Aviation Department
First Grade- Spring Semester

Statics- Moment of a Couple of force (Lecture 6)

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Lecture Content:

Moment of a Couple of Forces

Force and Couple System

Resultant of a Force and Couple System

Reduction of a Simple Distributed Loading

Learning Outcomes

<u>At the end of this lecture, you will be able to:</u>

Define the moment of a couple.

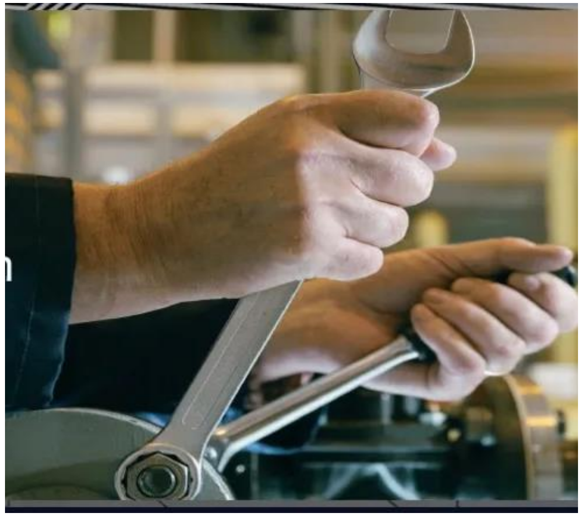
Provide a method for finding the moment of a couple of forces.

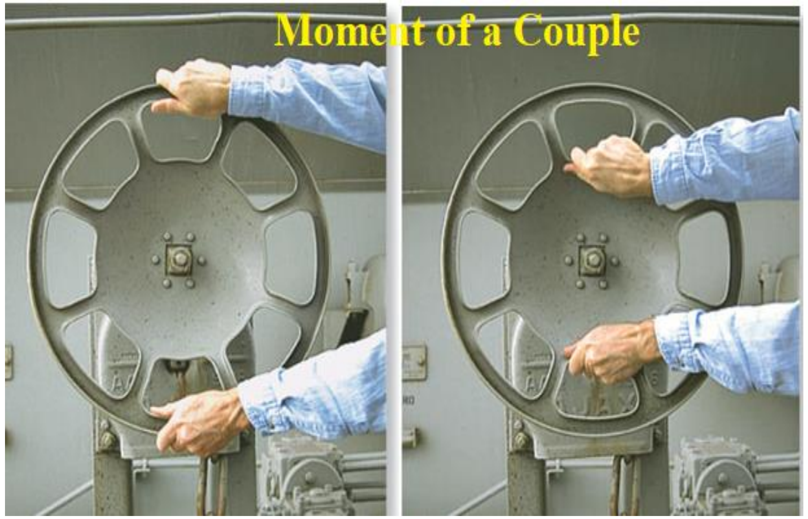
Find the resultants of nonconcurrent force systems.

Solve related problems

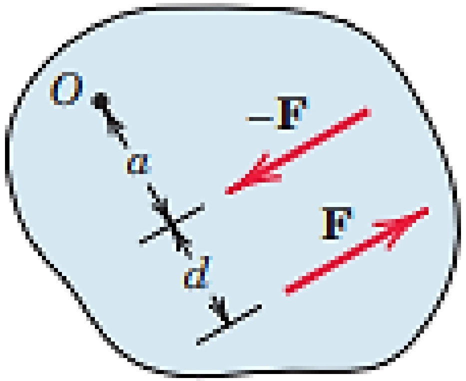
What is Couple?

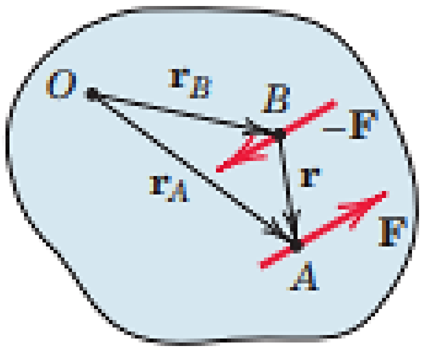
The moment produced by two equal, opposite, and noncollinear forces is called a ***couple***.

