



Aviation Department

First Grade- Spring Semester

Statics- Equilibrium (Lecture 7)

Lecturer: Ms. Jwan Khaleel M.

Lecture Content

- Equilibrium of a Particle
- Equilibrium of a Rigid body

- **Learning Outcomes:**

At the end of this lecture, you will be able to

Analyzing the forces acting on a body graphically (drawing free body diagram).

Prove that the system of forces applied of body is balance.

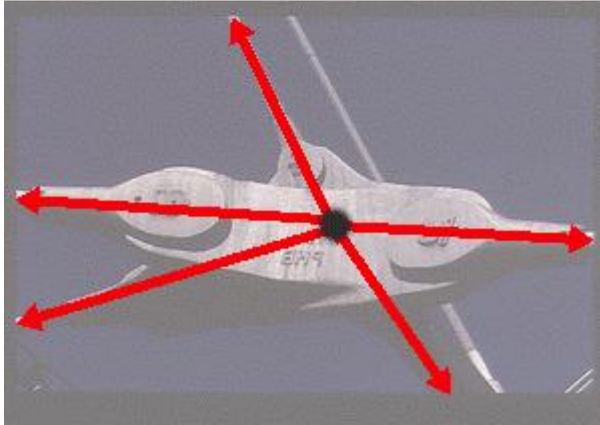
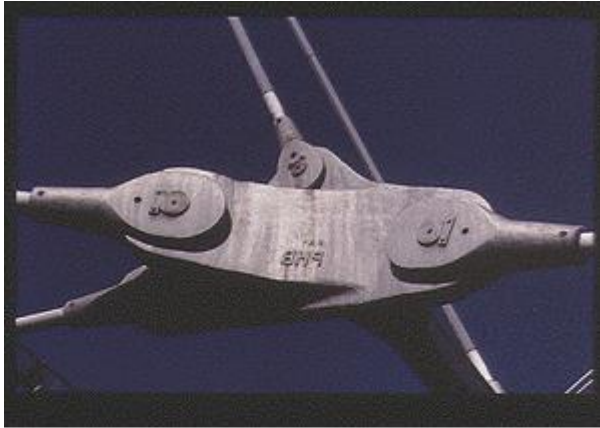
Evaluate that when the resultants are zero ($\Sigma F = 0$, $\Sigma M = 0$) the body is in complete equilibrium.

Develop the equations of equilibrium for a rigid body.

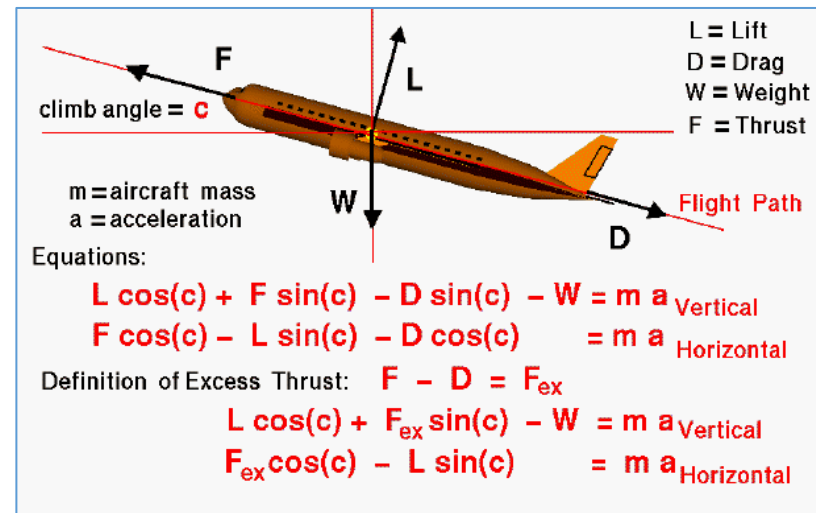
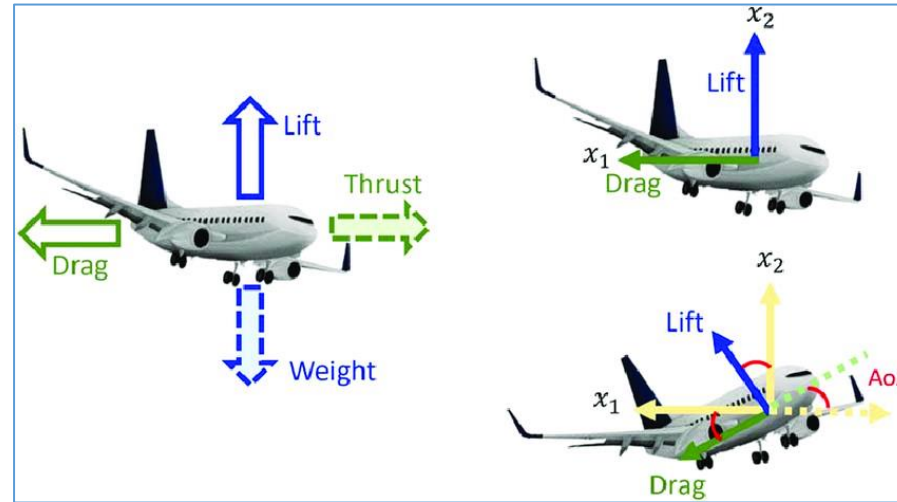
Explain the concept of the free-body diagram for a rigid body.

Solve equilibrium problems using the equations of equilibrium

• Condition for the Equilibrium of a Particle



To maintain equilibrium, it is *necessary* to satisfy Newton's first law of motion, which requires the *resultant force* acting on a particle to be equal to *zero*



$$\mathbf{R} = \Sigma \mathbf{F} = \mathbf{0}$$

$$\mathbf{M} = \Sigma \mathbf{M} = \mathbf{0}$$

- *Resultant vs. Equilibrium*
- The **resultant** is the properties of force, moment, and couple
- The resultant of a system of forces is the simplest force combination can replace the original forces without altering the external effect on the rigid body to which the forces are applied.
- **Equilibrium** of a body is the condition in which the resultant of all forces acting on the body is zero. When the body is in rest, the acceleration is **zero** which is the difference between the study of statics with dynamics.

Free-Body Diagram

➤ *A mechanical system is:*

Defined as a body or group of bodies which can be conceptually isolated from all other bodies.

A system may be a single body or a combination of connected bodies.

The bodies may be rigid or nonrigid.

➤ **To apply the equation of equilibrium:**

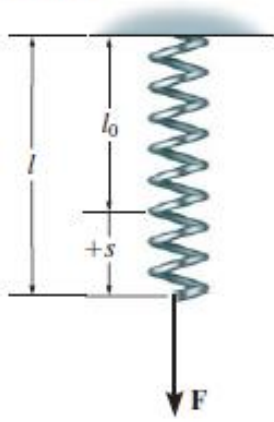
Account for *all* the **known and unknown forces** (F) which act *on* the particle.

Think of the particle as isolated and “free” from its surroundings.

A drawing that shows the particle with *all* the forces that act on it is called a *free-body diagram (FBD)*.

➤ **Free-body diagram**, which is a diagrammatic representation of the isolated system treated as a single body.

Springs.



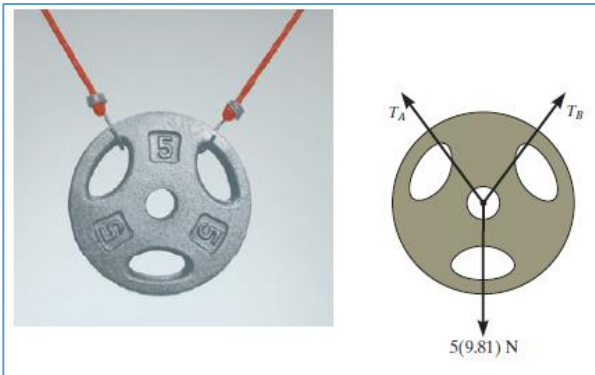
$$s = l - l_0$$

$$F = ks$$

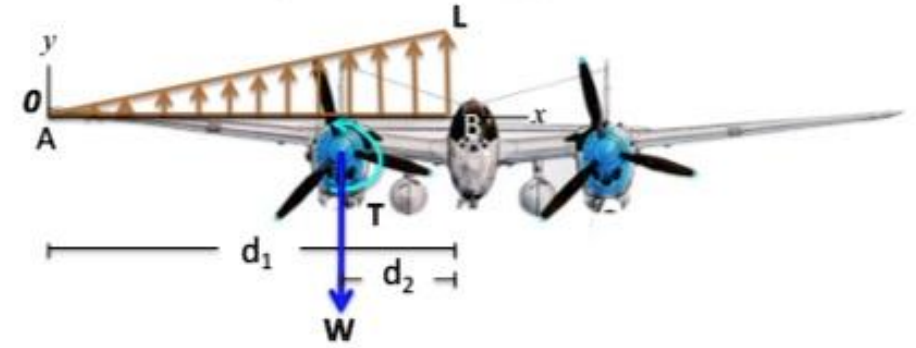
Cables and Pulleys.



Cable is in tension



Lift increases linearly from 0 at A to L at B



FBD

Lift increases linearly from 0 at A to L at B

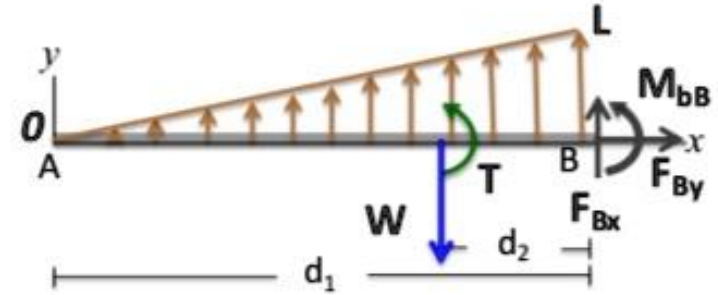


Figure 3 Loads on an airplane wing and its free body diagram

• Procedure for Drawing a Free-Body Diagram

To construct a free-body diagram, the following three steps are necessary.

- **Draw Outlined Shape.**
 - Imagine the particle to be *isolated* or cut “free” from its surroundings by drawing its outlined shape.
- **Show All Forces.**
 - Indicate on this sketch *all* the forces that act *on the particle* . These forces can be *active forces* , which tend to set the particle in motion, or they can be *reactive forces* which are the result of the constraints or supports that tend to prevent motion. To account for all these forces, it may be helpful to trace around the particle’s boundary, carefully noting each force acting on it.
- **Identify Each Force.**
 - The forces that are *known* should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and directions of forces that are unknown.

• *Procedure for Analysis*

Coplanar force equilibrium problems for a particle can be solved using the following procedure.

