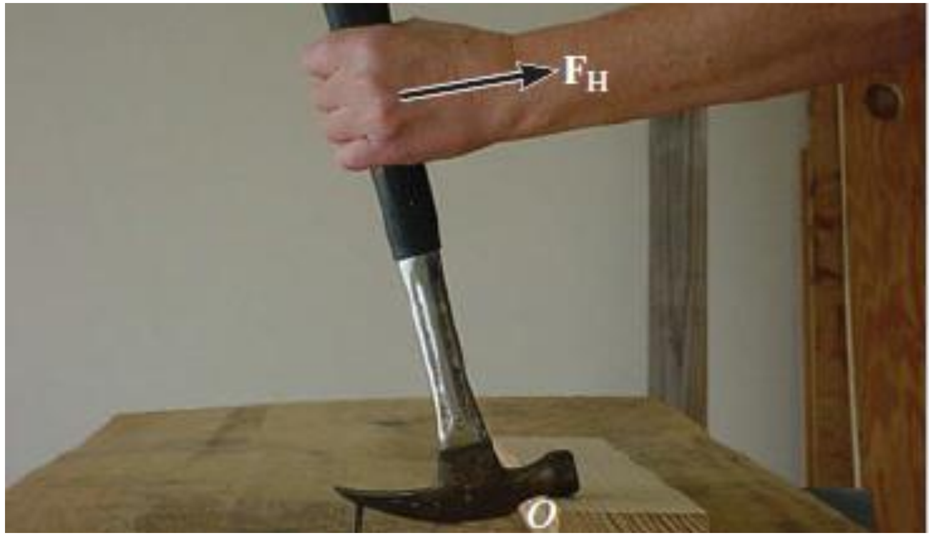
At the end of this lecture, you will be able to:

- Evaluate the concept of moment of a force.
- Compute a moment vector in terms of a force vector.
- Calculate the moment of a force in 2D and 3D.
- Solving problems using related equations

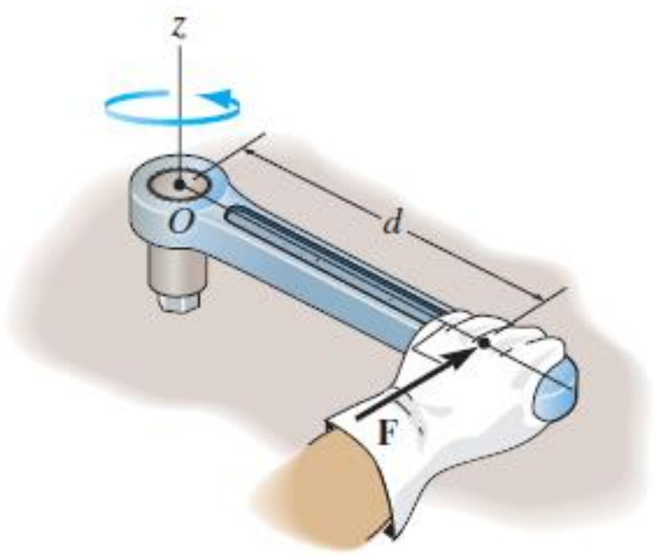
What is a Moment :

- A force can also tend to rotate a body about a point or an axis.
- The axis may be any line which neither intersects nor is parallel to the line of action of the force.
- This rotational tendency is known as the ***moment*** ***"M"*** of the force.
- Moment is also referred to as ***torque***.

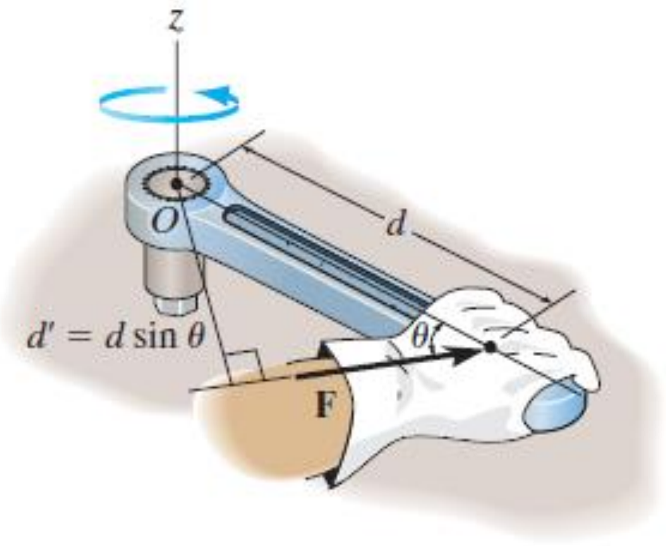




• Moment about, a Point (Scalar Formation):



(a)



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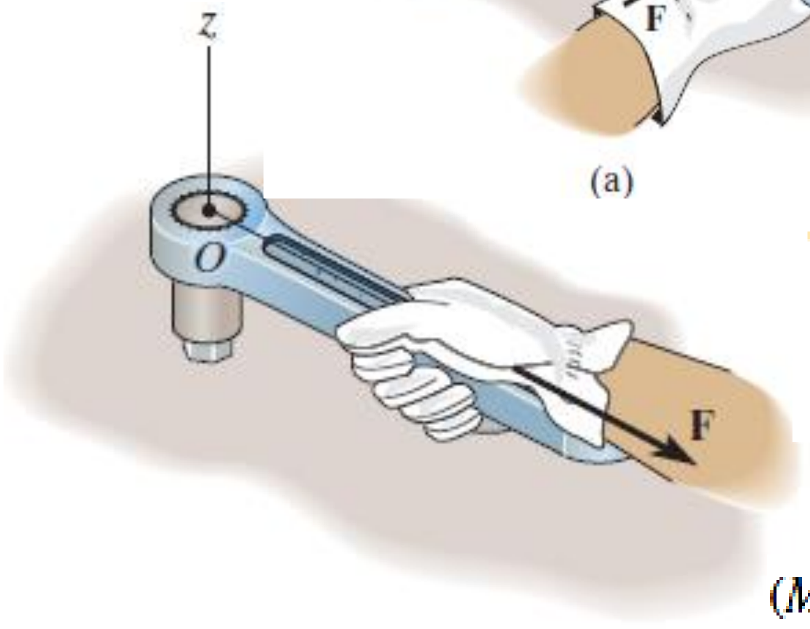
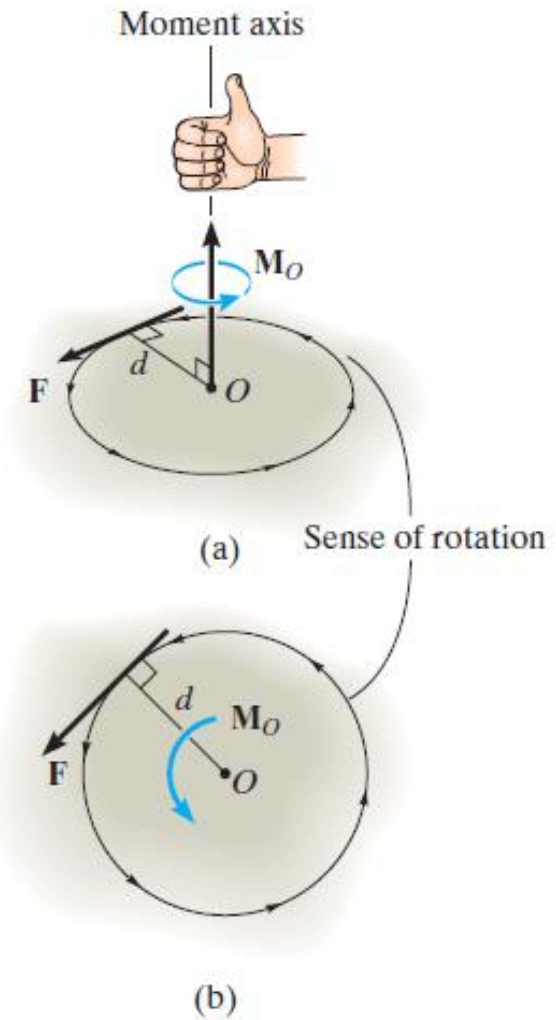