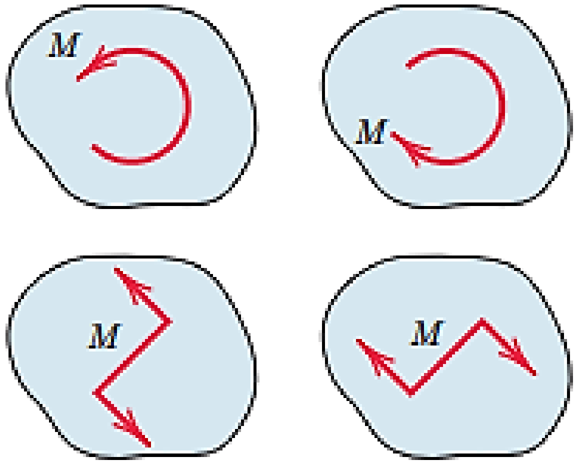




- ***<u>Vector Algebra Method:</u>***







Counterclockwise couple

Clockwise couple

$$M = F(a + d) - Fa$$

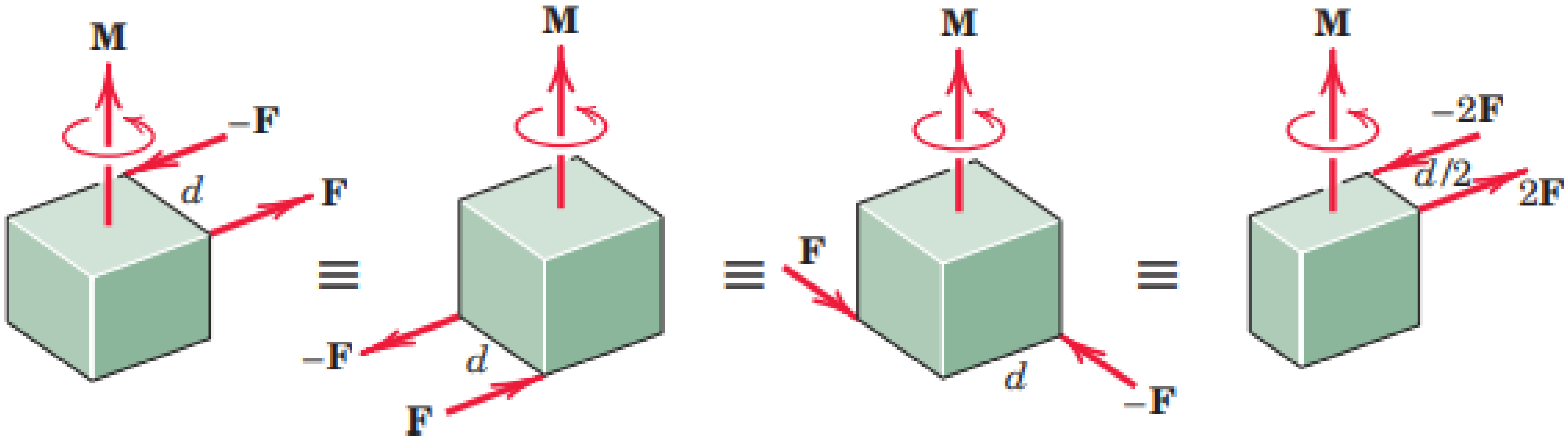
$$M = Fd$$

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F})$$

$$= (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

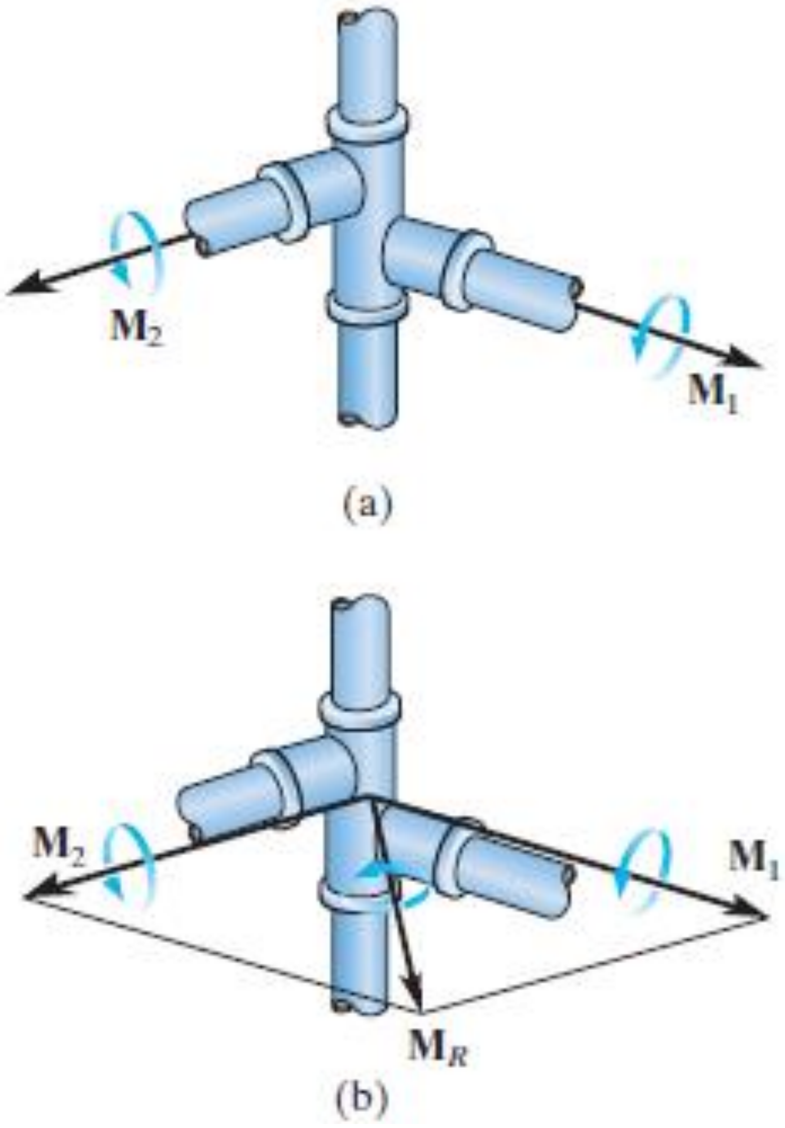
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Equivalent Couples:



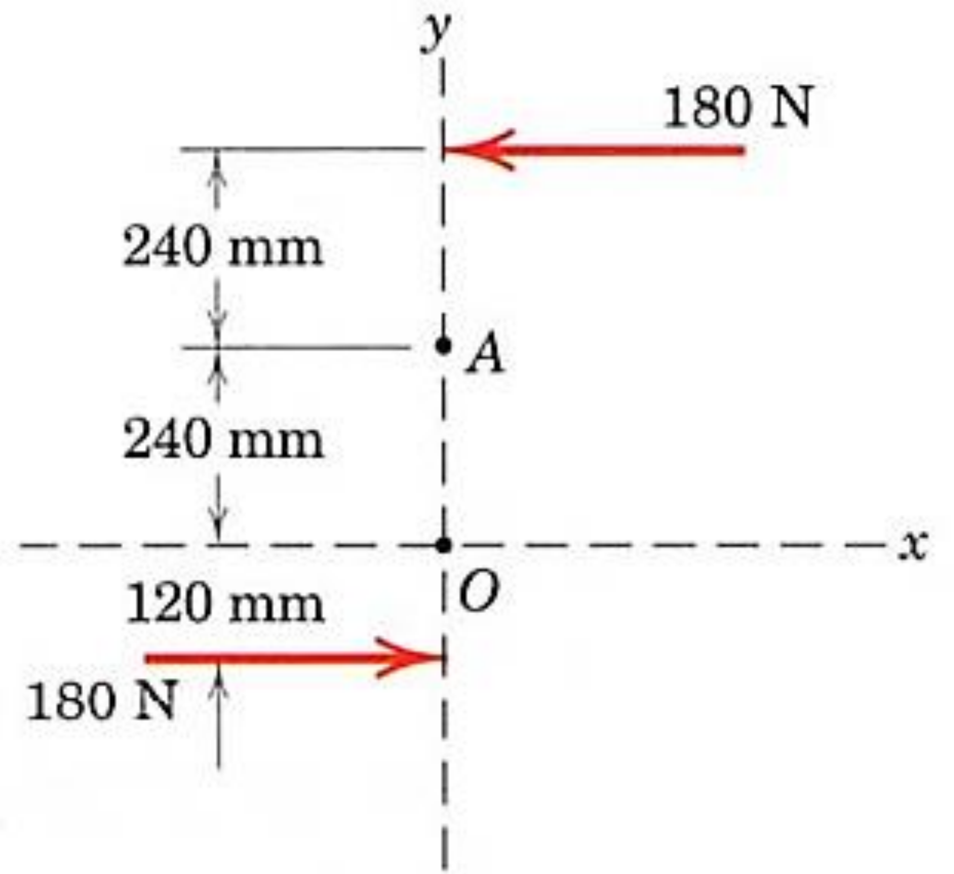
- ***Resultant Couple Moment.***

$$\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$$



Example 1:

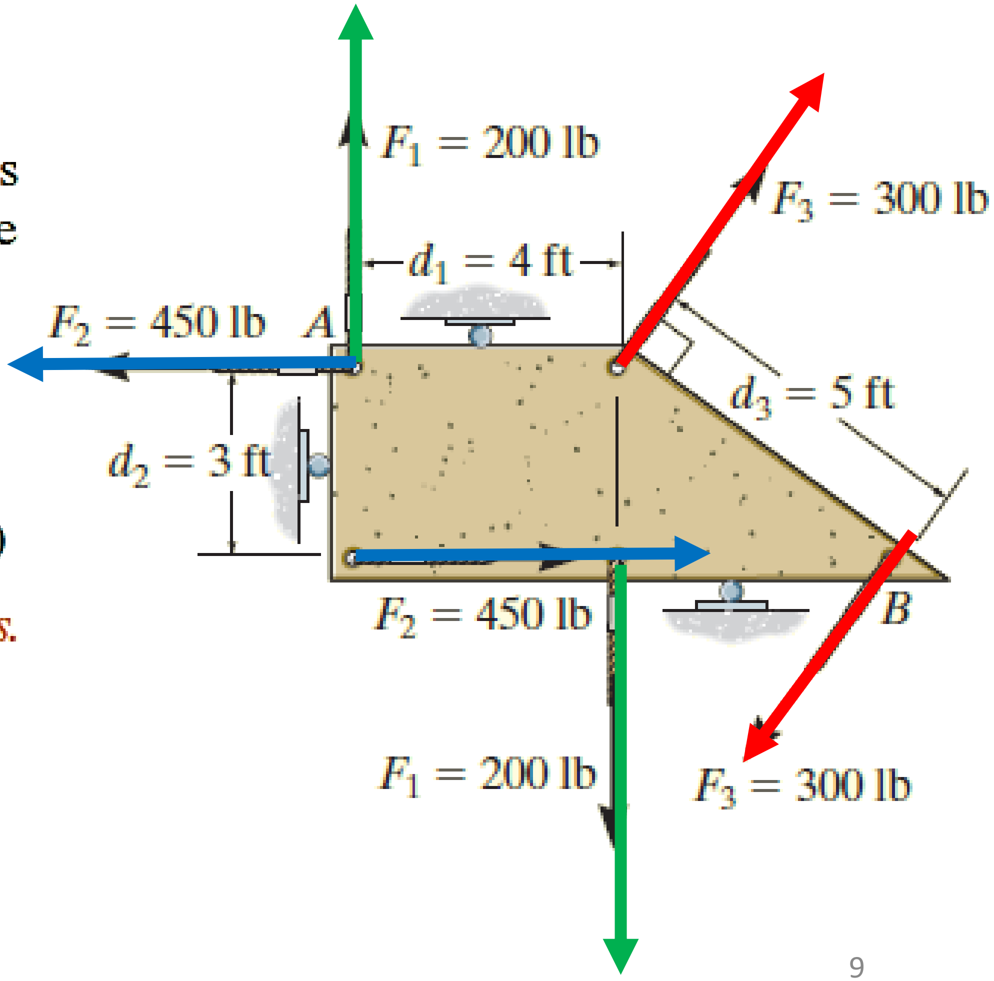
Compute the combined moment of the two 180 N forces about (a) point O and (b) point A.



$$\circlearrowleft + \; M = M_o = M_A = Fd = 180\,(0.24 + 0.24 + 0.12)$$
$$= 108 \; N.m \;\; CCW$$

Example 2:

Determine the resultant couple moment of the three couples acting on the plate in the Figure shown.



SOLUTION

As shown the perpendicular distances between each pair of couple forces are $d_1 = 4$ ft, $d_2 = 3$ ft, and $d_3 = 5$ ft. Considering counterclockwise couple moments as positive, we have

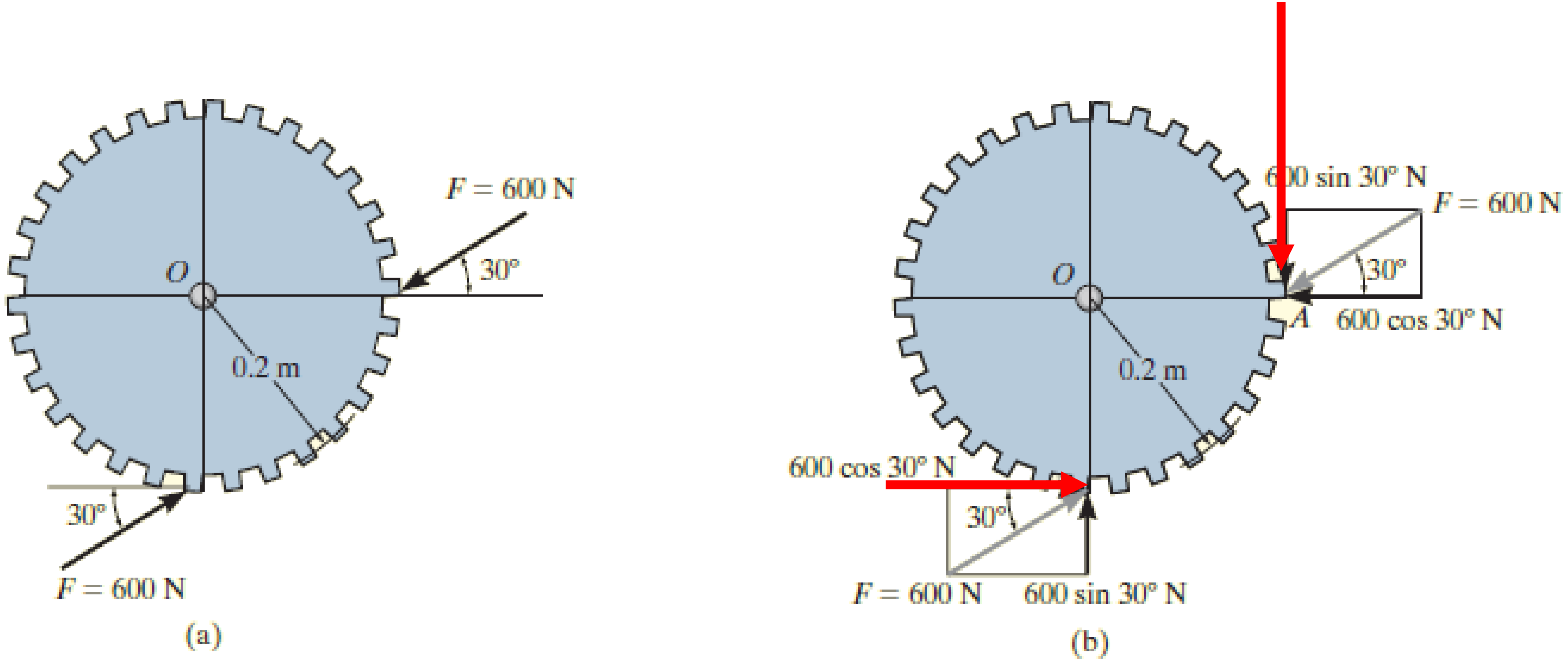
$$\circlearrowleft + M_R = \Sigma M; \quad M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

$$= -(200 \text{ lb})(4 \text{ ft}) + (450 \text{ lb})(3 \text{ ft}) - (300 \text{ lb})(5 \text{ ft})$$

$$= -950 \text{ lb} \cdot \text{ft} = 950 \text{ lb} \cdot \text{ft} \circlearrowright \qquad \textit{Ans.}$$

The negative sign indicates that M_R has a clockwise rotational sense.

Example 3: Determine the magnitude and direction of the couple moment acting on the gear in Figure shown.