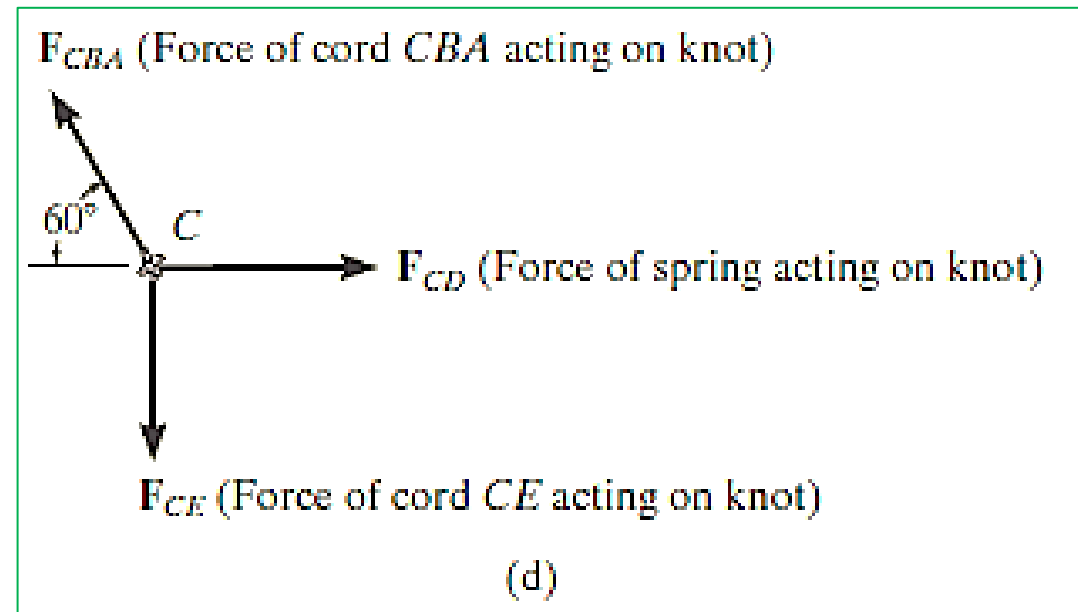
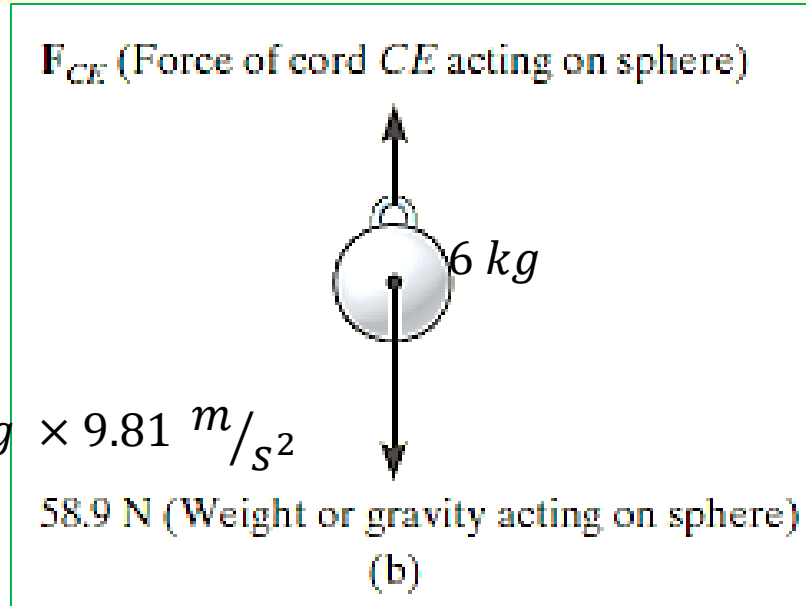
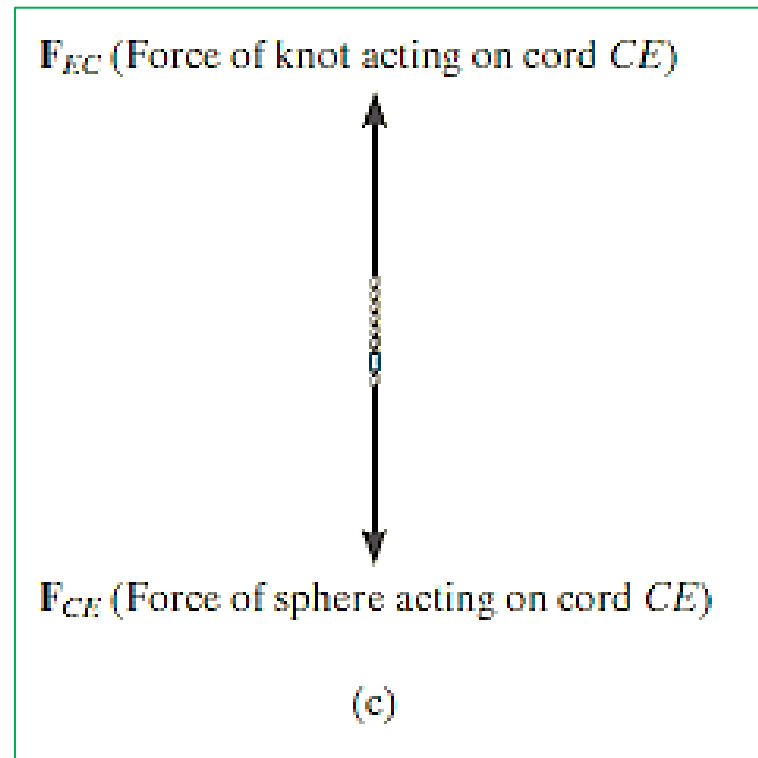
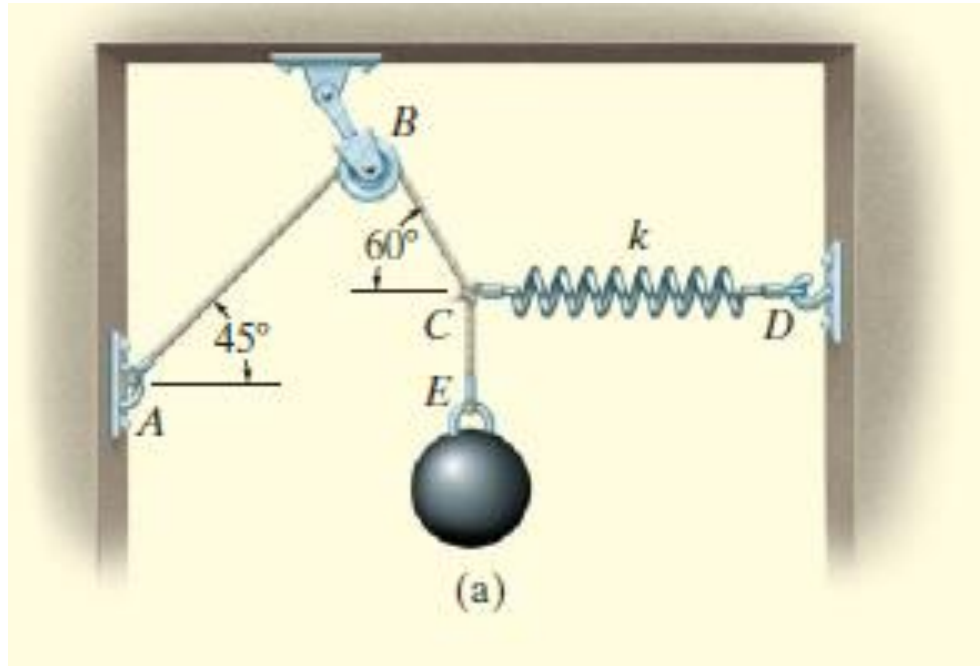
• **Free-Body Diagram.**

- Establish the x, y axes in any suitable orientation.
- Label all the known and unknown force magnitudes and directions on the diagram.
- The sense of a force having an unknown magnitude can be assumed.

• **Equations of Equilibrium.**

- Apply the equations of equilibrium, $\sum F_x = 0$ and $\sum F_y = 0$.
- Indicate the Components if they are directed along a positive axis, and negative
- If more than two unknowns exist and the problem involves a spring, apply $F = ks$ to relate the spring force to the deformation s of the spring.
- Since the magnitude of a force is always a positive quantity, then if the solution for a force yields a negative result, this indicates that its sense is the reverse of that shown on the free-body diagram.

Example// Draw the FBD of the sphere, the cord CE and the knot at C.



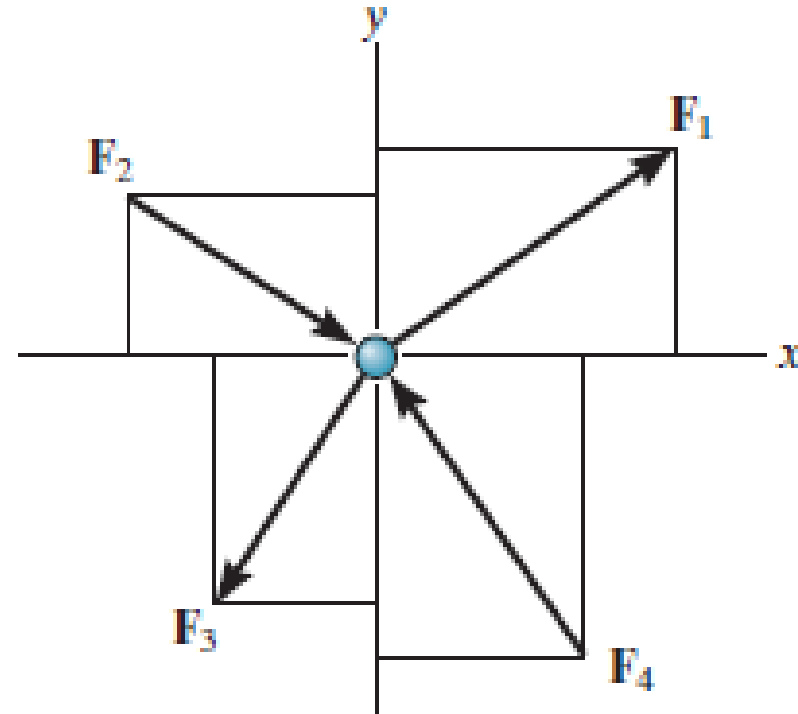
- *Coplanar Force Systems*

$$\Sigma \mathbf{F} = \mathbf{0}$$

$$\Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} = \mathbf{0}$$

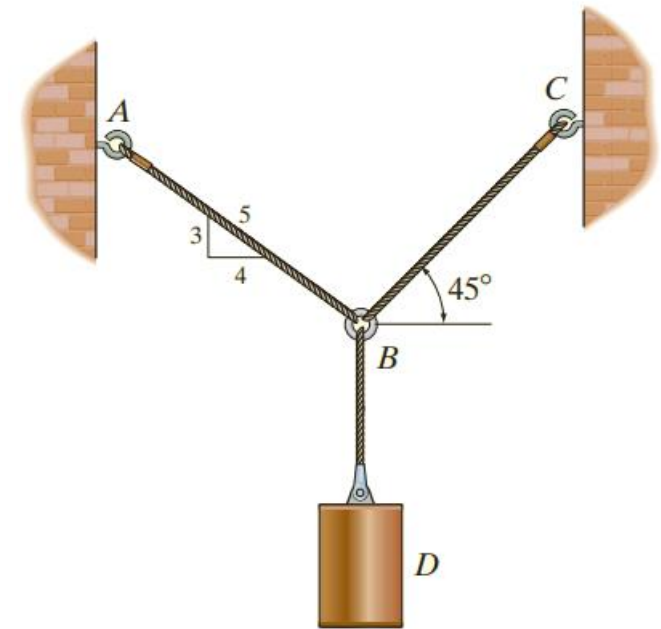
$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$



Example 1:

Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig. shown.



SOLUTION

Free-Body Diagram. Due to equilibrium, the weight of the cylinder causes the tension in cable BD to be $T_{BD} = 60(9.81) \text{ N}$, Fig. b.

The forces in cables BA and BC can be determined by investigating the equilibrium of ring B. Its free-body diagram is shown in Fig. c. The magnitudes of T_A and T_C are unknown, but their directions are known. Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes, we have

$$\rightarrow \Sigma F_x = 0; \quad T_C \cos 45^\circ - \left(\frac{4}{5}\right)T_A = 0 \quad (1)$$

$$+ \uparrow \Sigma F_y = 0; \quad T_C \sin 45^\circ + \left(\frac{3}{5}\right)T_A - 60(9.81) \text{ N} = 0 \quad (2)$$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

$$T_C \sin 45^\circ + \left(\frac{3}{5}\right)(0.8839T_C) - 60(9.81) \text{ N} = 0$$

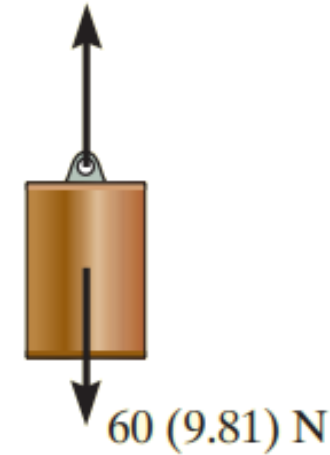
so that

$$T_C = 475.66 \text{ N} = 476 \text{ N} \quad \text{Ans.}$$

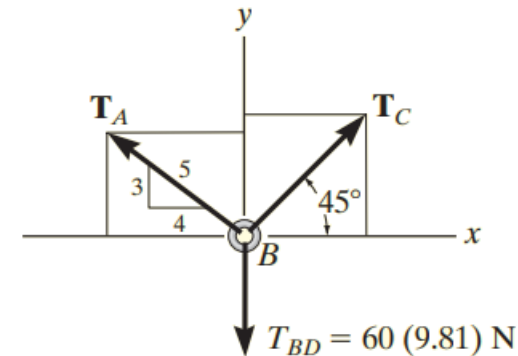
Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420 \text{ N} \quad \text{Ans.}$$

$$T_{BD} = 60(9.81) \text{ N}$$



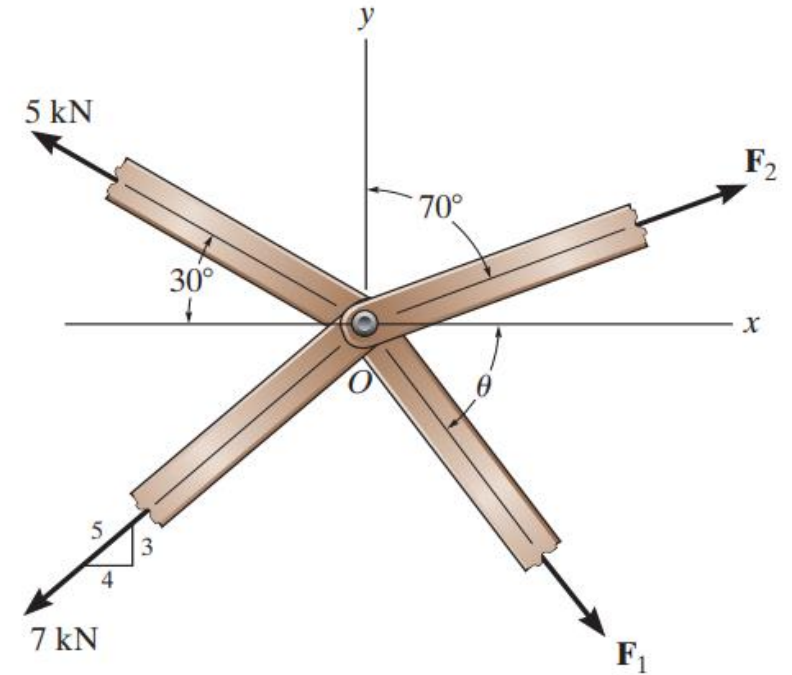
(b)



(c)

Example 2:

The members of a truss are pin connected at joint O. Determine the magnitudes of F_1 and F_2 for equilibrium. Set $\theta = 60^\circ$.