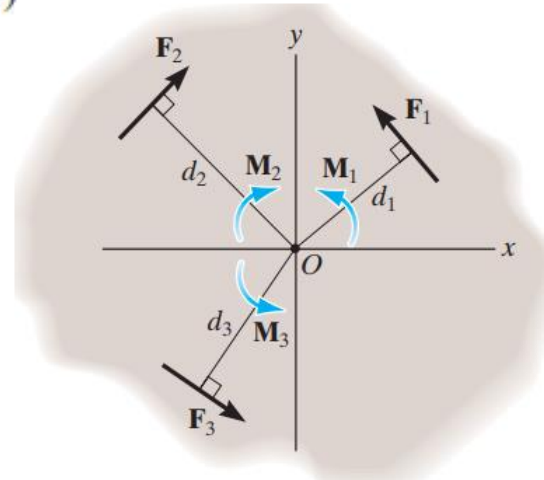


The magnitude of $\mathbf{M}_O$ is

$$M_O = Fd$$

$$\circlearrowleft + (M_R)_o = \Sigma Fd;$$

$$(M_R)_o = F_1 d_1 - F_2 d_2 + F_3 d_3$$

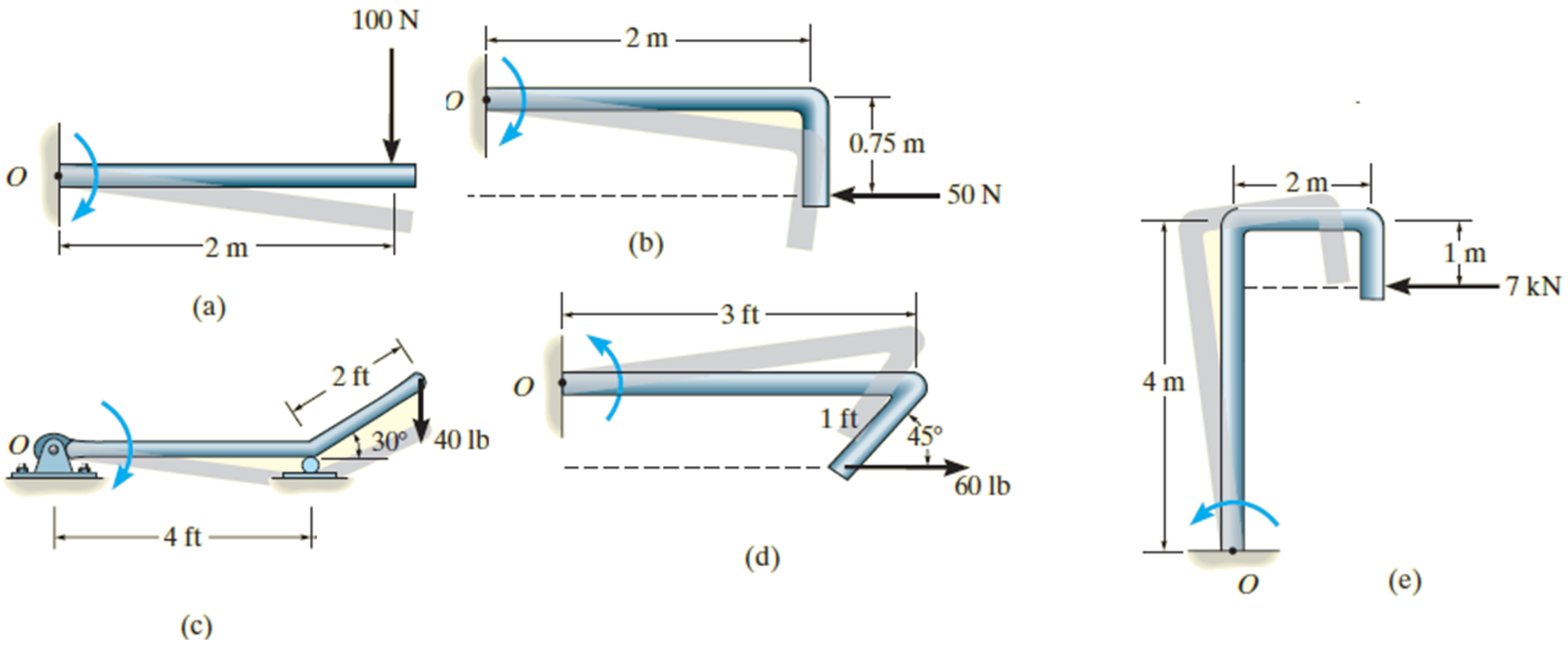


Its moment arm will be zero since the line of action of **F** will intersect point *O* (the *z* axis).

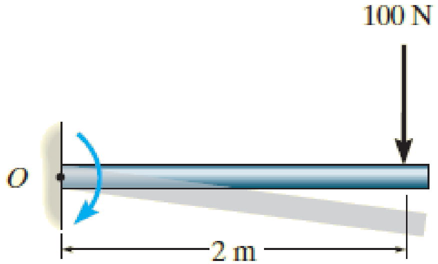
- Plus, sign (+) for counterclockwise moments and a minus sign (−) for clockwise moments.
- The basic units of moment in SI units are newton-meters (N. m), and in the U.S. customary system are pound-feet (lb-ft).

Example 1:

For each case illustrated in Figure shown, determine the moment of the force about point O .

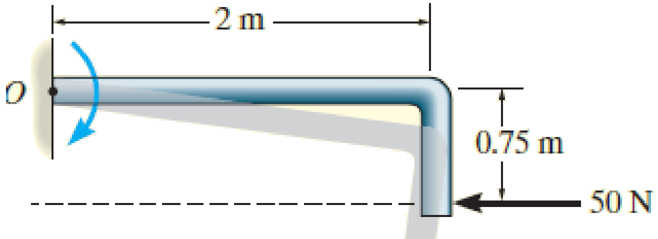


Solution:



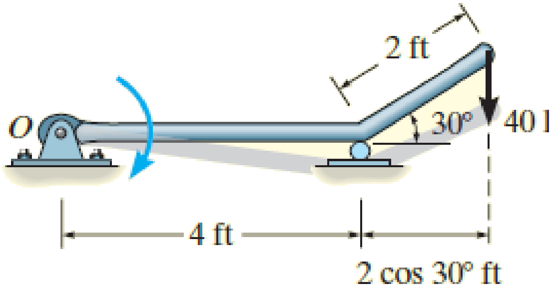
(a)

(a) $M_O = (100\text{ N})(2\text{ m}) = 200\text{ N}\cdot\text{m} \circlearrowright$



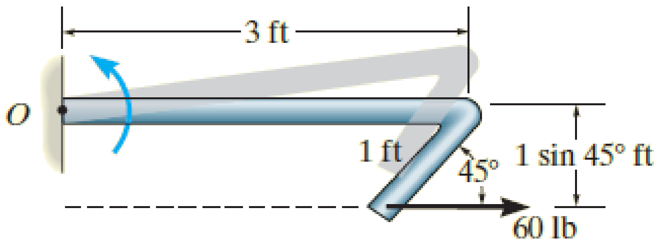
(b)

(b) $M_O = (50\text{ N})(0.75\text{ m}) = 37.5\text{ N}\cdot\text{m} \circlearrowright$