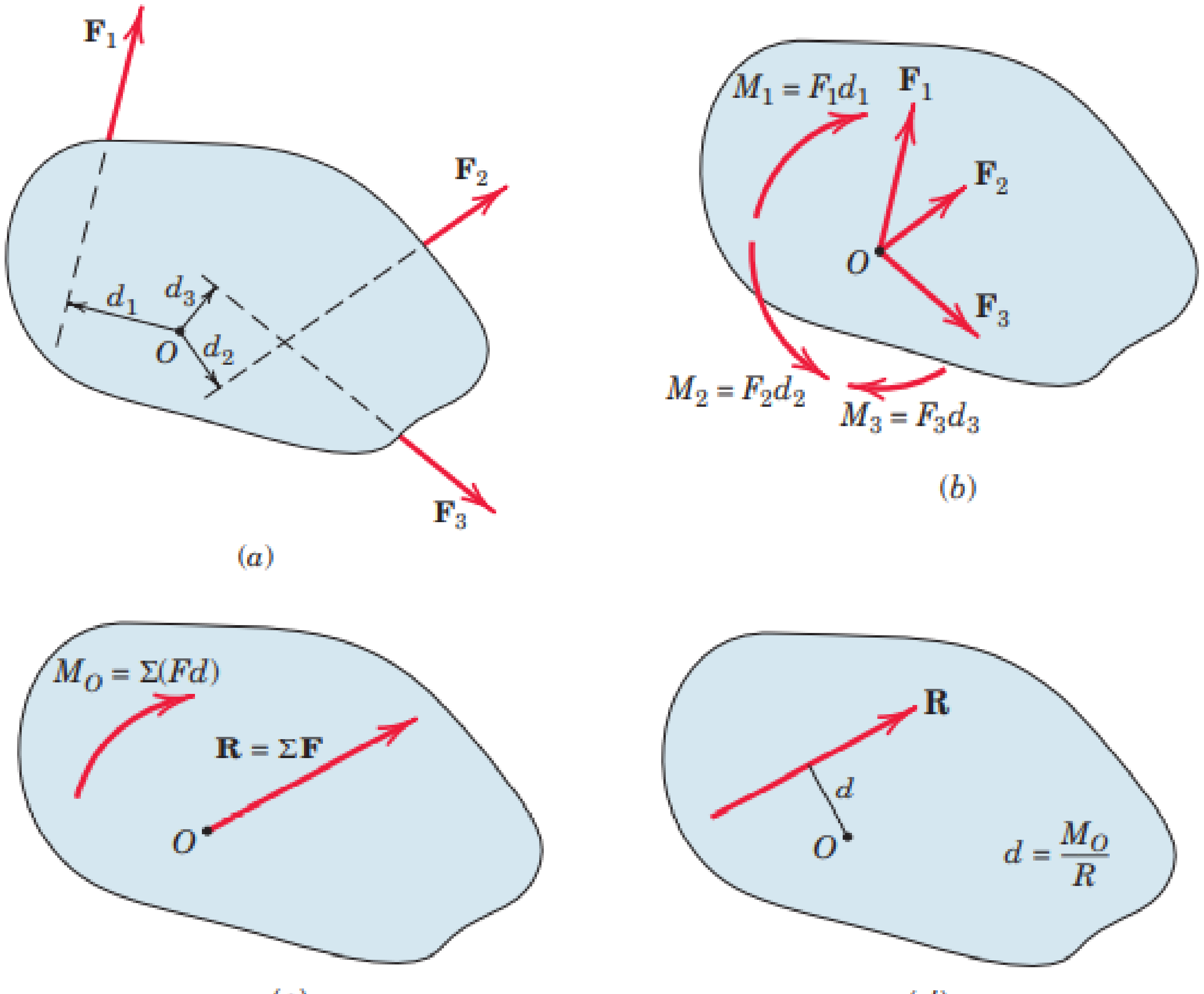
SOLUTION

$$\circlearrowleft + M = \Sigma M_O;\ M = (600 \cos 30^\circ \text{ N})(0.2 \text{ m}) - (600 \sin 30^\circ \text{ N})(0.2 \text{ m})$$

$$= 43.9 \text{ N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$

• Force–Couple Systems:

Resultant of a Force -Couple System (Simplification of a Force -Couple System)

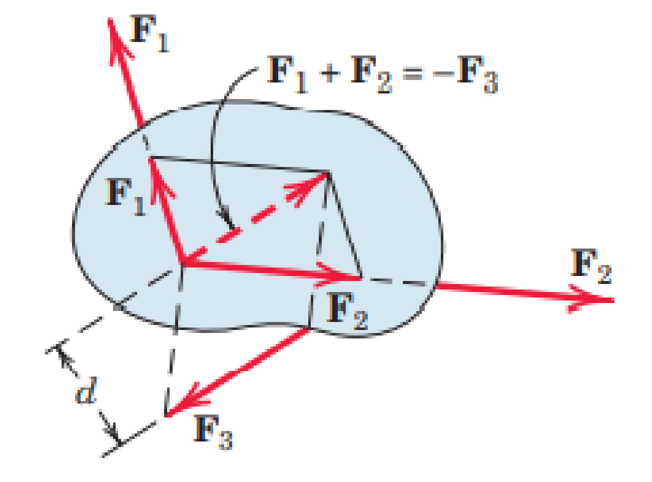


(a) (b) (c) (d)