SOLUTION

Free-Body Diagram.

$$\pm \rightarrow \Sigma F_x = 0; \quad F_2 \sin 70^\circ + F_1 \cos 60^\circ - 5 \cos 30^\circ - \frac{4}{5}(7) = 0$$

$$0.9397F_2 + 0.5F_1 = 9.930$$

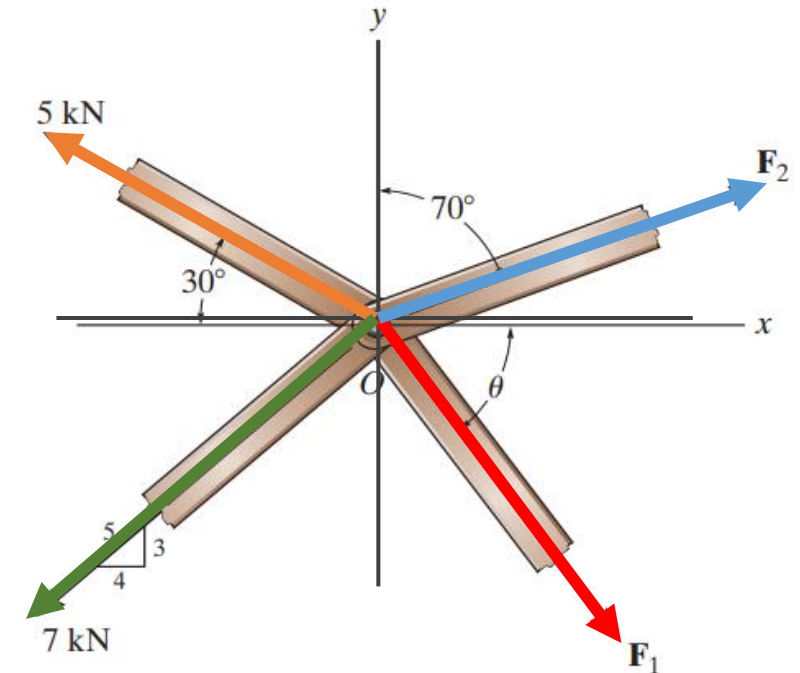
$$+\uparrow \Sigma F_y = 0; \quad F_2 \cos 70^\circ + 5 \sin 30^\circ - F_1 \sin 60^\circ - \frac{3}{5}(7) = 0$$

$$0.3420F_2 - 0.8660F_1 = 1.7$$

Solving:

$$F_2 = 9.60 \text{ kN}$$

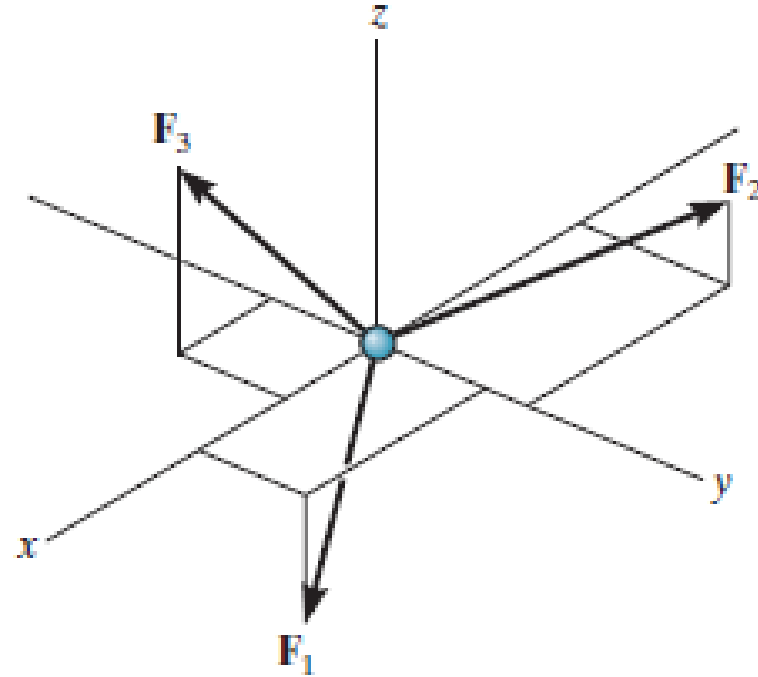
$$F_1 = 1.83 \text{ kN}$$



EQUILIBRIUM IN THREE DIMENSIONS

$$\Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = \mathbf{0}$$

$$\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma F_z &= 0 \end{aligned}$$



• Procedure for Analysis

Three-dimensional force equilibrium problems for a particle can be solved using the following procedure.

• Free-Body Diagram.

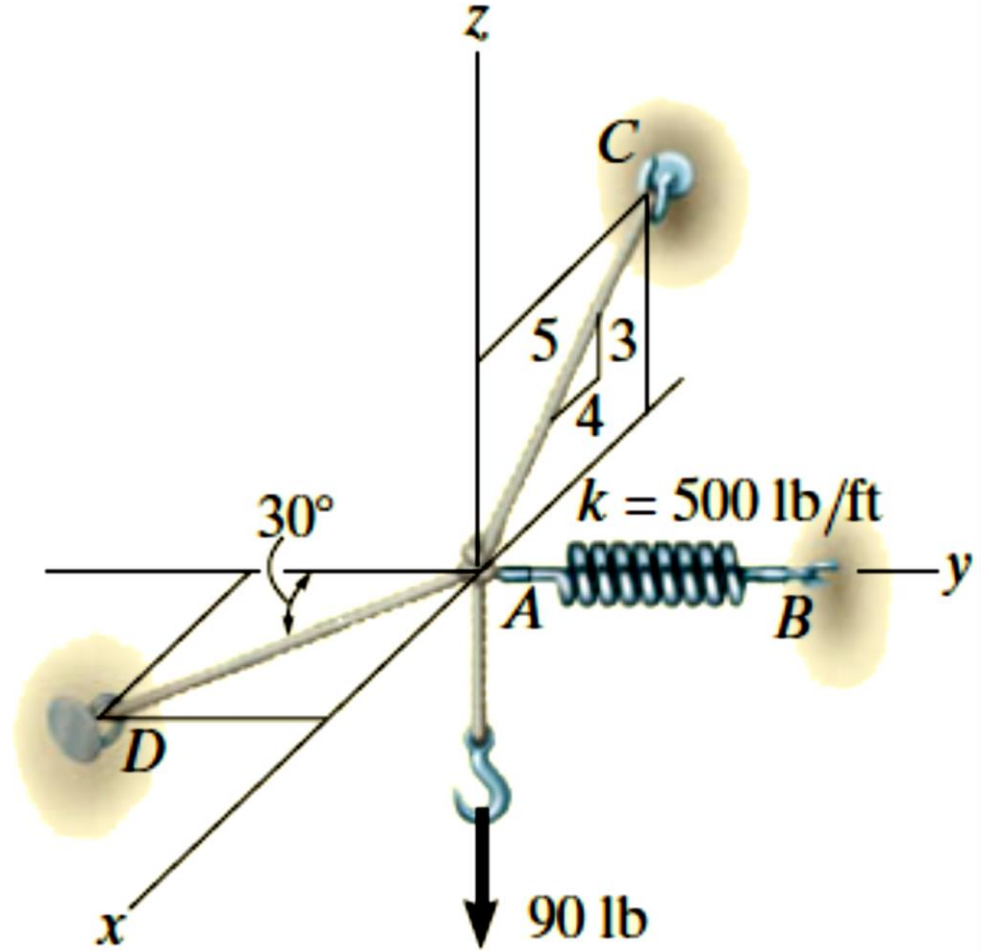
- Establish the x, y, z axes in any suitable orientation.
- Label all the known and unknown force magnitudes and directions on the diagram.
- The sense of a force having an unknown magnitude can be assumed.

• Equations of Equilibrium.

- Use the scalar equations of equilibrium, $F_x = 0, F_y = 0, F_z = 0$, in cases where it is easy to resolve each force into its x, y, z components.
- If the three-dimensional geometry appears difficult, then first express each force on the free-body diagram as a Cartesian vector, substitute these vectors into $\mathbf{F} = \mathbf{0}$, and then set the $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components equal to zero.
- If the solution for a force yields a negative result, this indicates that its sense is the reverse of that shown on the free-body diagram.

Example 2:

A 90-lb load is suspended from the hook shown in Fig. If the load is supported by two cables and a spring having a stiffness $k = 500 \text{ lb./ft}$, determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x - y plane and cable AC lies in the x - z plane.



Solution:

$$\Sigma F_x = 0; \quad F_D \sin 30^\circ - \left(\frac{4}{5}\right)F_C = 0 \quad (1)$$

$$\Sigma F_y = 0; \quad -F_D \cos 30^\circ + F_B = 0 \quad (2)$$

$$\Sigma F_z = 0; \quad \left(\frac{3}{5}\right)F_C - 90 \text{ lb} = 0 \quad (3)$$

Solving Eq. (3) for F_C , then Eq. (1) for F_D , and finally Eq. (2) for F_B , yields

$$F_C = 150 \text{ lb}$$

$$F_D = 240 \text{ lb}$$

$$F_B = 207.8 \text{ lb} = 208 \text{ lb}$$

The stretch of the spring is therefore

$$F_B = k s_{AB}$$

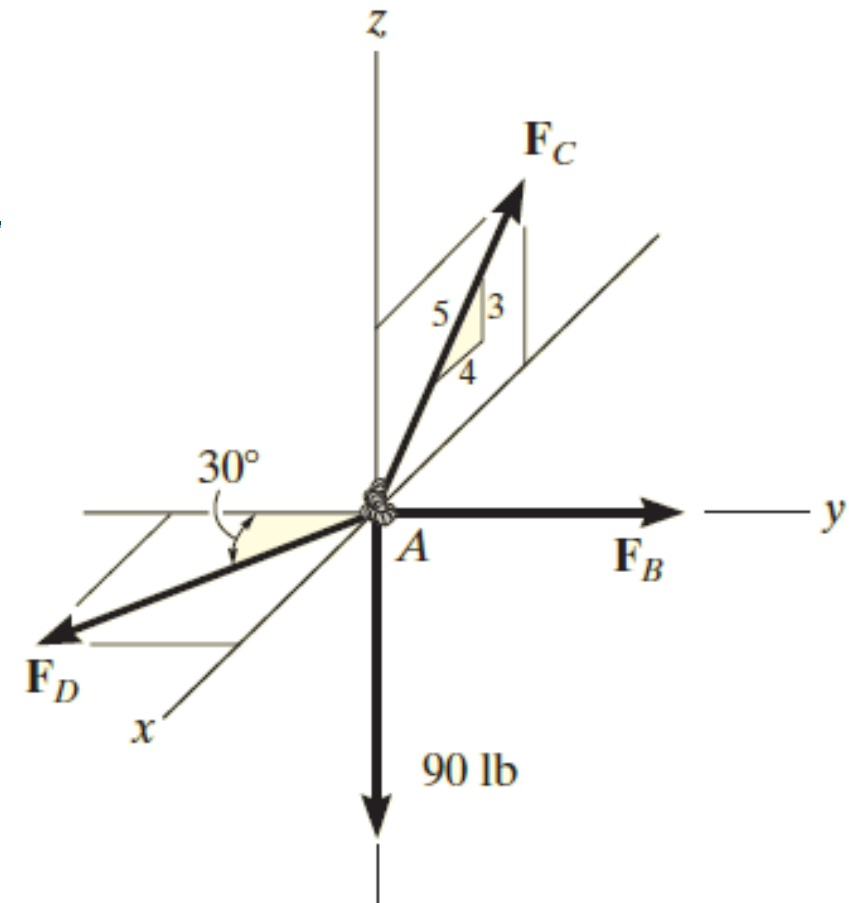
$$207.8 \text{ lb} = (500 \text{ lb/ft})(s_{AB})$$

$$s_{AB} = 0.416 \text{ ft}$$

Ans.

Ans.

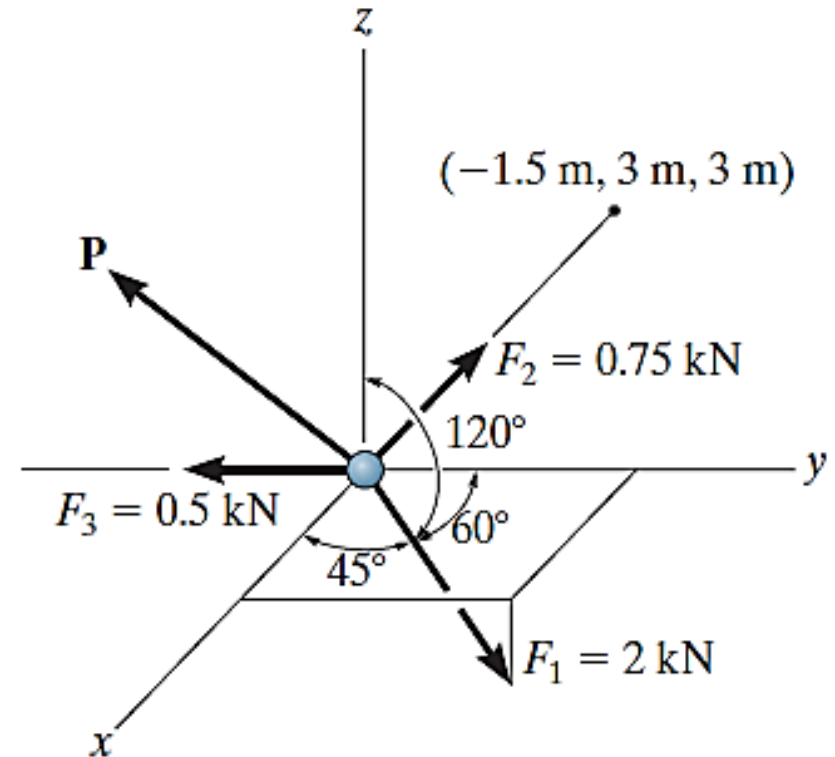
Ans.



Ans.

Example 3:

Determine the magnitude and direction of the force P required to keep the concurrent force system in equilibrium.



Solution

Cartesian Vector Notation:

$$\mathbf{F}_1 = 2\{\cos 45^\circ\mathbf{i} + \cos 60^\circ\mathbf{j} + \cos 120^\circ\mathbf{k}\} \text{ kN} = \{1.414\mathbf{i} + 1.00\mathbf{j} - 1.00\mathbf{k}\} \text{ kN}$$

$$\mathbf{F}_2 = 0.75\left(\frac{-1.5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}}{\sqrt{(-1.5)^2 + 3^2 + 3^2}}\right) = \{-0.250\mathbf{i} + 0.50\mathbf{j} + 0.50\mathbf{k}\} \text{ kN}$$

$$\mathbf{F}_3 = \{-0.50\mathbf{j}\} \text{ kN}$$

$$\mathbf{P} = P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}$$

Equations of Equilibrium:

$$\Sigma\mathbf{F} = \mathbf{0}; \quad \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{P} = \mathbf{0}$$

$$(P_x + 1.414 - 0.250)\mathbf{i} + (P_y + 1.00 + 0.50 - 0.50)\mathbf{j} + (P_z - 1.00 + 0.50)\mathbf{k} = \mathbf{0}$$

Equating \mathbf{i} , \mathbf{j} , and \mathbf{k} components, we have

$$P_x + 1.414 - 0.250 = 0 \quad P_x = -1.164 \text{ kN}$$

$$P_y + 1.00 + 0.50 - 0.50 = 0 \quad P_y = -1.00 \text{ kN}$$

$$P_z - 1.00 + 0.50 = 0 \quad P_z = 0.500 \text{ kN}$$

Solution cont.