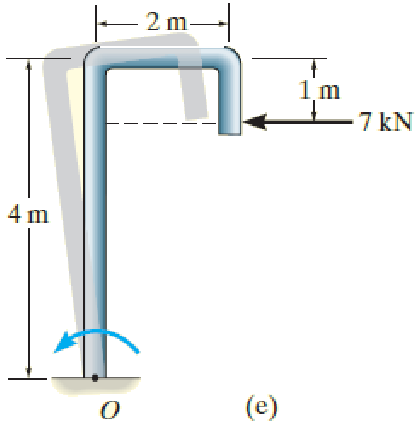
(c)

(c) $M_O = (40\text{ lb})(4\text{ ft} + 2\cos 30^\circ\text{ ft}) = 229\text{ lb}\cdot\text{ft} \circlearrowright$



(d)

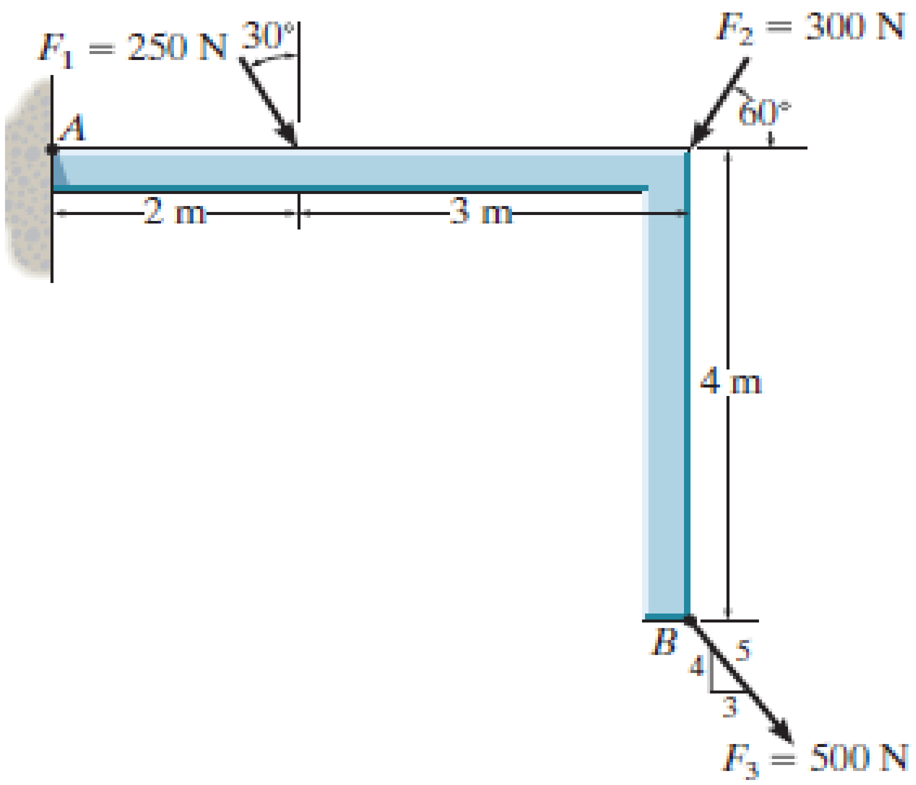
(d) $M_O = (60\text{ lb})(1\sin 45^\circ\text{ ft}) = 42.4\text{ lb}\cdot\text{ft} \circlearrowleft$



(e)

(e) $M_O = (7\text{ kN})(4\text{ m} - 1\text{ m}) = 21.0\text{ kN}\cdot\text{m} \circlearrowleft$

Example 2: Determine the moment of each of the three forces about point A.



SOLUTION

The moment arm measured perpendicular to each force from point A is

$$d_1 = 2\sin 60^\circ = 1.732\text{ m}$$

$$d_2 = 5\sin 60^\circ = 4.330\text{ m}$$

$$d_3 = 2\sin 53.13^\circ = 1.60\text{ m}$$

Using each force where $M_A = Fd$, we have

$$\circlearrowleft + (M_{F_1})_A = -250(1.732)$$

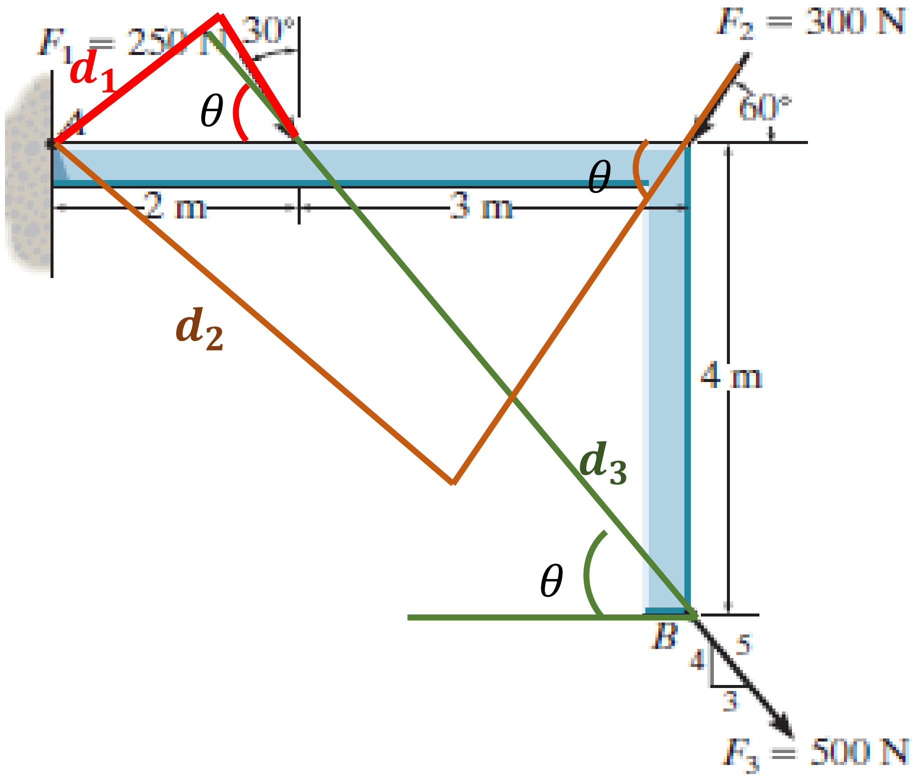
$$= -433\text{ N}\cdot\text{m} = 433\text{ N}\cdot\text{m} \quad \textit{(Clockwise)}$$

$$\circlearrowleft + (M_{F_2})_A = -300(4.330)$$

$$= -1299\text{ N}\cdot\text{m} = 1.30\text{ kN}\cdot\text{m} \quad \textit{(Clockwise)}$$

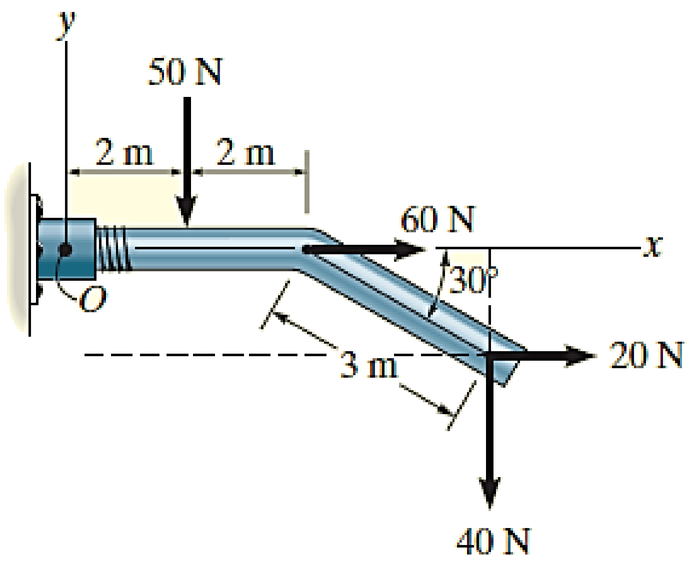
$$\circlearrowleft + (M_{F_3})_A = -500(1.60)$$

$$= -800\text{ N}\cdot\text{m} = 800\text{ N}\cdot\text{m} \quad \textit{(Clockwise)}$$



Example 3:

Determine the resultant moment of the four forces acting on the rod shown in Figure about point O.