$$\mathbf{R} = \Sigma \mathbf{F}$$

$$M_O = \Sigma M = \Sigma(Fd)$$

$$Rd = M_O$$



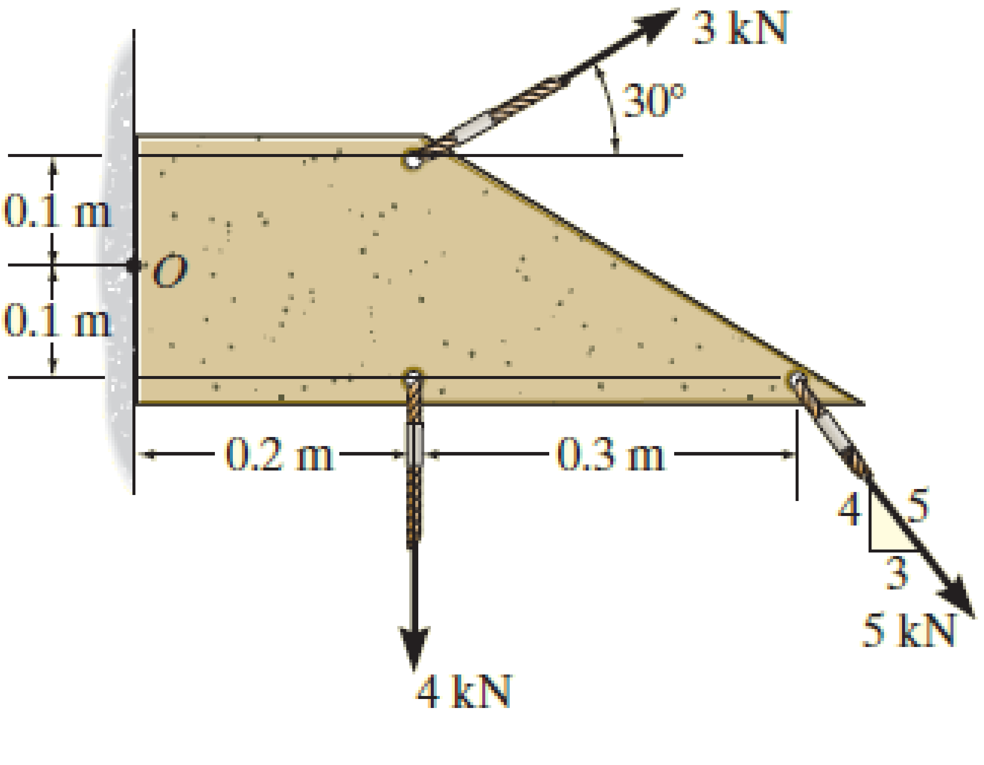
$$M = F_3 d$$

Force-couple system

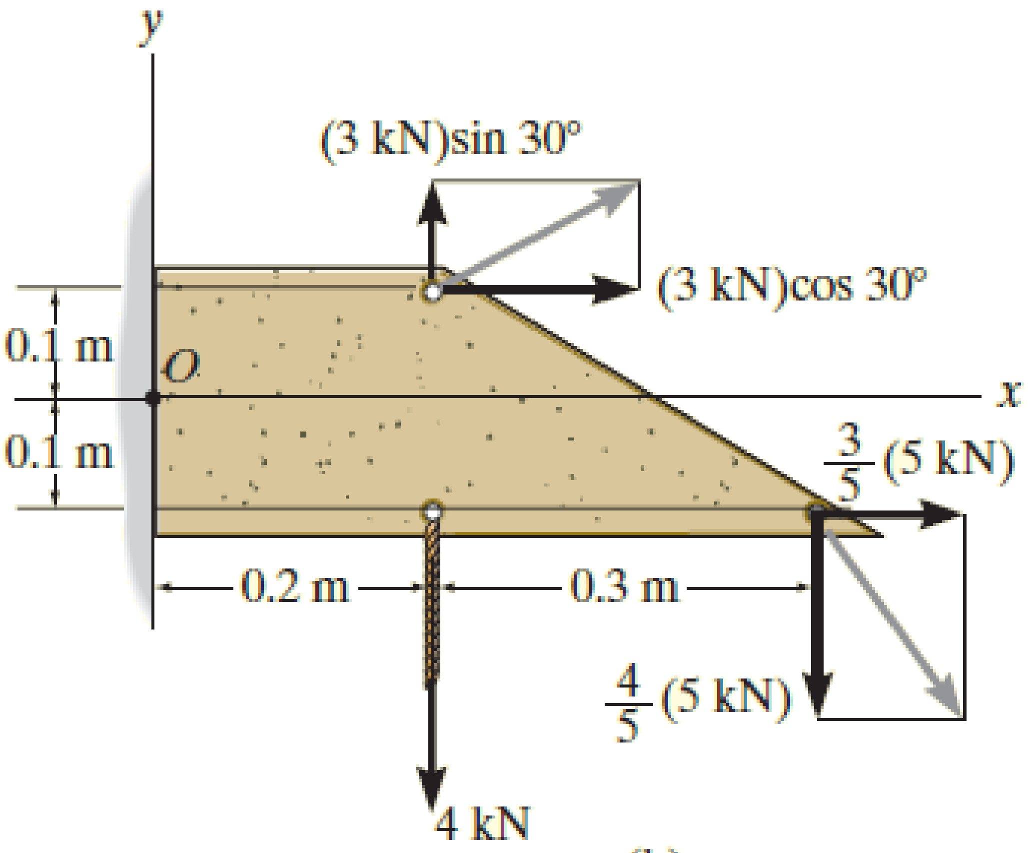
(a) (b) (c)

(a) (b) (c)

Example 5: Replace the force and couple system shown in Fig. *a* by an equivalent resultant force and couple moment acting at point *O*.



(a)



(b)

SOLUTION

Force Summation. The 3 kN and 5 kN forces are resolved into their x and y components as shown in Fig. b. We have

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = (3\text{ kN})\cos 30^\circ + \left(\tfrac{3}{5}\right)(5\text{ kN}) = 5.598\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = (3\text{ kN})\sin 30^\circ - \left(\tfrac{4}{5}\right)(5\text{ kN}) - 4\text{ kN} = -6.50\text{ kN} = 6.50\text{ kN}\downarrow$$

Using the Pythagorean theorem, Fig. 4–37c, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{(5.598\text{ kN})^2 + (6.50\text{ kN})^2} = 8.58\text{ kN} \qquad \textit{Ans.}$$

Its direction θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50\text{ kN}}{5.598\text{ kN}}\right) = 49.3^\circ \qquad \textit{Ans.}$$

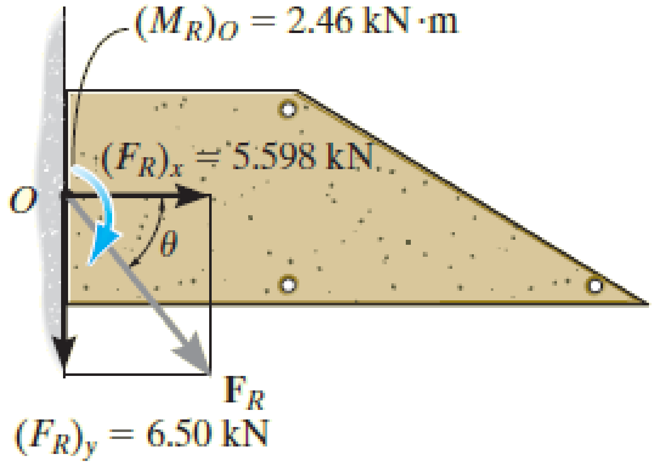
Moment Summation. The moments of 3 kN and 5 kN about point O will be determined using their x and y components. Referring to Fig. b, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O;$$

$$(M_R)_O = (3\text{ kN})\sin 30^\circ(0.2\text{ m}) - (3\text{ kN})\cos 30^\circ(0.1\text{ m}) + \left(\tfrac{3}{5}\right)(5\text{ kN})(0.1\text{ m}) - \left(\tfrac{4}{5}\right)(5\text{ kN})(0.5\text{ m}) - (4\text{ kN})(0.2\text{ m})$$

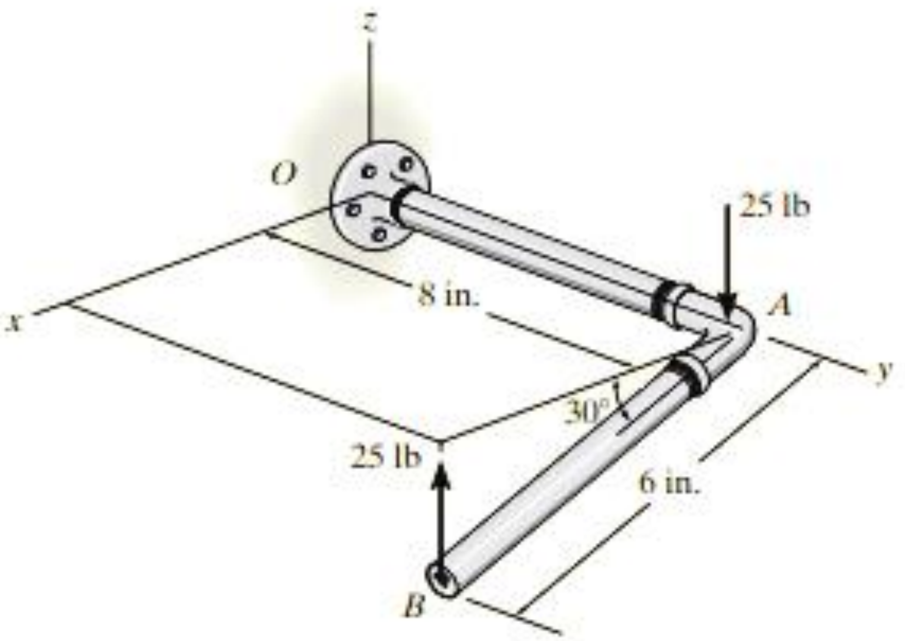
$$= -2.46\text{ kN}\cdot\text{m} = 2.46\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

This clockwise moment is shown in Fig. c.