The magnitude of \mathbf{P} is

$$\begin{aligned} P &= \sqrt{P_x^2 + P_y^2 + P_z^2} \\ &= \sqrt{(-1.164)^2 + (-1.00)^2 + (0.500)^2} \\ &= 1.614 \text{ kN} = 1.61 \text{ kN} \end{aligned} \quad \text{Ans.}$$

The coordinate direction angles are

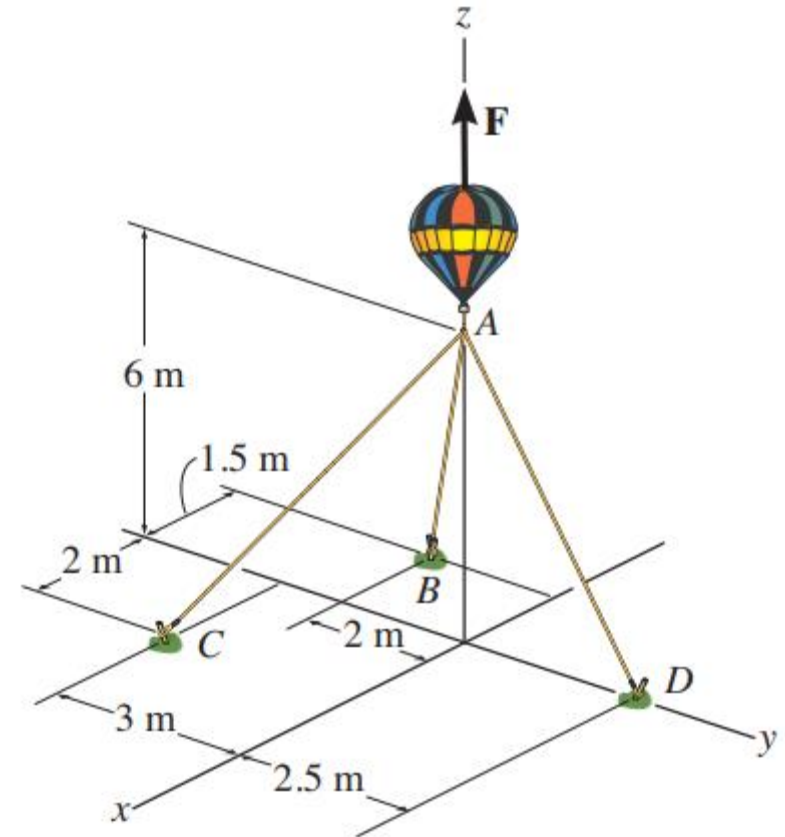
$$\alpha = \cos^{-1}\left(\frac{P_x}{P}\right) = \cos^{-1}\left(\frac{-1.164}{1.614}\right) = 136^\circ \quad \text{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{P_y}{P}\right) = \cos^{-1}\left(\frac{-1.00}{1.614}\right) = 128^\circ \quad \text{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{P_z}{P}\right) = \cos^{-1}\left(\frac{0.500}{1.614}\right) = 72.0^\circ \quad \text{Ans.}$$

Example 4:

If the balloon is subjected to a net uplift force of $F = 800 \text{ N}$, determine the tension developed in ropes AB, AC, AD.



SOLUTION

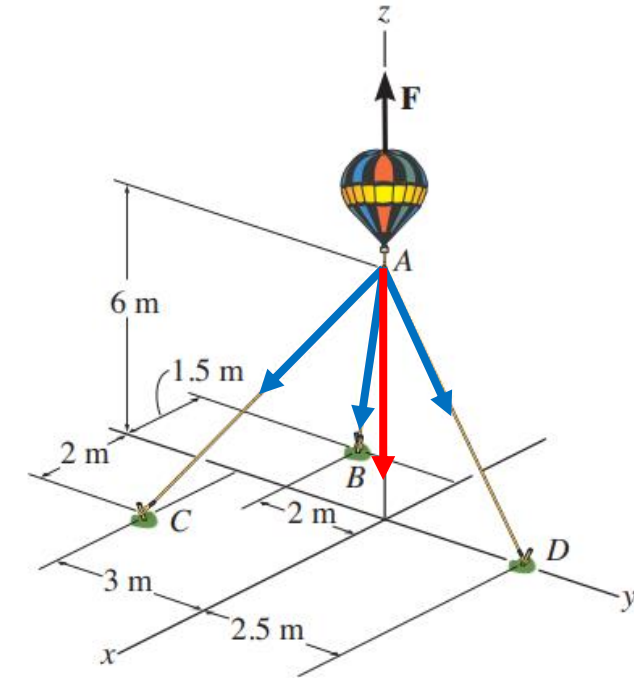
Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (a) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB} \left[\frac{(-1.5 - 0)\mathbf{i} + (-2 - 0)\mathbf{j} + (-6 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0)^2 + (-2 - 0)^2 + (-6 - 0)^2}} \right] = -\frac{3}{13} F_{AB} \mathbf{i} - \frac{4}{13} F_{AB} \mathbf{j} - \frac{12}{13} F_{AB} \mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC} \left[\frac{(2 - 0)\mathbf{i} + (-3 - 0)\mathbf{j} + (-6 - 0)\mathbf{k}}{\sqrt{(2 - 0)^2 + (-3 - 0)^2 + (-6 - 0)^2}} \right] = \frac{2}{7} F_{AC} \mathbf{i} - \frac{3}{7} F_{AC} \mathbf{j} - \frac{6}{7} F_{AC} \mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD} \left[\frac{(0 - 0)\mathbf{i} + (2.5 - 0)\mathbf{j} + (-6 - 0)\mathbf{k}}{\sqrt{(0 - 0)^2 + (2.5 - 0)^2 + (-6 - 0)^2}} \right] = \frac{5}{13} F_{AD} \mathbf{j} - \frac{12}{13} F_{AD} \mathbf{k}$$

$$\mathbf{W} = \{800\mathbf{k}\}\text{N}$$



Equations of Equilibrium: Equilibrium requires

$$\Sigma \mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$\left(-\frac{3}{13} F_{AB} \mathbf{i} - \frac{4}{13} F_{AB} \mathbf{j} - \frac{12}{13} F_{AB} \mathbf{k} \right) + \left(\frac{2}{7} F_{AC} \mathbf{i} - \frac{3}{7} F_{AC} \mathbf{j} - \frac{6}{7} F_{AC} \mathbf{k} \right) + \left(\frac{5}{13} F_{AD} \mathbf{j} - \frac{12}{13} F_{AD} \mathbf{k} \right) + 800 \mathbf{k} = 0$$

$$\left(-\frac{3}{13} F_{AB} + \frac{2}{7} F_{AC} \right) \mathbf{i} + \left(-\frac{4}{13} F_{AB} - \frac{3}{7} F_{AC} - \frac{5}{13} F_{AD} \right) \mathbf{j} + \left(-\frac{12}{13} F_{AB} - \frac{6}{7} F_{AC} - \frac{12}{13} F_{AD} + 800 \right) \mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{3}{13} F_{AB} + \frac{2}{7} F_{AC} = 0 \quad (1)$$

$$-\frac{4}{13} F_{AB} - \frac{3}{7} F_{AC} + \frac{5}{13} F_{AD} = 0 \quad (2)$$

$$-\frac{12}{13} F_{AB} - \frac{6}{7} F_{AC} - \frac{12}{13} F_{AD} + 800 = 0 \quad (3)$$

Solving Eqs. (1) through (3) yields

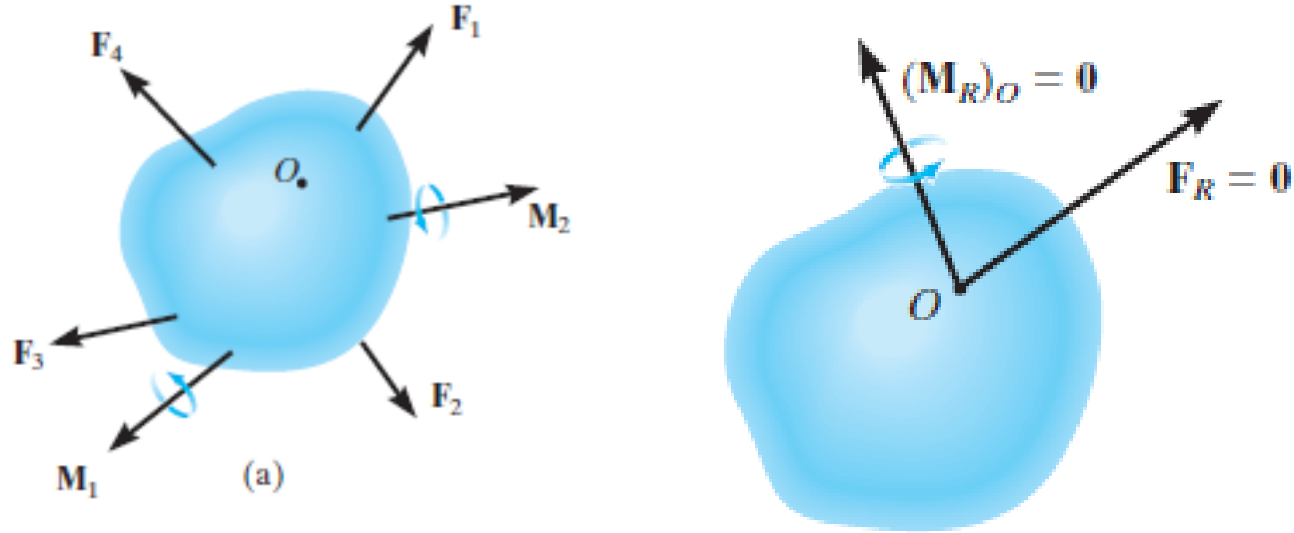
$$\mathbf{F}_{AC} = 203 \text{ N}$$

$$\mathbf{F}_{AB} = 251 \text{ N}$$

$$\mathbf{F}_{AD} = 427 \text{ N}$$

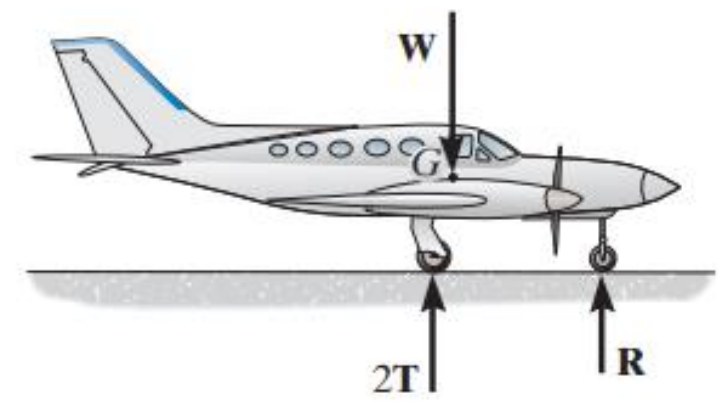
Equilibrium of a Rigid Body

- The Concept of Equilibrium of a Rigid Body***



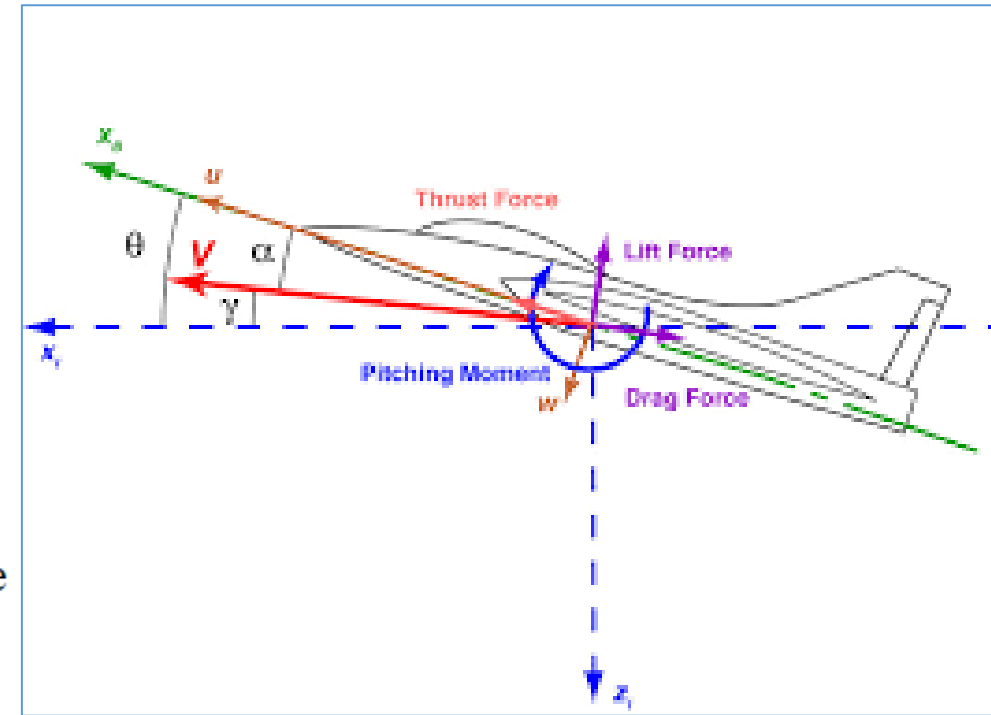
$$\mathbf{R} = \sum \mathbf{F} = \mathbf{0} \quad \mathbf{M} = \sum \mathbf{M} = \mathbf{0}$$

For example, the airplane in Fig. has a plane of symmetry through its center axis, and so the loads acting on the airplane are symmetrical with respect to this plane. Thus, each of the two wing tires will support the same load T , which is represented on the side (two-dimensional) view of the plane as $2T$



4 FORCES ACTING ON AIRPLANE

- Model airplane as rigid body with four natural forces acting on it
 1. **Lift, L**
 - Acts perpendicular to flight path (always perpendicular to relative wind)
 2. **Drag, D**
 - Acts parallel to flight path direction (parallel to *incoming* relative wind)
 3. **Propulsive Thrust, T**
 - For most airplanes propulsive thrust acts in flight path direction
 - May be inclined with respect to flight path angle, α_T , usually small angle
 4. **Weight, W**
 - Always acts vertically toward center of earth
 - Inclined at angle, θ , with respect to lift direction
- Apply Newton's Second Law ($\mathbf{F}=\mathbf{ma}$) to curvilinear flight path
 - Force balance in direction parallel to flight path
 - Force balance in direction perpendicular to flight path



Free Body Diagram

Support Reactions: Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider the various types of reactions that occur at supports and points of contact between bodies subjected to coplanar force systems. As a general rule, If a support prevents the translation of a body in a given direction, then a force is developed on the body in that direction.