SOLUTION

Assuming that positive moments act in the $+\mathbf{k}$ direction, i.e., counterclockwise, we have

$$\circlearrowleft + \quad (M_R)_o = \Sigma Fd;$$

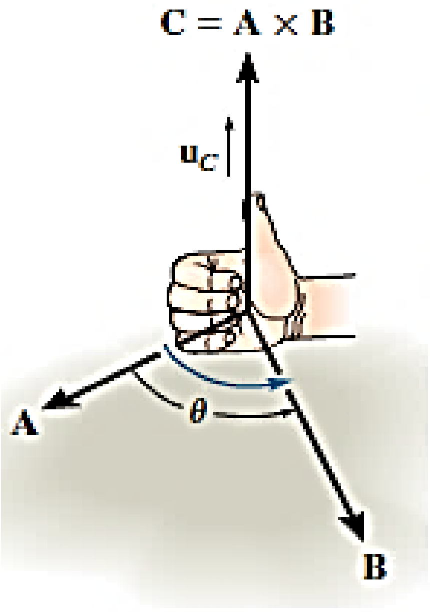
$$(M_R)_o = -50\text{ N}(2\text{ m}) + 60\text{ N}(0) + 20\text{ N}(3\sin 30^\circ\text{ m})$$

$$-40\text{ N}(4\text{ m} + 3\cos 30^\circ\text{ m})$$

$$(M_R)_o = -334\text{ N}\cdot\text{m} = 334\text{ N}\cdot\text{m}\circlearrowright \qquad \textit{Ans.}$$

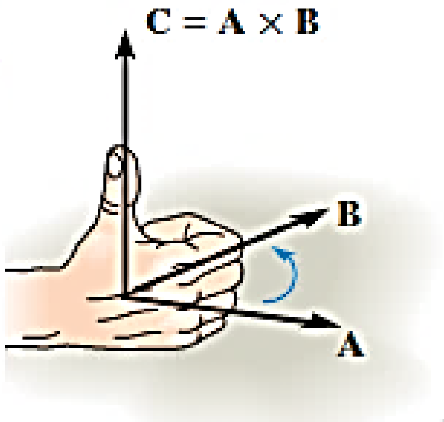
• Cross Product, click here:

It is a useful approach to find the vector which is perpendicular to a plane formed by two vectors.



$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

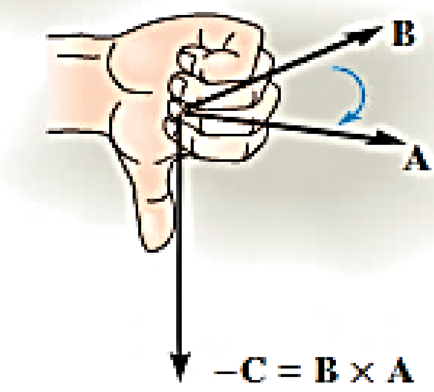
Magnitude. The *magnitude* of **C** is defined as the product of the magnitudes of **A** and **B** and the sine of the angle θ between their tails ($0° \le \theta \le 180°$). Thus, $C = AB \sin \theta$.



Laws of Operation.

- The commutative law is *not* valid; i.e., $\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$. Rather,

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$



- If the cross product is multiplied by a scalar a, it obeys the associative law;

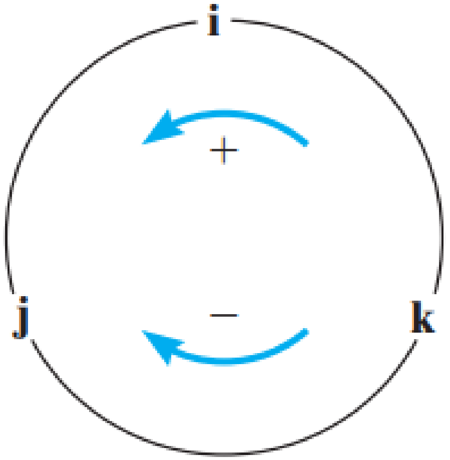
$$a(\mathbf{A} \times \mathbf{B}) = (a\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (a\mathbf{B}) = (\mathbf{A} \times \mathbf{B})a$$

- The vector cross product also obeys the distributive law of addition.

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$$

- ## ***Cartesian Vector Formulation:***

$$\mathbf{i}\times\mathbf{j}=\mathbf{k} \quad \mathbf{i}\times\mathbf{k}=-\mathbf{j} \quad \mathbf{i}\times\mathbf{i}=0$$
$$\mathbf{j}\times\mathbf{k}=\mathbf{i} \quad \mathbf{j}\times\mathbf{i}=-\mathbf{k} \quad \mathbf{j}\times\mathbf{j}=0$$
$$\mathbf{k}\times\mathbf{i}=\mathbf{j} \quad \mathbf{k}\times\mathbf{j}=-\mathbf{i} \quad \mathbf{k}\times\mathbf{k}=0$$



$$\begin{aligned}\mathbf{A}\times\mathbf{B} &= (A_x\mathbf{i}+A_y\mathbf{j}+A_z\mathbf{k})\times(B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k})\\ &= A_xB_x(\mathbf{i}\times\mathbf{i})+A_xB_y(\mathbf{i}\times\mathbf{j})+A_xB_z(\mathbf{i}\times\mathbf{k})\\ &\quad + A_yB_x(\mathbf{j}\times\mathbf{i})+A_yB_y(\mathbf{j}\times\mathbf{j})+A_yB_z(\mathbf{j}\times\mathbf{k})\\ &\quad + A_zB_x(\mathbf{k}\times\mathbf{i})+A_zB_y(\mathbf{k}\times\mathbf{j})+A_zB_z(\mathbf{k}\times\mathbf{k})\end{aligned}$$

$$\mathbf{A}\times\mathbf{B}=(A_yB_z-A_zB_y)\mathbf{i}-(A_xB_z-A_zB_x)\mathbf{j}+(A_xB_y-A_yB_x)\mathbf{k}$$

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

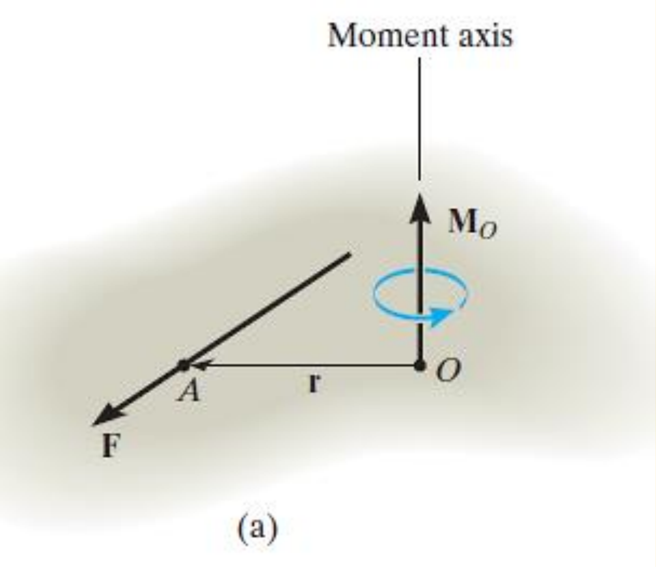
For element **i**: $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$