(c)

Example 6: Determine the couple moment acting on the pipe shown in Fig.a . Segment AB is directed 30° below the $x\!-\!y$ plane.



(a)

Solution:

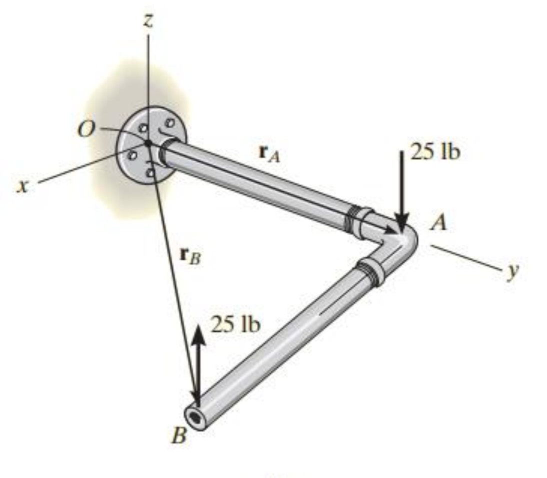
SOLUTION I *(VECTOR ANALYSIS)*

The moment of the two couple forces can be found about *any point*. If point O is considered, Fig. *b*, we have

$$\mathbf{M} = \mathbf{r}_A \times (-25\mathbf{k}) + \mathbf{r}_B \times (25\mathbf{k})$$
$$= (8\mathbf{j}) \times (-25\mathbf{k}) + (6\cos 30^\circ\mathbf{i} + 8\mathbf{j} - 6\sin 30^\circ\mathbf{k}) \times (25\mathbf{k})$$
$$= -200\mathbf{i} - 129.9\mathbf{j} + 200\mathbf{i}$$
$$= \{-130\mathbf{j}\}\ \text{lb}\cdot\text{in.} \qquad \textit{Ans.}$$

It is *easier* to take moments of the couple forces about a point lying on the line of action of one of the forces, e.g., point A, Fig. *c*. In this case the moment of the force at A is zero, so that

$$\mathbf{M} = \mathbf{r}_{AB} \times (25\mathbf{k})$$
$$= (6\cos 30^\circ\mathbf{i} - 6\sin 30^\circ\mathbf{k}) \times (25\mathbf{k})$$
$$= \{-130\mathbf{j}\}\ \text{lb}\cdot\text{in.} \qquad \textit{Ans.}$$



(b)

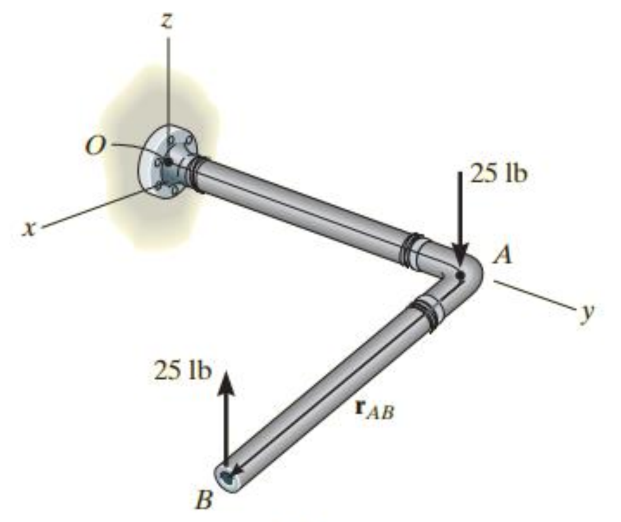
SOLUTION II *(SCALAR ANALYSIS)*

Although this problem is shown in three dimensions, the geometry is simple enough to use the scalar equation $M = Fd$. The perpendicular distance between the lines of action of the couple forces is $d = 6\cos 30^\circ = 5.196$ in., Fig. *d*. Hence, taking moments of the forces about either point A or point B yields

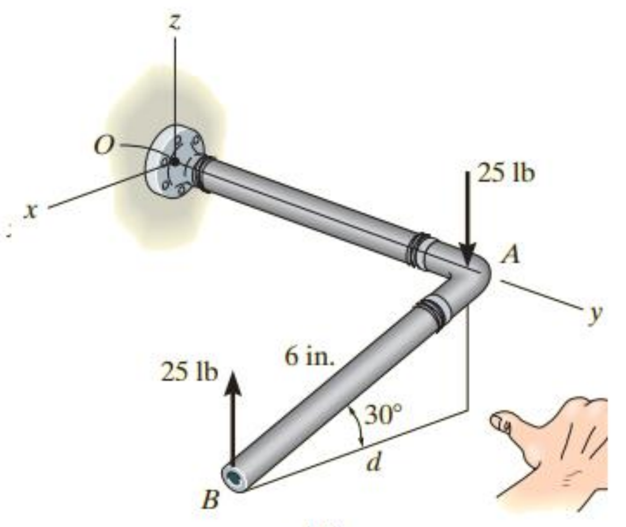
$$M = Fd = 25\ \text{lb}\ (5.196\ \text{in.}) = 129.9\ \text{lb}\cdot\text{in.}$$

Applying the right-hand rule, $\mathbf{M}$ acts in the $-\mathbf{j}$ direction. Thus,

$$\mathbf{M} = \{-130\mathbf{j}\}\ \text{lb}\cdot\text{in.} \qquad \textit{Ans.}$$

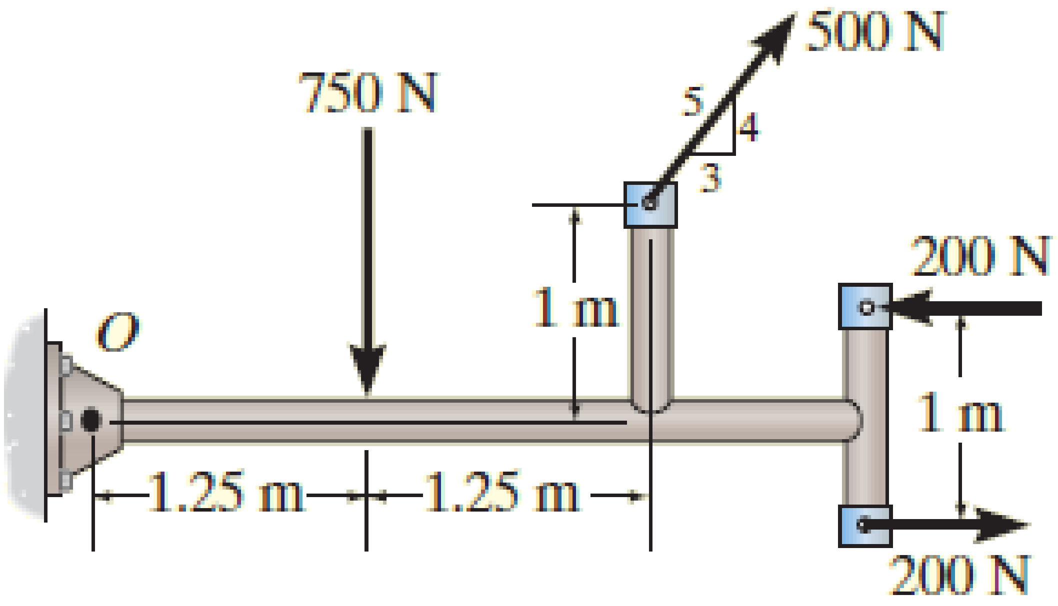


(c)



(d)

Example 7: Replace the force and couple system acting on the member in Fig. *a* by an equivalent resultant force and couple moment acting at point *O*.