- If rotation is prevented, a couple moment is exerted on the body.

Typical examples of actual supports are shown in the following sequence of photos. The numbers refer to the connection types in Table 5-1.



The cable exerts a force on the bracket in the direction of the cable. (1)

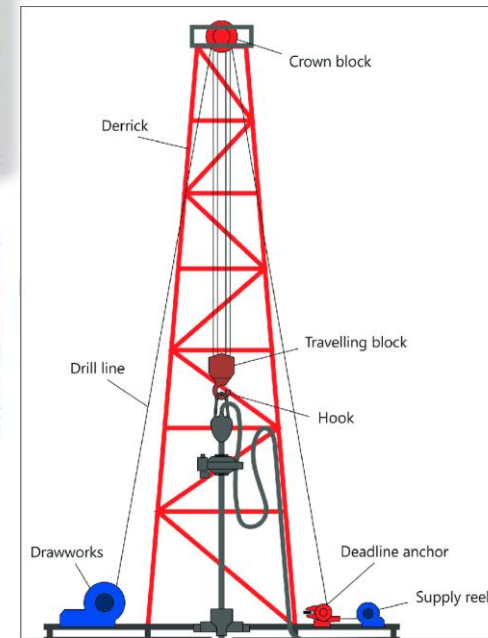


The rocker support for this bridge girder allows horizontal movement so the bridge is free to expand and contract due to a change in temperature. (4)

This concrete girder rests on the ledge that is assumed to act as a smooth contacting surface. (5)



This utility building is pin supported at the top of the column. (8)

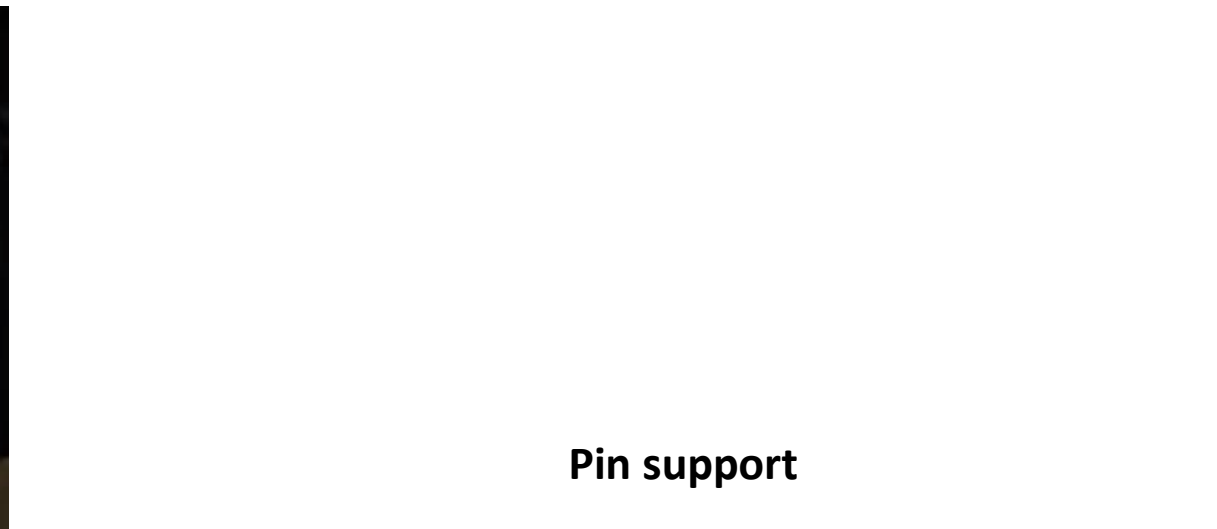


The floor beams of this building are welded together and thus form fixed connections. (10)





Roller support



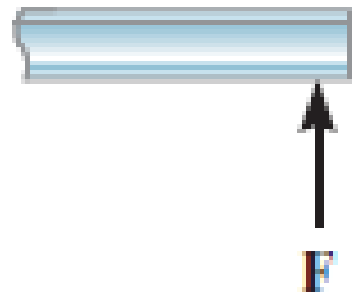
Pin support





roller

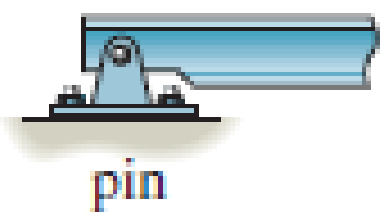
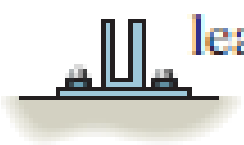
(a)



(b)

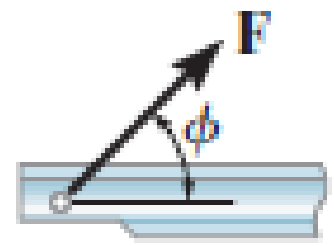
T member
pin

leaves



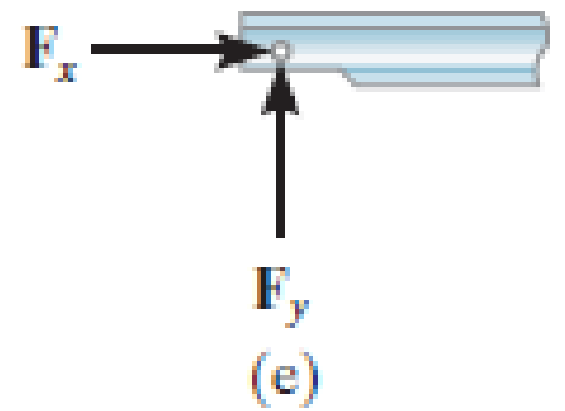
(c)

pin

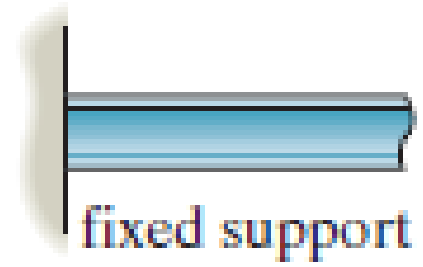


(d)

or

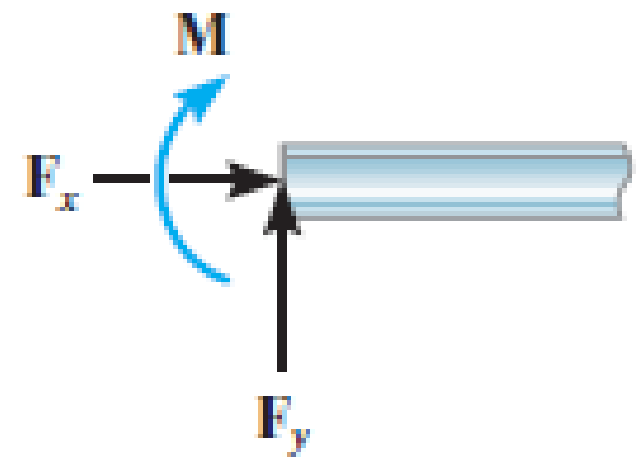


(e)



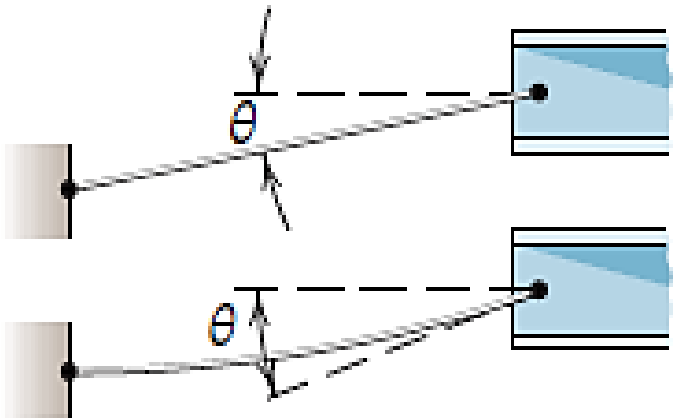
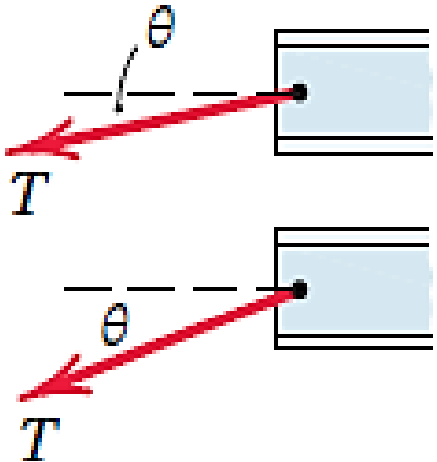
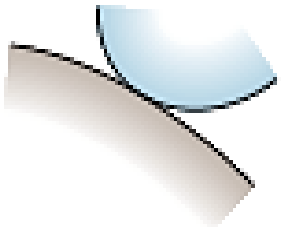
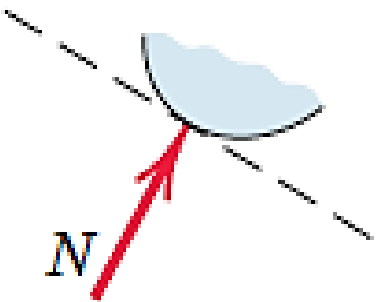
fixed support

(f)

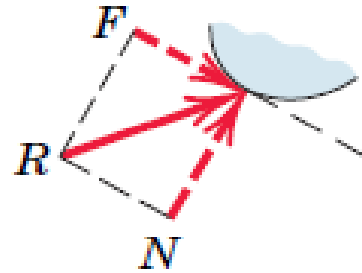


(g)

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS

Type of Contact and Force Origin	Action on Body to Be Isolated
<p>1. Flexible cable, belt, chain, or rope</p> <p>Weight of cable negligible</p> <p>Weight of cable not negligible</p> 	 <p>Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.</p>
<p>2. Smooth surfaces</p> 	 <p>Contact force is compressive and is normal to the surface.</p>

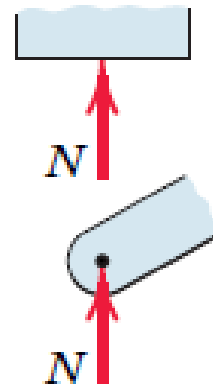
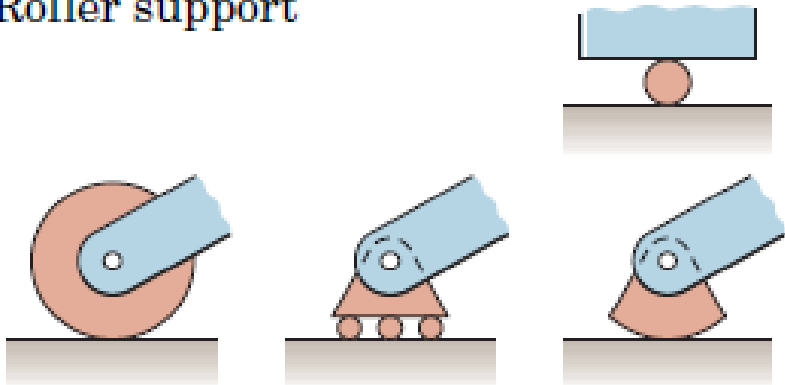
3. Rough surfaces



Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R .

<https://www.youtube.com/watch?v=Y4861IUnUUA>

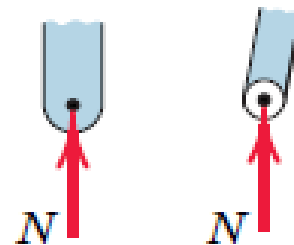
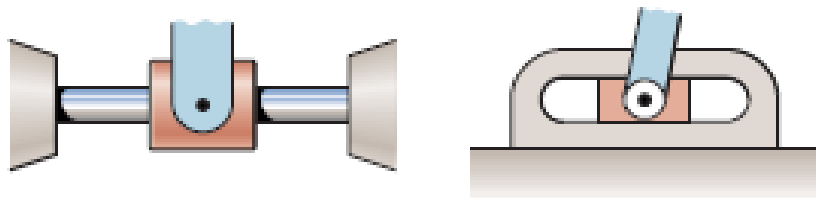
4. Roller support



Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

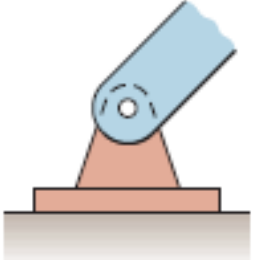
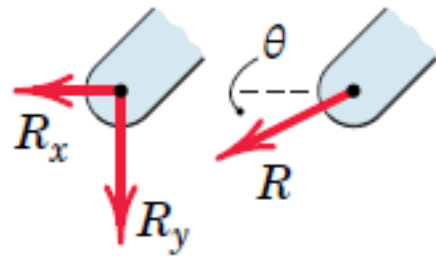
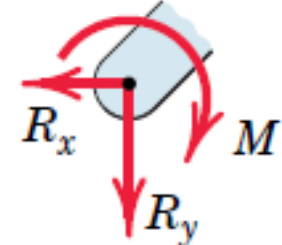
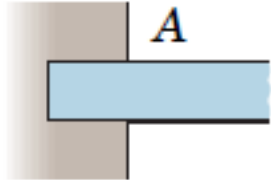
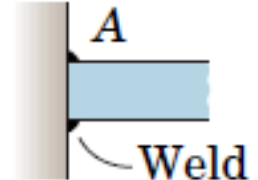
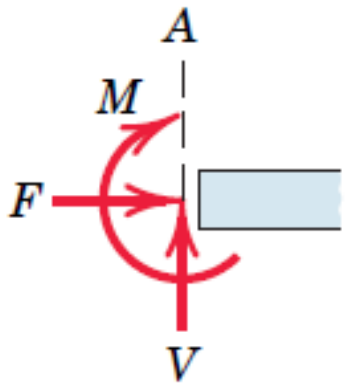
<https://www.youtube.com/watch?v=Jy2IUzGO4Gg>