For element **j**: $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$

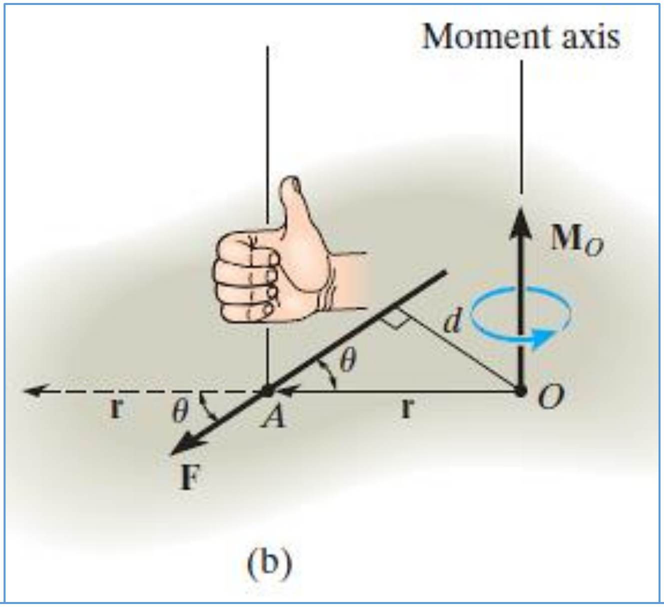
Remember the negative sign

For element **k**: $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$

• Moment of an axis passing through point O (Vector Formulation):



(a)



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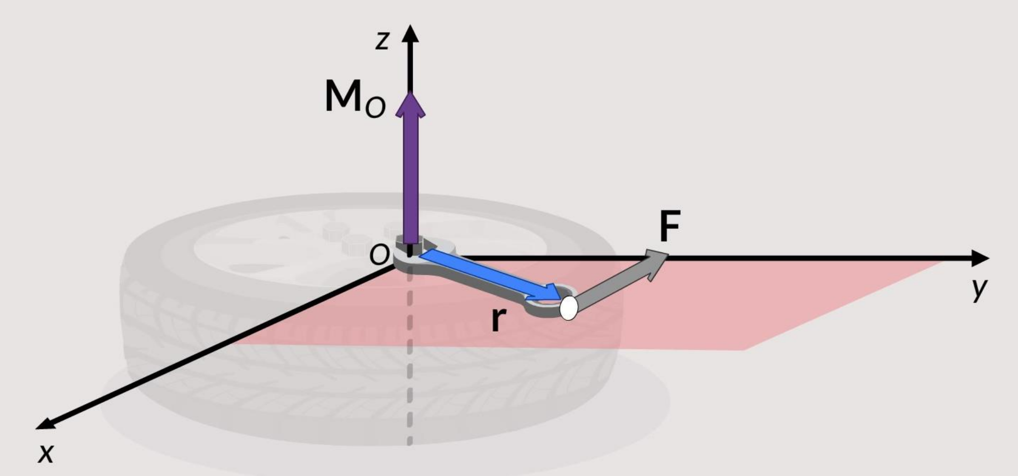
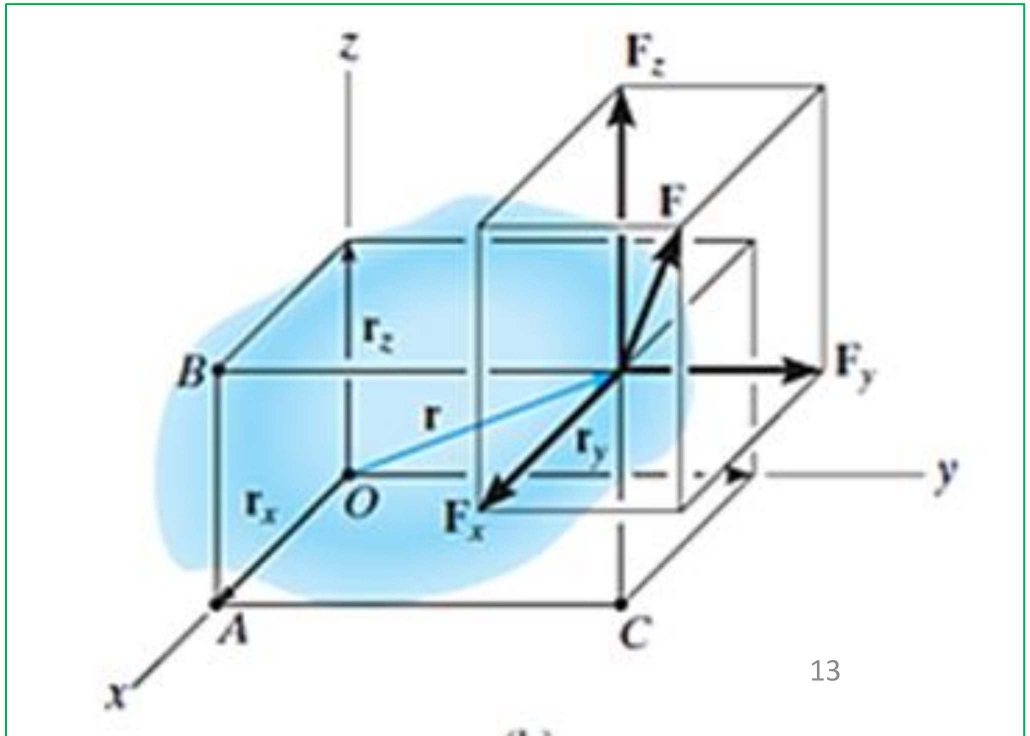


$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$M_O = rF\sin\theta = F(r\sin\theta) = Fd$$

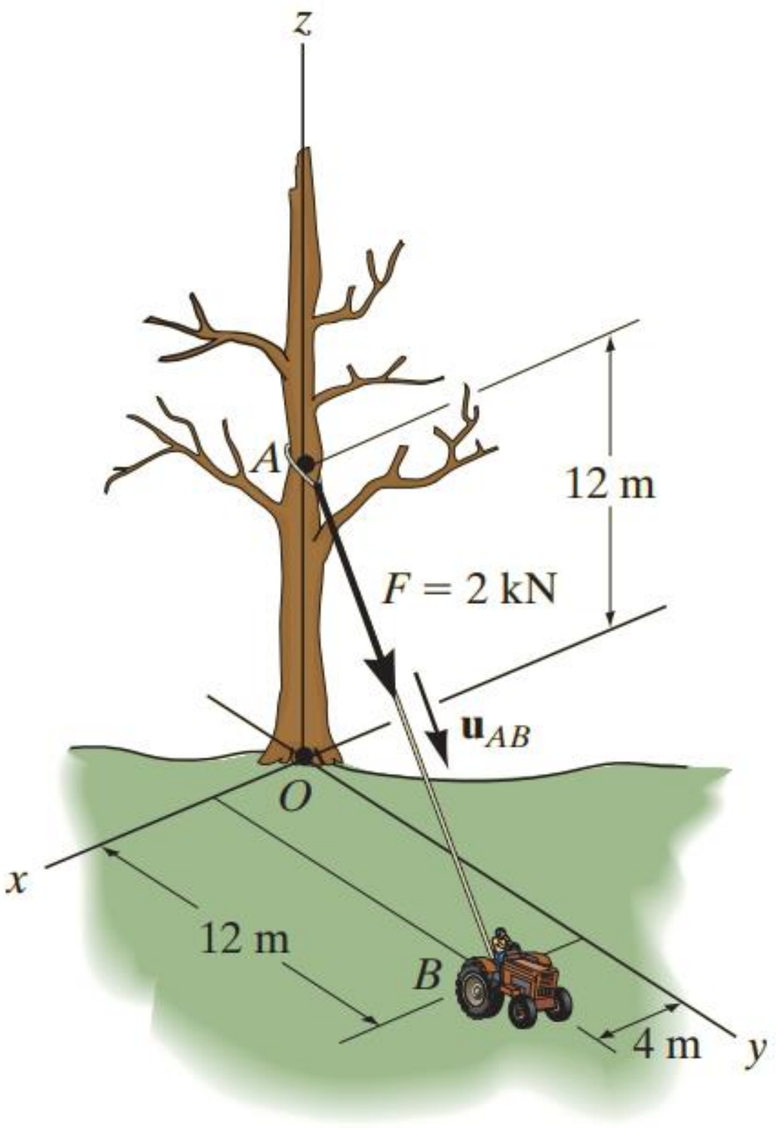
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\mathbf{M}_O = (r_yF_z - r_zF_y)\mathbf{i} - (r_xF_z - r_zF_x)\mathbf{j} + (r_xF_y - r_yF_x)\mathbf{k}$$