(a)

SOLUTION

Force Summation.

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \; (F_R)_x = \left(\tfrac{3}{5}\right)(500 \text{ N}) = 300 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \; (F_R)_y = (500 \text{ N})\left(\tfrac{4}{5}\right) - 750 \text{ N} = -350 \text{ N} = 350 \text{ N}\downarrow$$

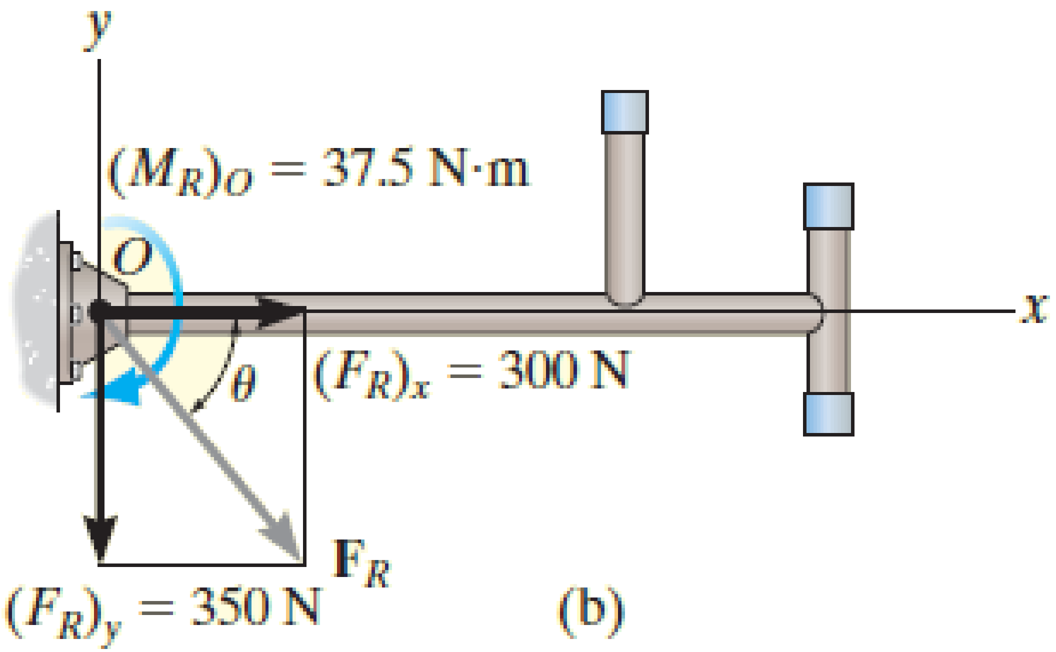
From Fig. 4–15*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(300 \text{ N})^2 + (350 \text{ N})^2} = 461 \text{ N}$$

And the angle θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350 \text{ N}}{300 \text{ N}}\right) = 49.4^\circ$$



Moment Summation

$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

$$(M_R)_O = (500 \text{ N})\left(\tfrac{4}{5}\right)(2.5 \text{ m}) - (500 \text{ N})\left(\tfrac{3}{5}\right)(1 \text{ m})$$

$$- (750 \text{ N})(1.25 \text{ m}) + 200 \text{ N}\cdot\text{m}$$

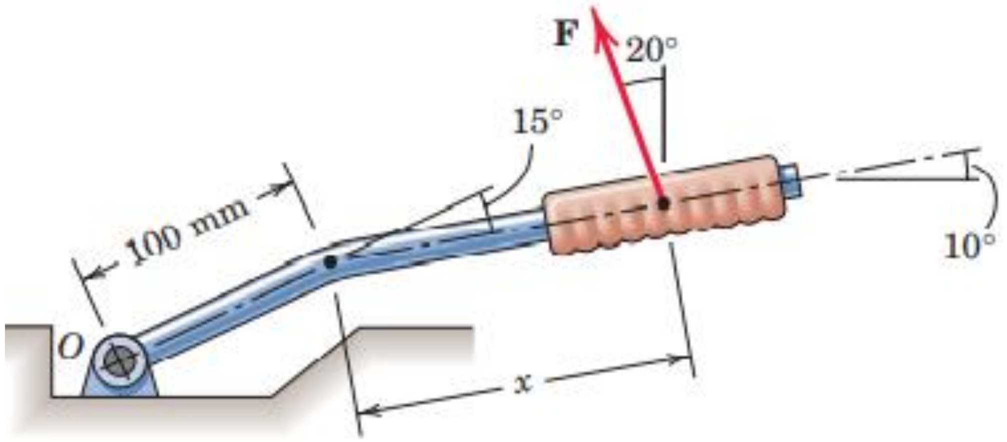
$$= -37.5 \text{ N}\cdot\text{m} = 37.5 \text{ N}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

This clockwise moment is shown in Fig. *b*.

Assignment 1

(Solve this problems then submit your answer)

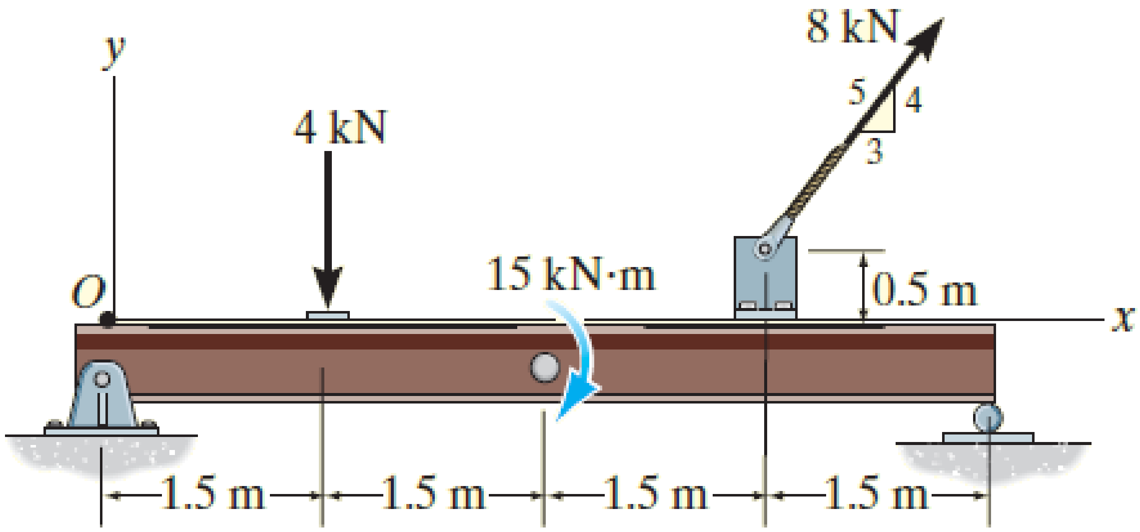
A force F of magnitude 50N is exerted on the automobile parking-brake lever at the position $x = 250\ m$. Replace the force by an equivalent force-couple system at the pivot point O.