5. Freely sliding guide

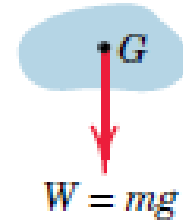
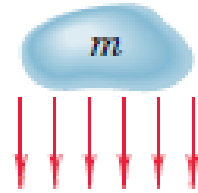


Collar or slider free to move along smooth guides; can support force normal to guide only.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS *(cont.)*

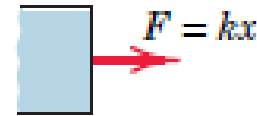
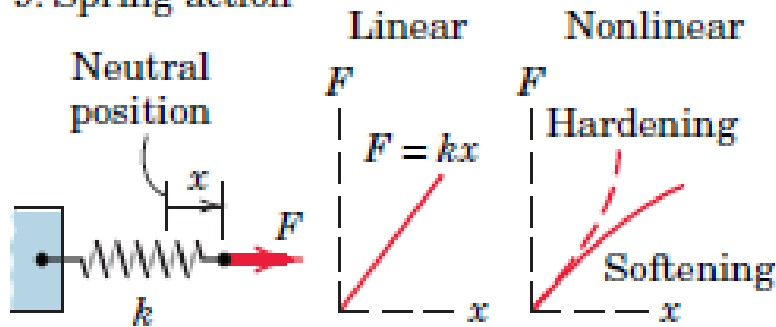
Type of Contact and Force Origin	Action on Body to Be Isolated
<p>6. Pin connection</p> <div style="text-align: center;">  </div>	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p>Pin free to turn</p>  <p>Pin not free to turn</p>  </div> <div style="width: 50%; padding-left: 20px;"> <p>A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.</p> </div> </div>
<p>7. Built-in or fixed support</p> <div style="display: flex; align-items: center; justify-content: center;">  or  </div>	<div style="text-align: center;">  </div> <p>A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.</p>

8. Gravitational attraction



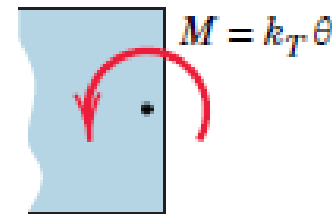
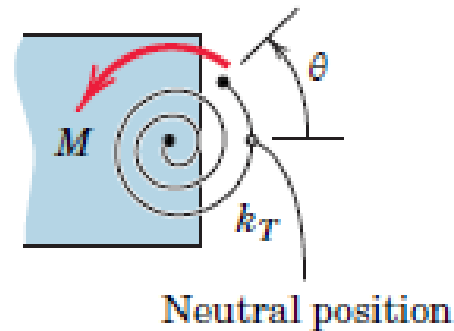
The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center of gravity G .

9. Spring action



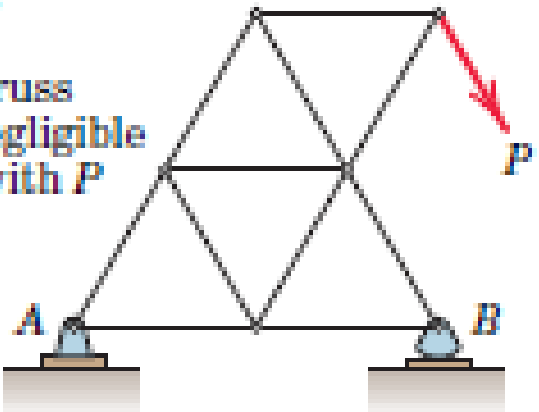
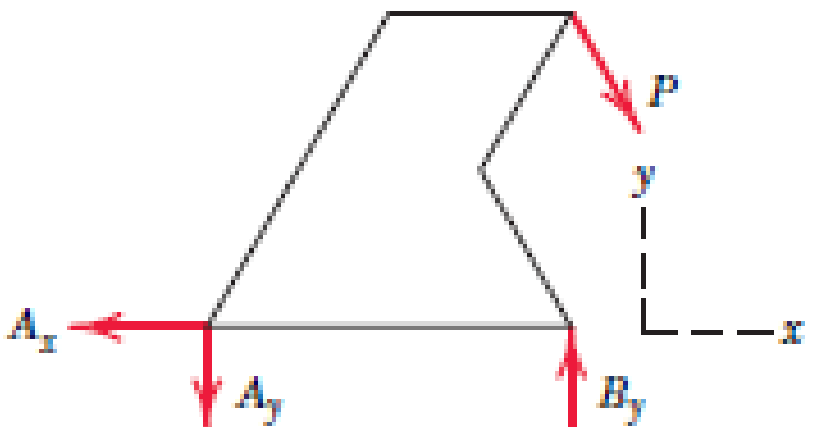
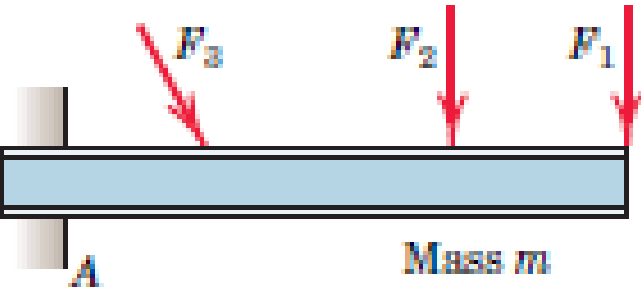
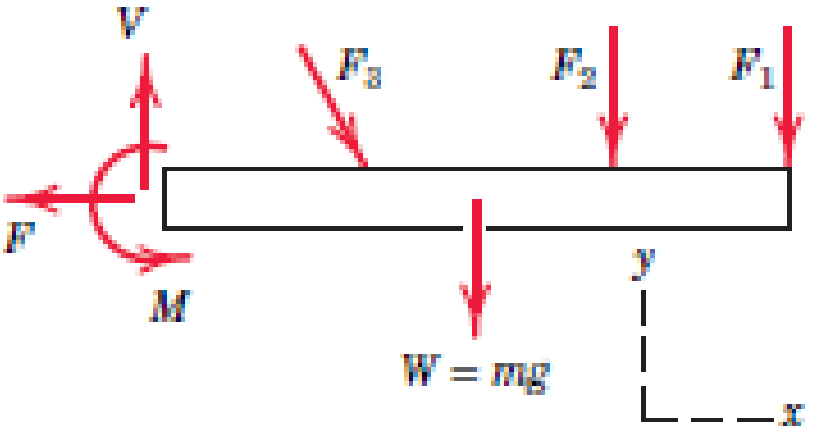
Spring force is tensile if the spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

10. Torsional spring action



For a linear torsional spring, the applied moment M is proportional to the angular deflection θ from the neutral position. The stiffness k_T is the moment required to deform the spring one radian.

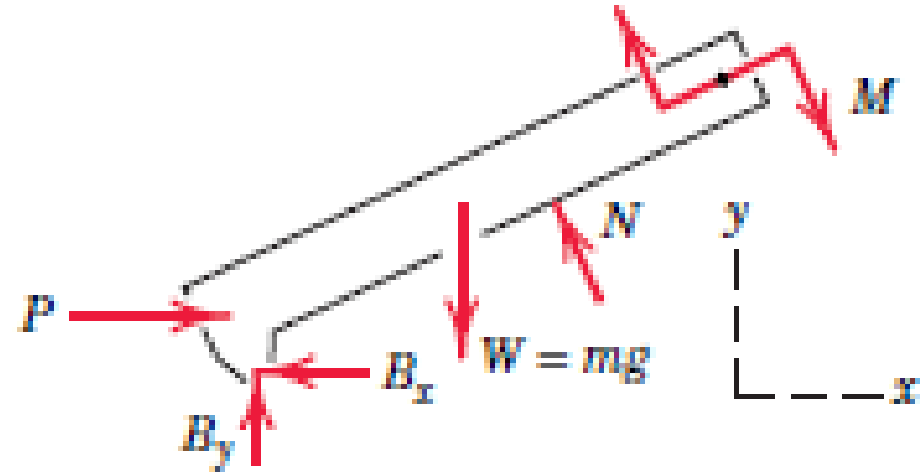
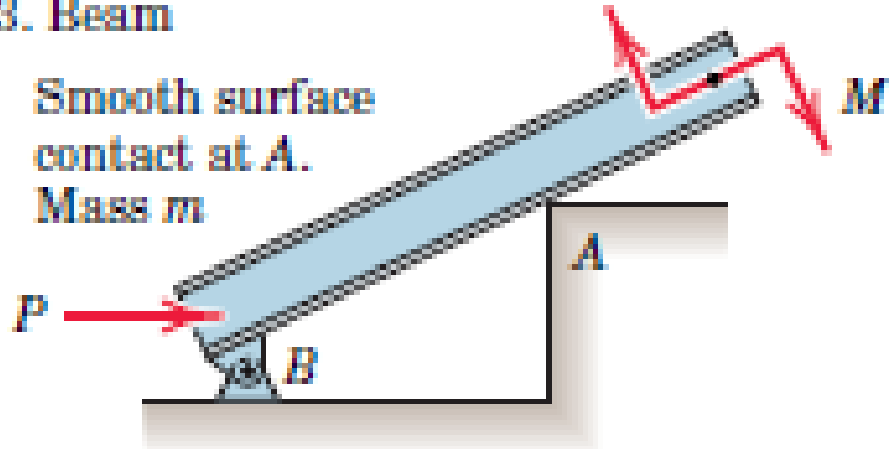
Construction of Free Body Diagram

SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	Free-Body Diagram of Isolated Body
<p>1. Plane truss</p> <p>Weight of truss assumed negligible compared with P</p> 	
<p>2. Cantilever beam</p> 	

Construction of Free Body Diagram

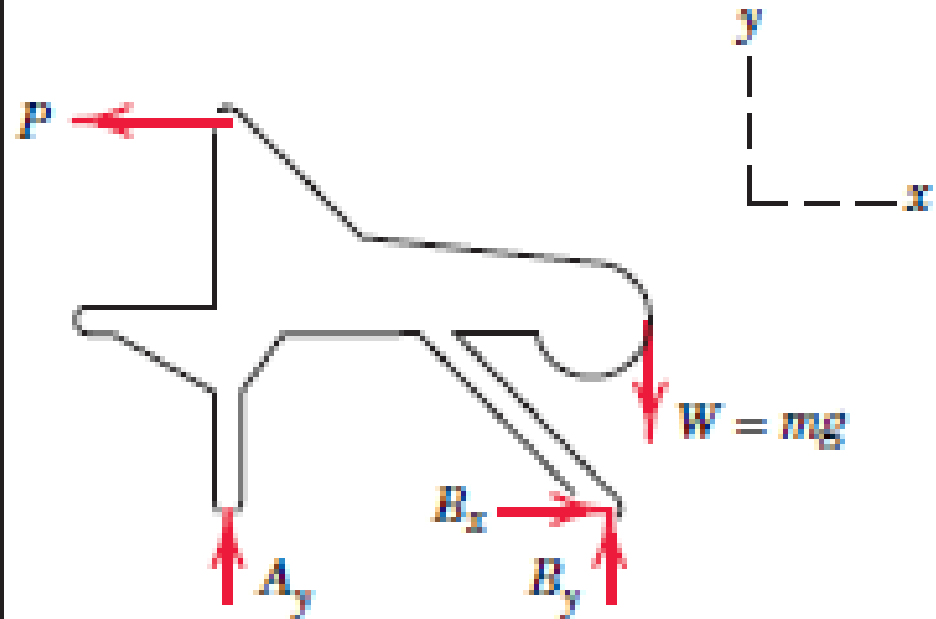
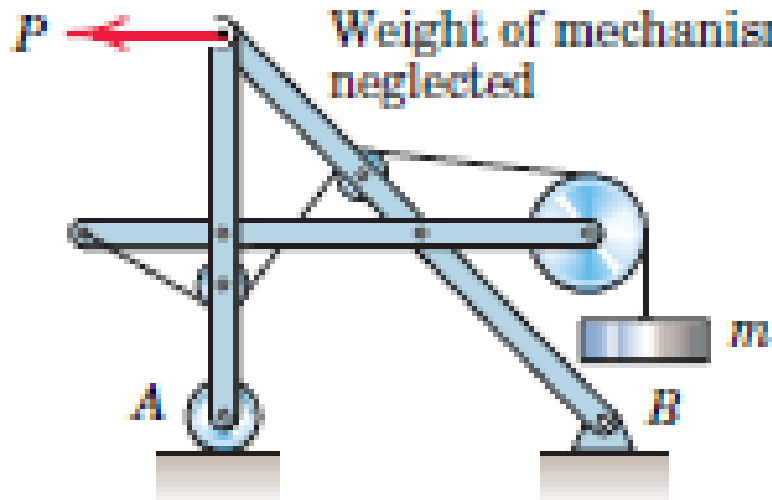
3. Beam

Smooth surface contact at A.
Mass m

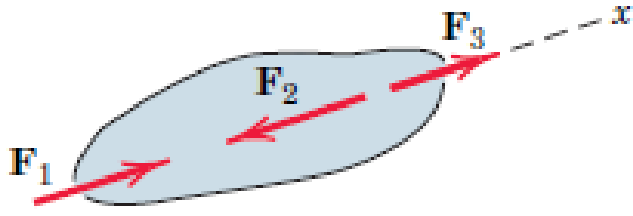
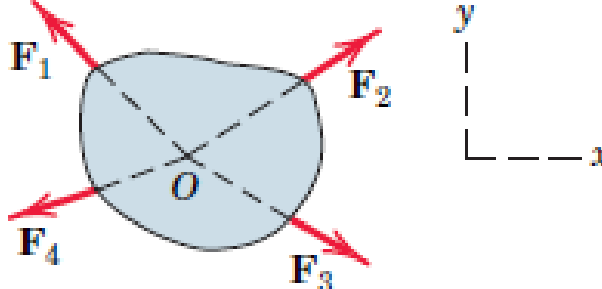
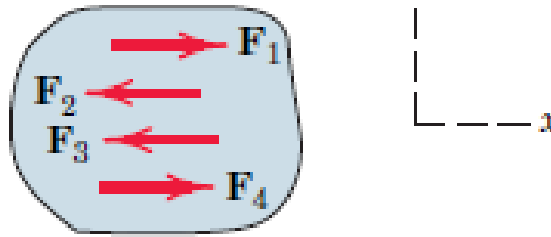
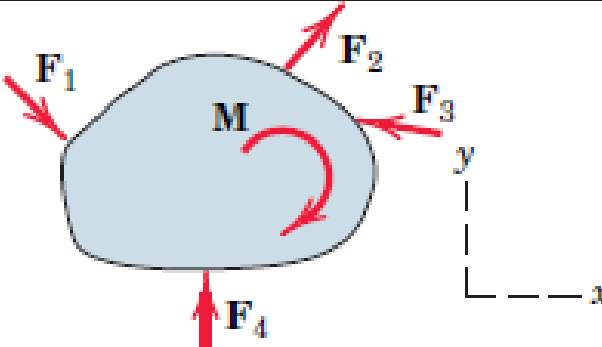