Example 4:

Determine the moment produced by the force F in Fig. a about point O . Express the result as a Cartesian vector



(a)

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SOLUTION

As shown in Fig. either $\mathbf{r}_A$ or $\mathbf{r}_B$ can be used to determine the moment about point O. These position vectors are

$$\mathbf{r}_A = \{12\mathbf{k}\}\text{ m} \quad \text{and} \quad \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\text{ m}$$

Force $\mathbf{F}$ expressed as a Cartesian vector is

$$\mathbf{F} = F\mathbf{u}_{AB} = 2\text{ kN}\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}\text{ m}}{\sqrt{(4\text{ m})^2 + (12\text{ m})^2 + (-12\text{ m})^2}}\right]$$

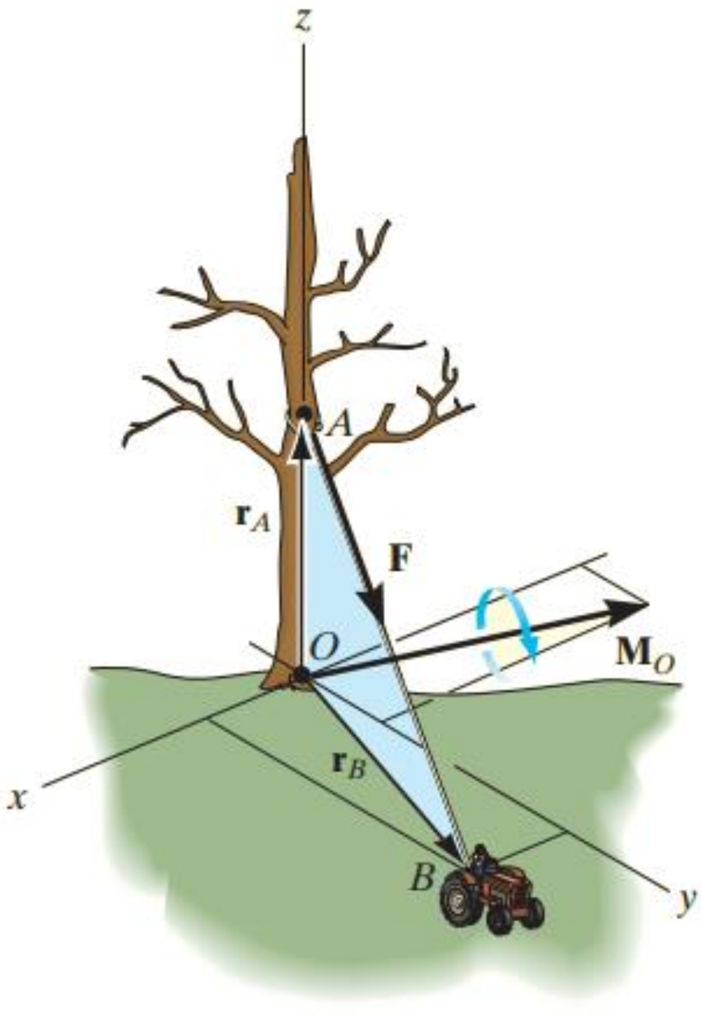
$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\text{ kN}$$

Thus

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [0(-1.376) - 12(1.376)]\mathbf{i} - [0(-1.376) - 12(0.4588)]\mathbf{j} + [0(1.376) - 0(0.4588)]\mathbf{k}$$

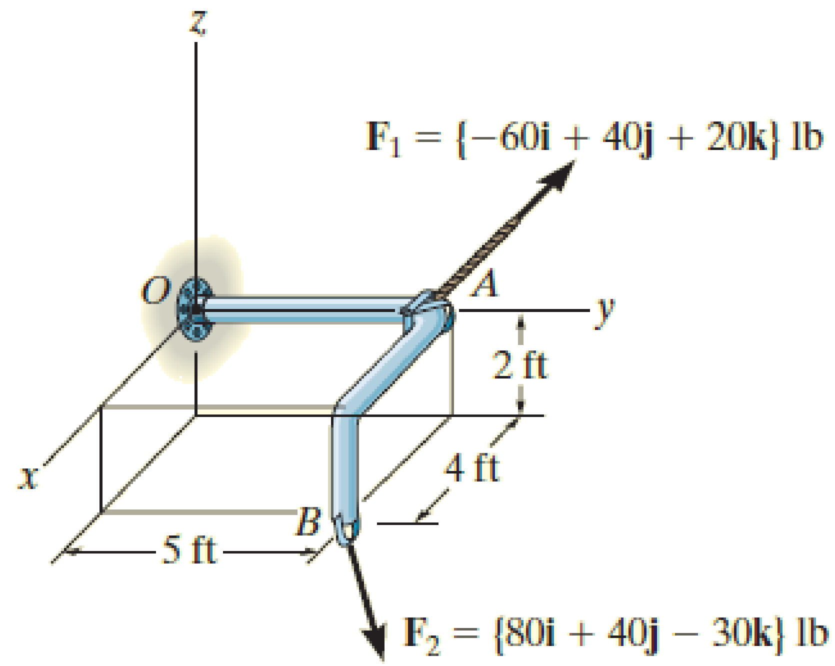
$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\text{ kN}\cdot\text{m}$$ *Ans.*



(b)

Example 5:

Two forces act on the rod shown in Figure. Determine the resultant moment they create about the flange at O . Express the result as a Cartesian vector.



Solution:

Position vectors are directed from point O to each force as shown in Figure These vectors are