Assignment 2

(Solve this problems then submit your answer)

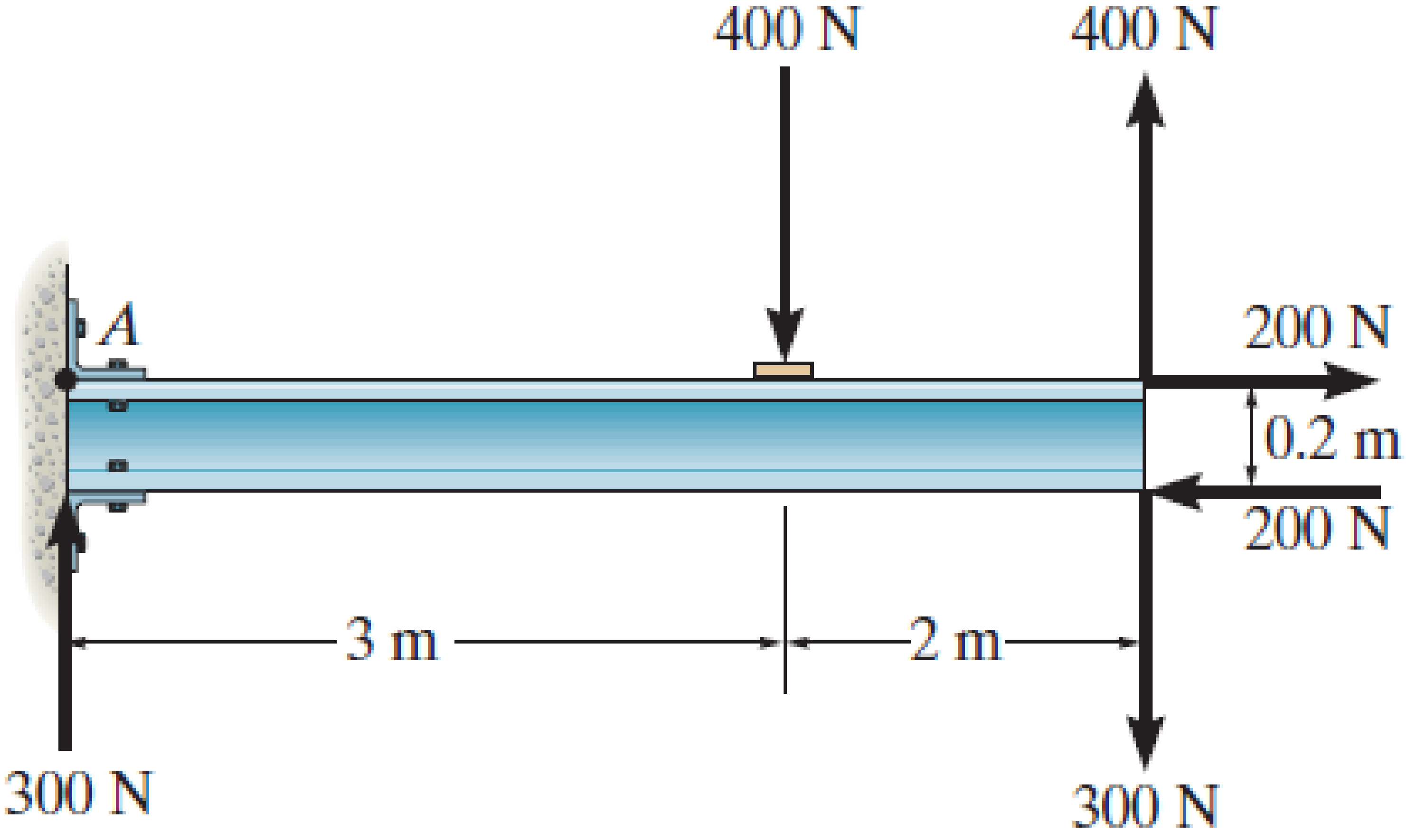
Replace the force and couple moment system acting on the beam in Figure shown by an equivalent resultant force, and find where its line of action intersects the beam, measured from point O.



Assignment 3

(Solve this problems then submit your answer)

Determine the resultant couple moment acting on the beam.



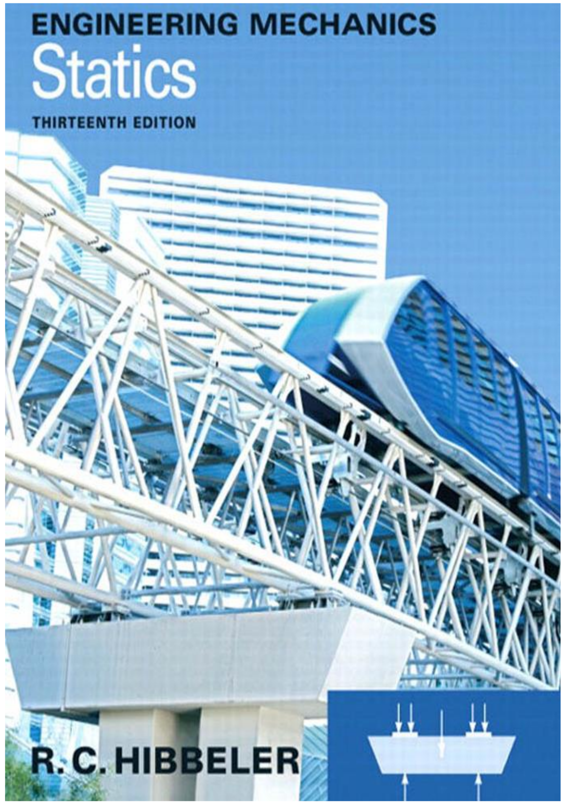
Next Lecture:

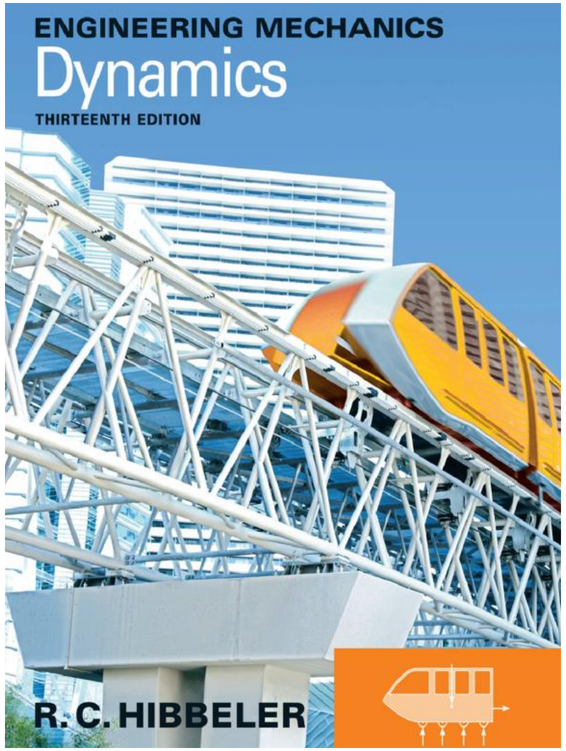
- Equilibrium of a particle

- Equilibrium of a rigid body

References:

Engineering Mechanics R.C. Hibbeler 13th edition (Statics and Dynamics).





The end of the lecture
Enjoy your time