4. Rigid system of interconnected bodies analyzed as a single unit

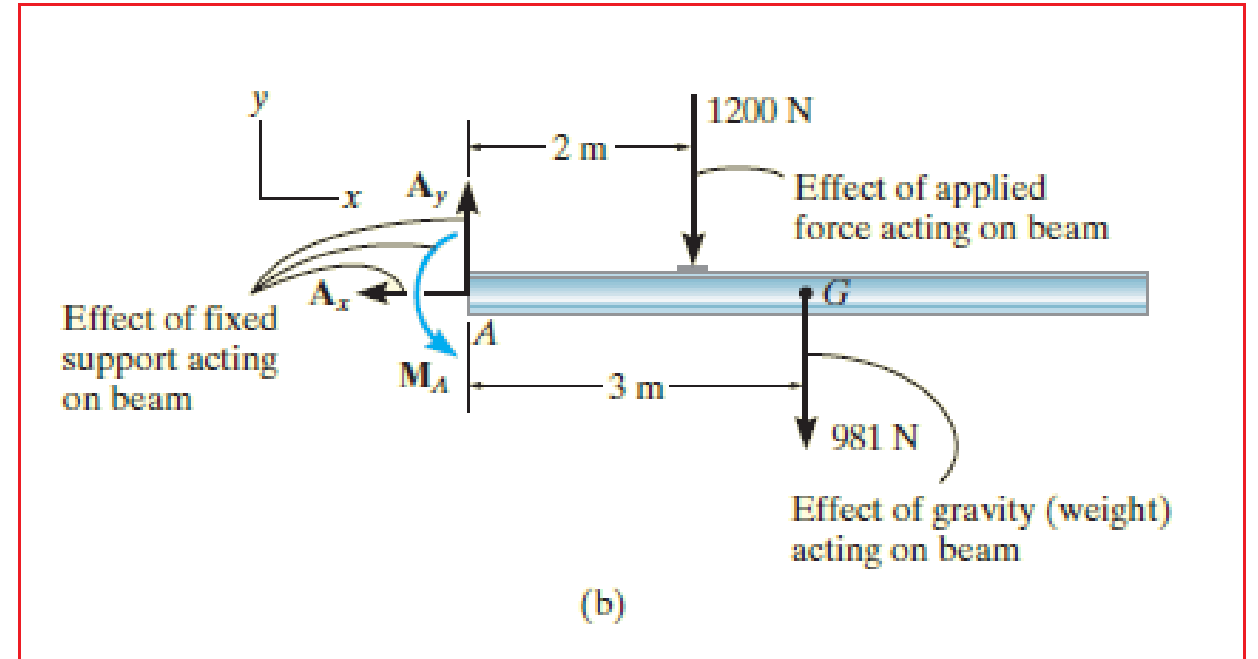
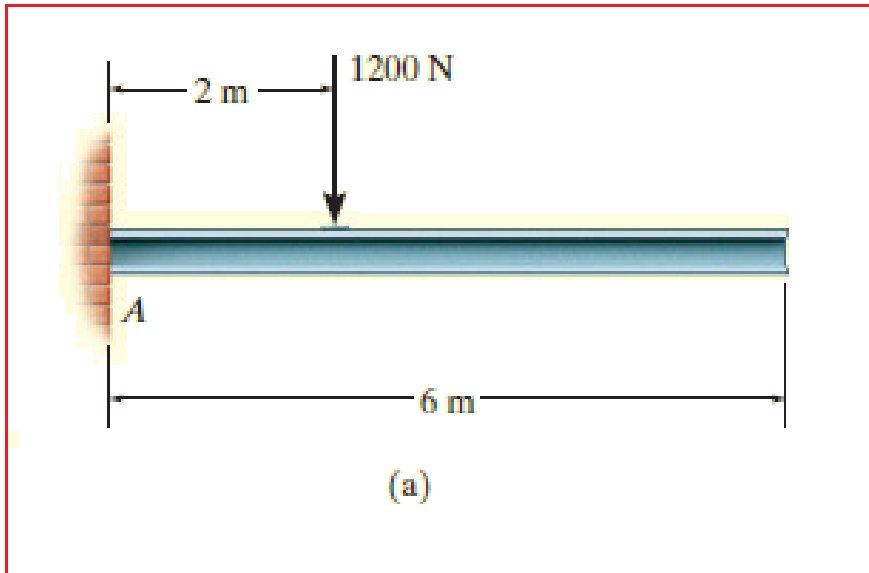
Weight of mechanism neglected



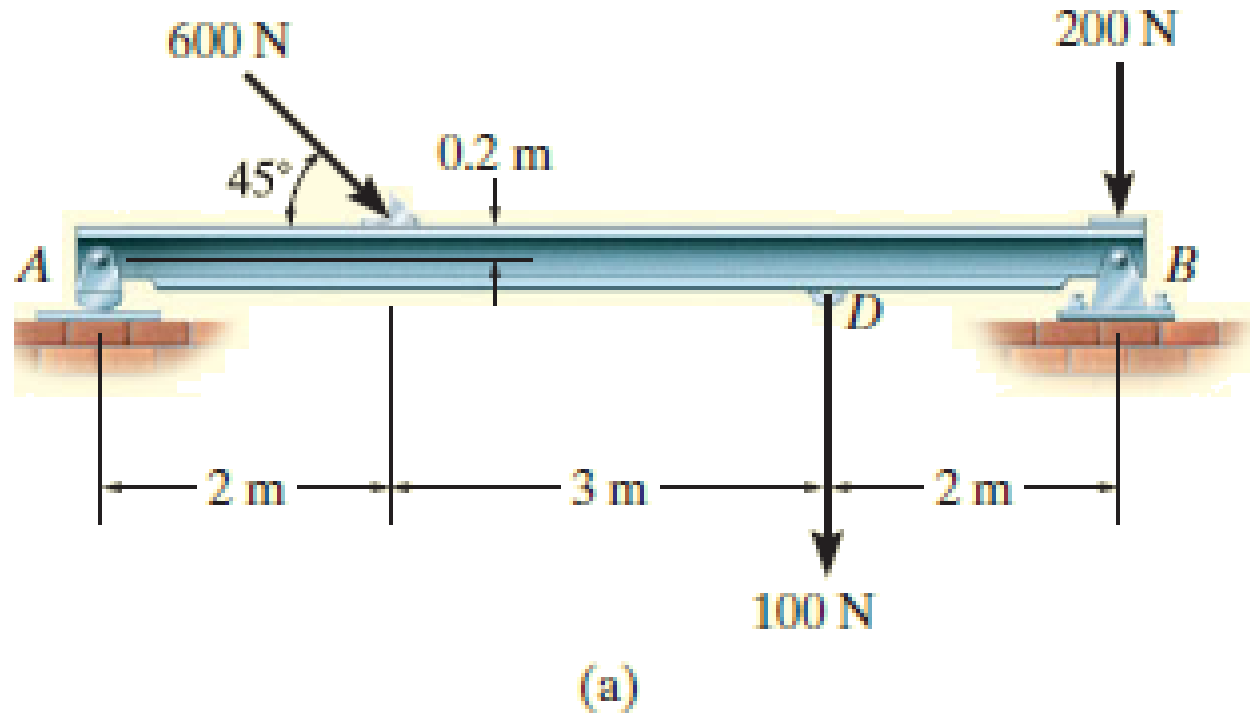
Equilibrium Conditions

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

Example 1// Draw the free-body diagram of the uniform beam shown in Fig. *a* . The beam has a mass of 100 kg.



Example 2 // Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the rocker at A as shown in Fig. a . Neglect the weight of the beam.



SOLUTION

Free-Body Diagram. Fig. *b*.

Equations of Equilibrium. Summing forces in the *x* direction yields

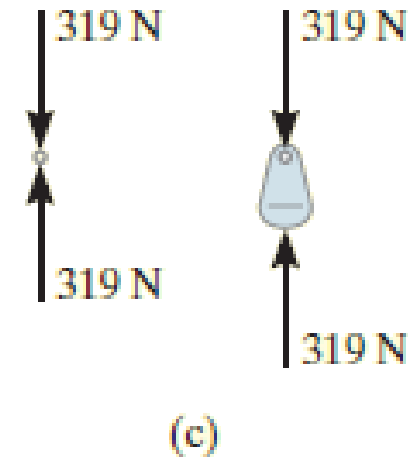
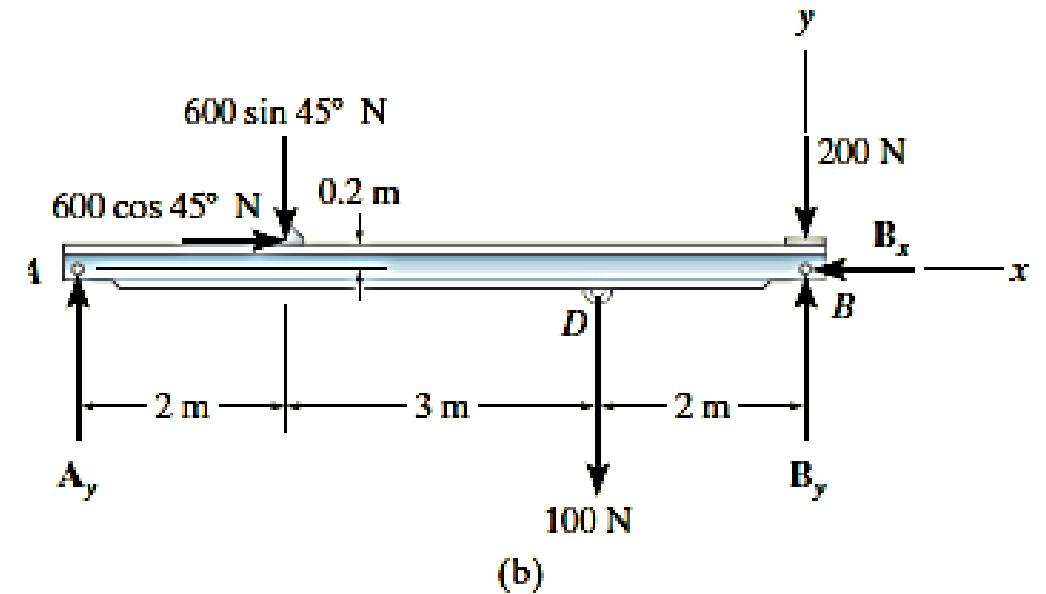
$$\begin{aligned} \rightarrow \Sigma F_x = 0; \quad 600 \cos 45^\circ \text{ N} - B_x &= 0 \\ B_x &= 424 \text{ N} \quad \text{Ans.} \end{aligned}$$

A direct solution for A_y can be obtained by applying the moment equation $\Sigma M_B = 0$ about point *B*.

$$\begin{aligned} \zeta + \Sigma M_B = 0; \quad 100 \text{ N}(2 \text{ m}) + (600 \sin 45^\circ \text{ N})(5 \text{ m}) \\ - (600 \cos 45^\circ \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) &= 0 \\ A_y &= 319 \text{ N} \quad \text{Ans.} \end{aligned}$$

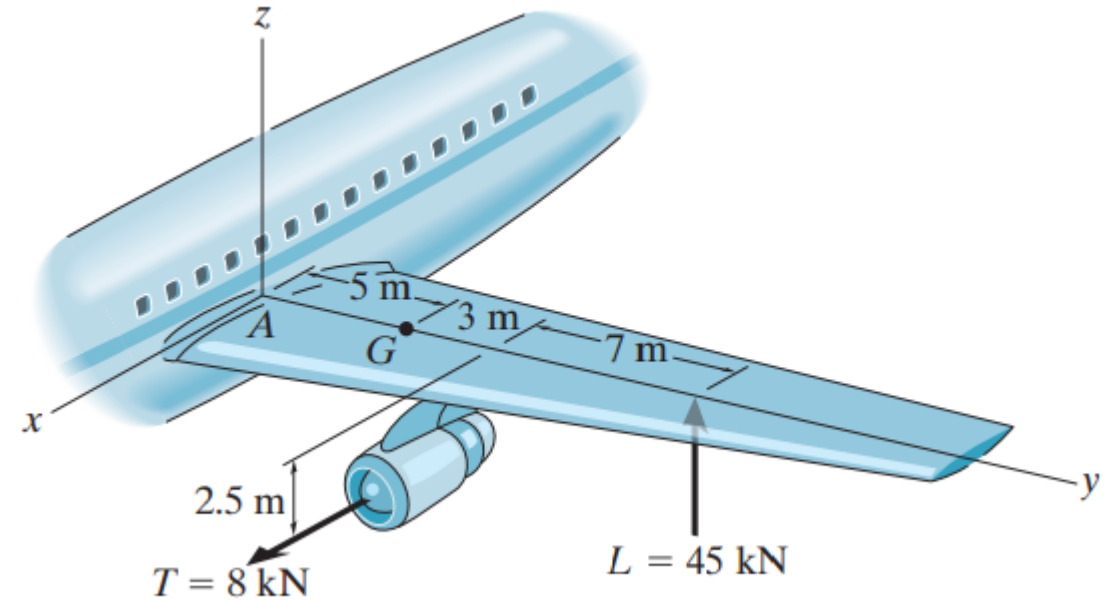
Summing forces in the *y* direction, using this result, gives

$$\begin{aligned} + \uparrow \Sigma F_y = 0; \quad 319 \text{ N} - 600 \sin 45^\circ \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y &= 0 \\ B_y &= 405 \text{ N} \quad \text{Ans.} \end{aligned}$$



Example 3:

The wing of the jet aircraft is subjected to a thrust of $T = 8 \text{ kN}$ from its engine and the resultant lift force $L = 45 \text{ kN}$. If the mass of the wing is 2.1 Mg and the mass center is at G , determine the x, y, z components of reaction where the wing is fixed to the fuselage at A .