$O = (0i, 0j, 0k)$
$A = (0i, 5j, 0k)$
$B = (4i, 5j, -2k)$

$$\mathbf{r}_A = \{5\mathbf{j}\}\ \text{ft}$$

$$\mathbf{r}_B = \{4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$

The resultant moment about O is therefore

$$(\mathbf{M}_R)_O = \Sigma(\mathbf{r} \times \mathbf{F})$$

$$= \mathbf{r}_A \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$

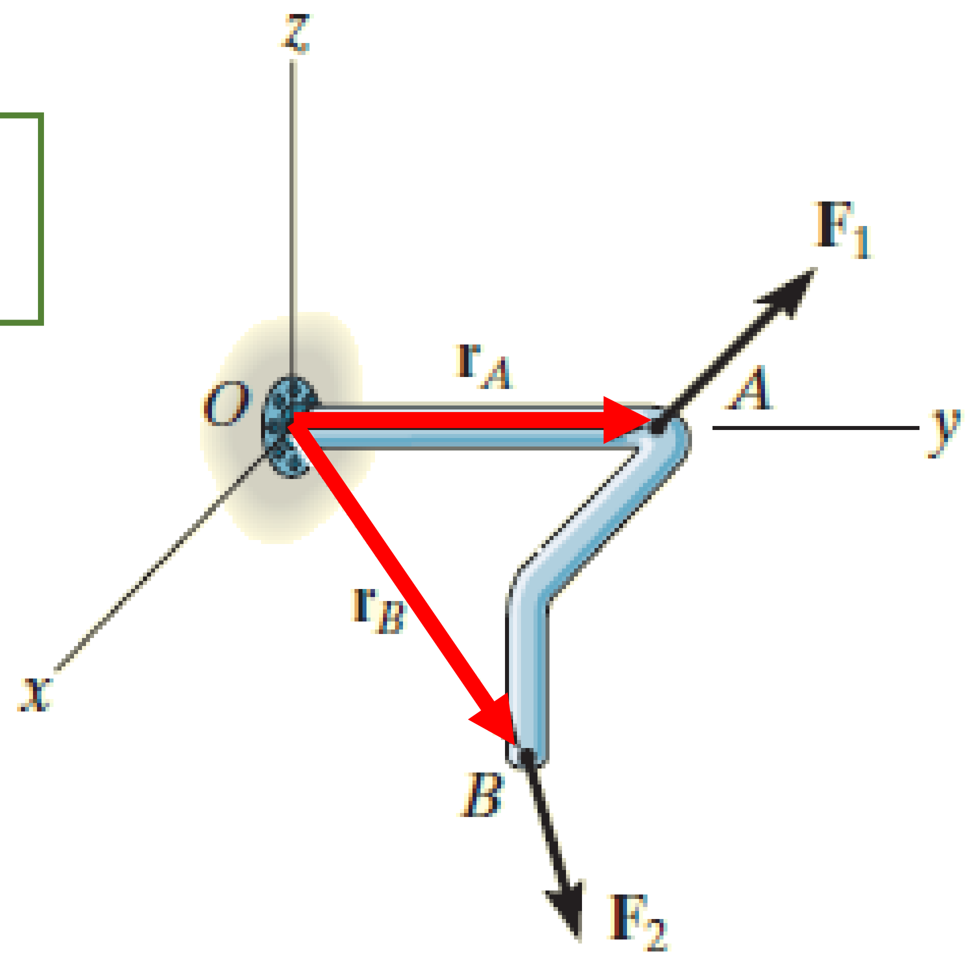
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

$$= [5(20) - 0(40)]\mathbf{i} - [0]\mathbf{j} + [0(40) - (5)(-60)]\mathbf{k}$$

$$+ [5(-30) - (-2)(40)]\mathbf{i} - [4(-30) - (-2)(80)]\mathbf{j} + [4(40) - 5(80)]\mathbf{k}$$

$$= \{30\mathbf{i} - 40\mathbf{j} + 60\mathbf{k}\}\ \text{lb} \cdot \text{ft}$$

Ans.



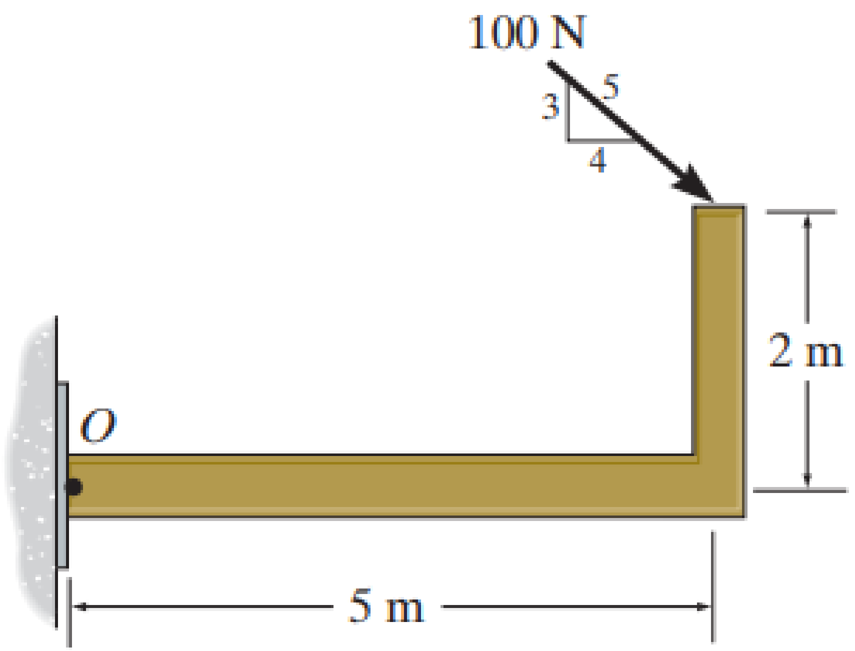
Next Lecture:

- Moment of a Couple of Forces
- Simplification of a Force and Couple System
- Further Simplification of a Force and Couple System
- Reduction of a Simple Distributed Loading

Assignment 1

(solve this problems then submit your answer)

Determine the moment of the force about point O



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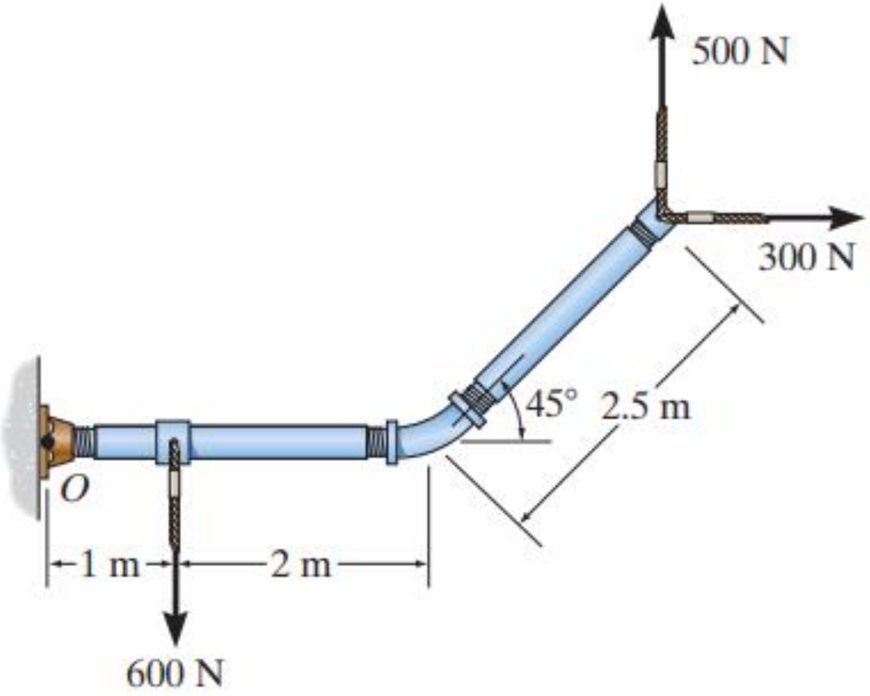
Assignment 2

(solve this problems then submit your answer)

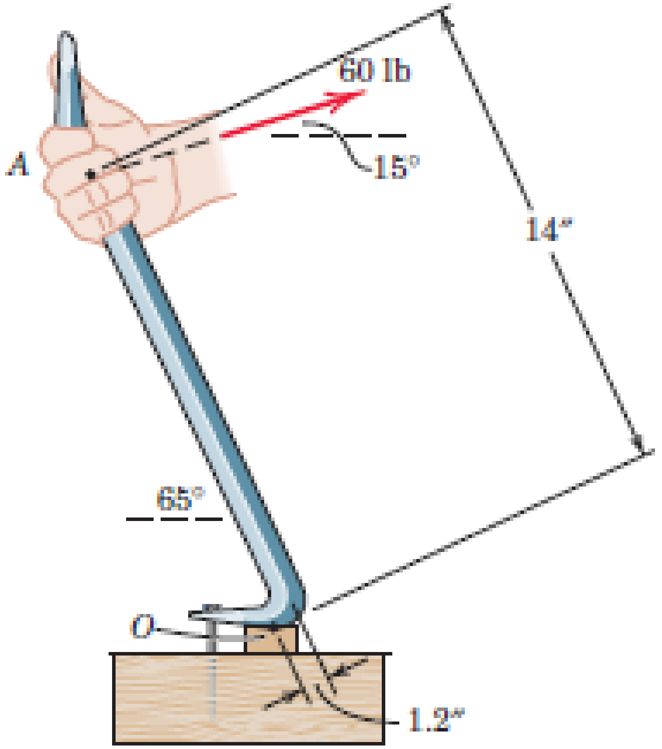
Determine the resultant moment produced by the forces about point O



Assignment 3

(solve this problems then submit your answer)

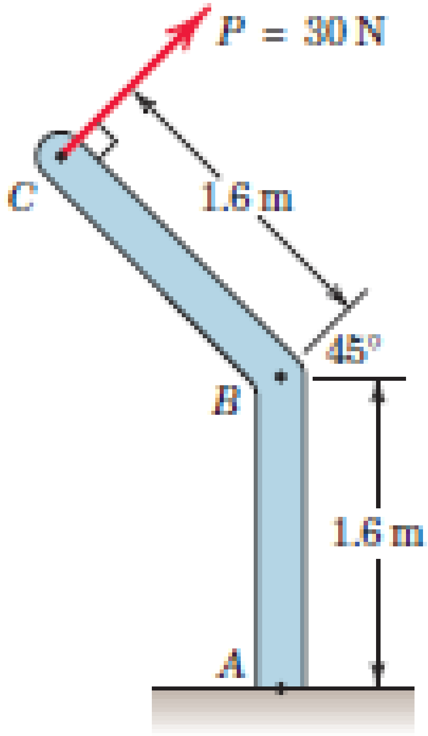
A pry bar is used to remove a nail as shown. Determine the moment of the 60-lb force about the point O of contact between the pry bar and the small support block.



Assignment 4

(solve this problems then submit your answer)

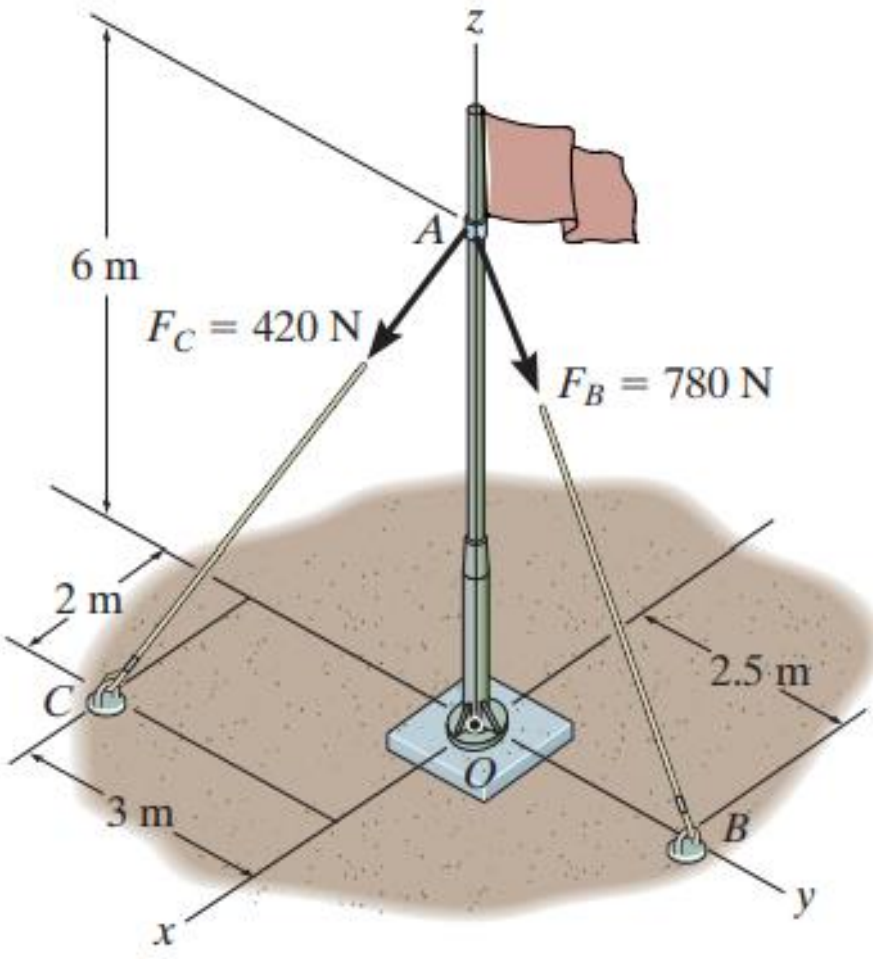
The 30-N force **P** is applied perpendicular to the portion *BC* of the bent bar. Determine the moment of **P** about point *B* and about point *A*.



Assignment 5

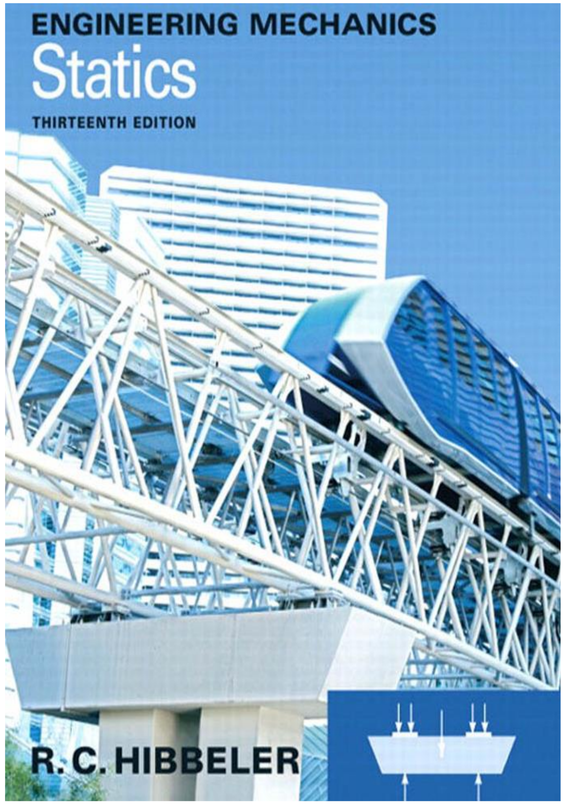
(solve this problems then submit your answer)

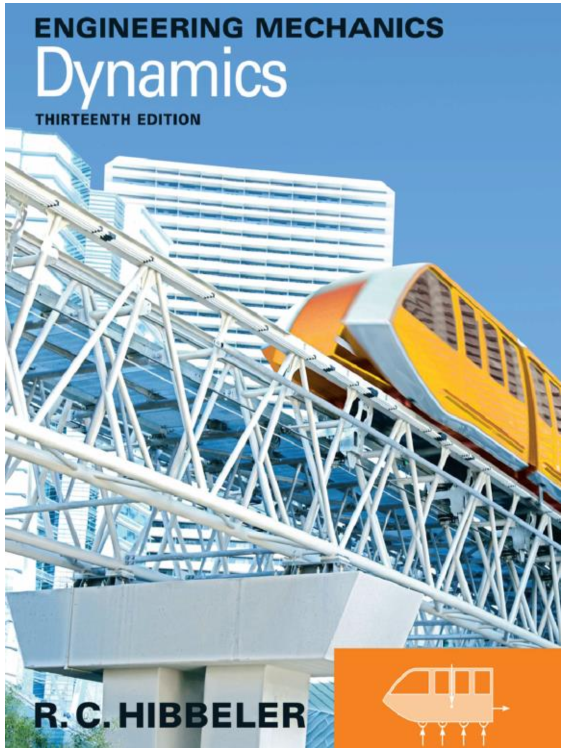
Determine the resultant moment produced by force FB and FC about point O . Express the result as a Cartesian vector



References:

Engineering Mechanics R.C. Hibbeler 13th edition (Statics and Dynamics).





The end of the lecture
Enjoy your time