SOLUTION

$$\Sigma F_x = 0; \quad -A_x + 8000 = 0$$

$$A_x = 8.00 \text{ kN}$$

$$\Sigma F_y = 0; \quad A_y = 0$$

$$\Sigma F_z = 0; \quad -A_z - 20\,601 + 45\,000 = 0$$

$$A_z = 24.4 \text{ kN}$$

$$\Sigma M_y = 0; \quad M_y - 2.5(8000) = 0$$

$$M_y = 20.0 \text{ kN} \cdot \text{m}$$

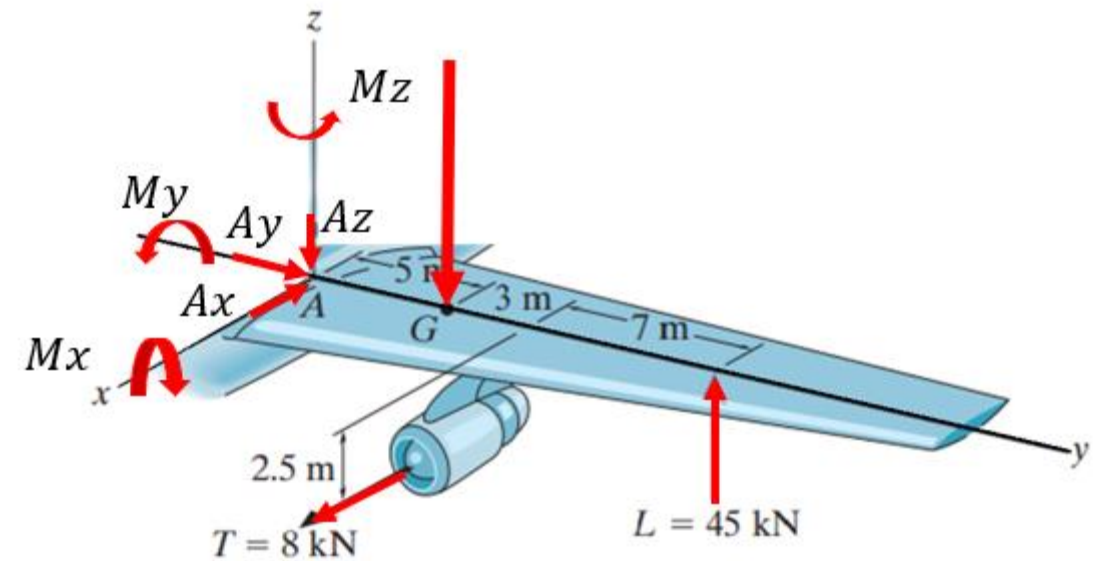
$$\Sigma M_x = 0; \quad 45\,000(15) - 20\,601(5) - M_x = 0$$

$$M_x = 572 \text{ kN} \cdot \text{m}$$

$$\Sigma M_z = 0; \quad M_z - 8000(8) = 0$$

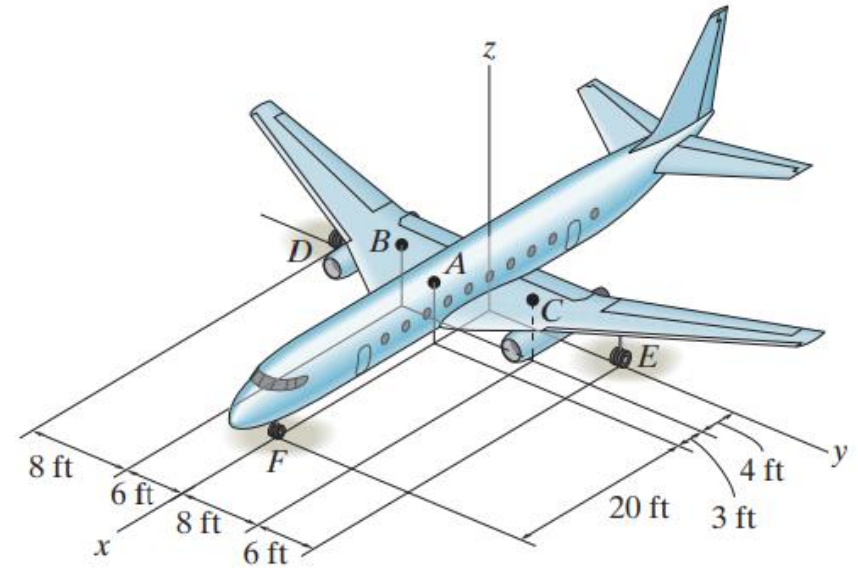
$$M_z = 64.0 \text{ kN} \cdot \text{m}$$

The weight of the wing = mass x acceleration = $2.1 \times 1000 \times 9.81 = 20601 \text{ N}$



Example 4:

Due to an unequal distribution of fuel in the wing tanks, the centers of gravity for the airplane fuselage A and wings B and C are located as shown. If these components have $W_A = 45\,000\text{ lb}$, $W_B = 8\,000\text{ lb}$, $W_C = 6\,000\text{ lb}$ weights and determine the normal reactions of the wheels D, E, and F on the ground.



SOLUTION

$$\Sigma M_x = 0; \quad 8000(6) - R_D(14) - 6000(8) + R_E(14) = 0$$

$$\Sigma M_y = 0; \quad 8000(4) + 45\,000(7) + 6000(4) - R_F(27) = 0$$

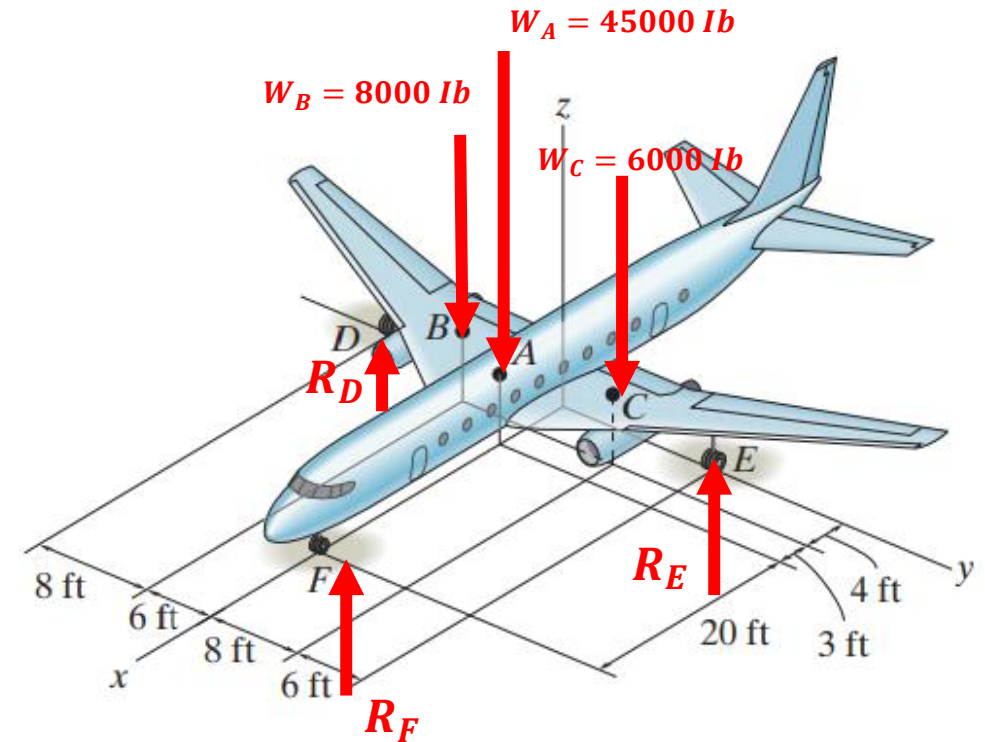
$$\Sigma F_z = 0; \quad R_D + R_E + R_F - 8000 - 6000 - 45\,000 = 0$$

Solving,

$$R_D = 22.6 \text{ kip}$$

$$R_E = 22.6 \text{ kip}$$

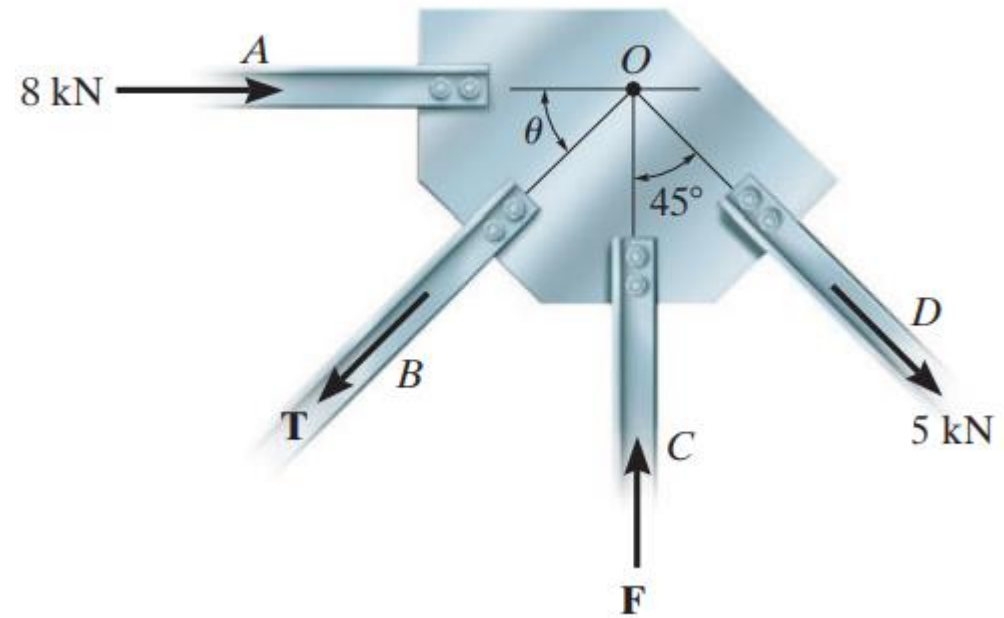
$$R_F = 13.7 \text{ kip}$$



Assignment 1

(Solve this problem then submit your answer)

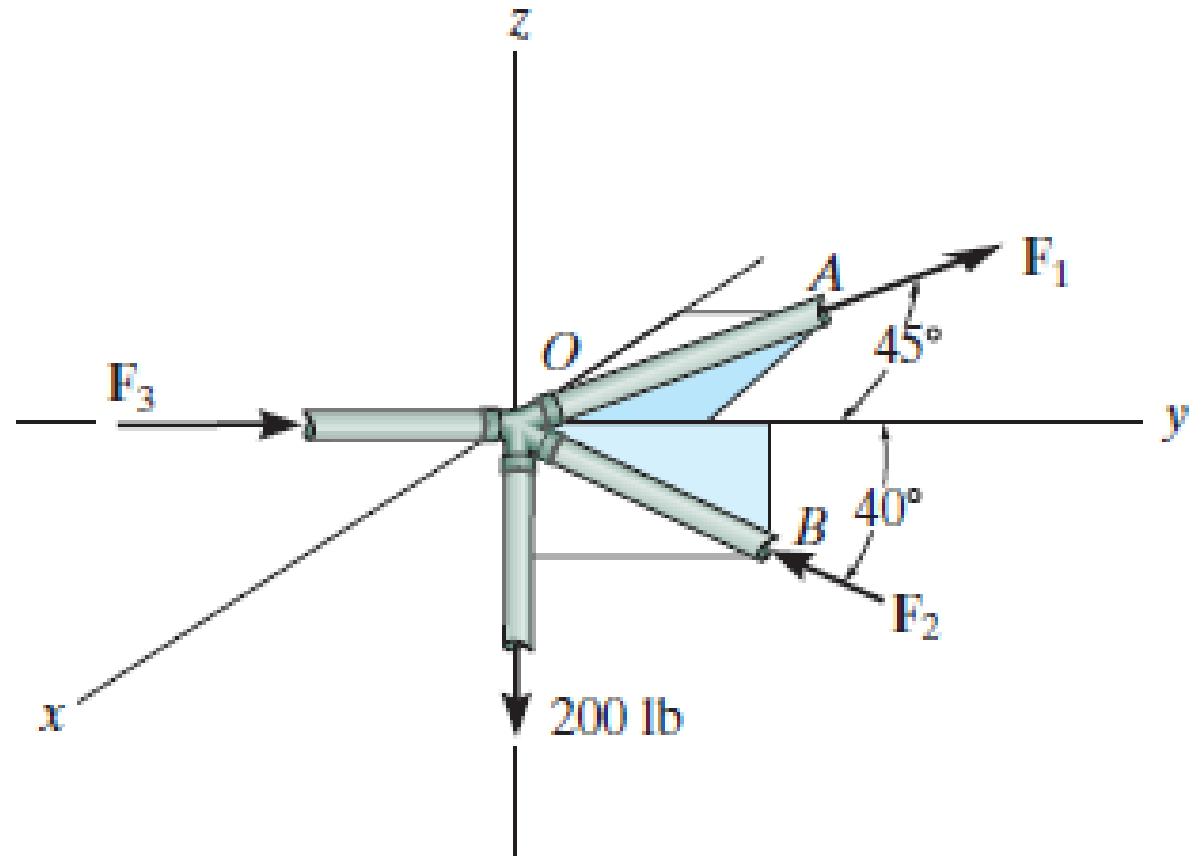
The gusset plate is subjected to the forces of four members. Determine the force in member B and its proper orientation θ for equilibrium. The forces are concurrent at point O . Take $F = 12 \text{ kN}$.



Assignment 2

(Solve this problems then submit your answer)

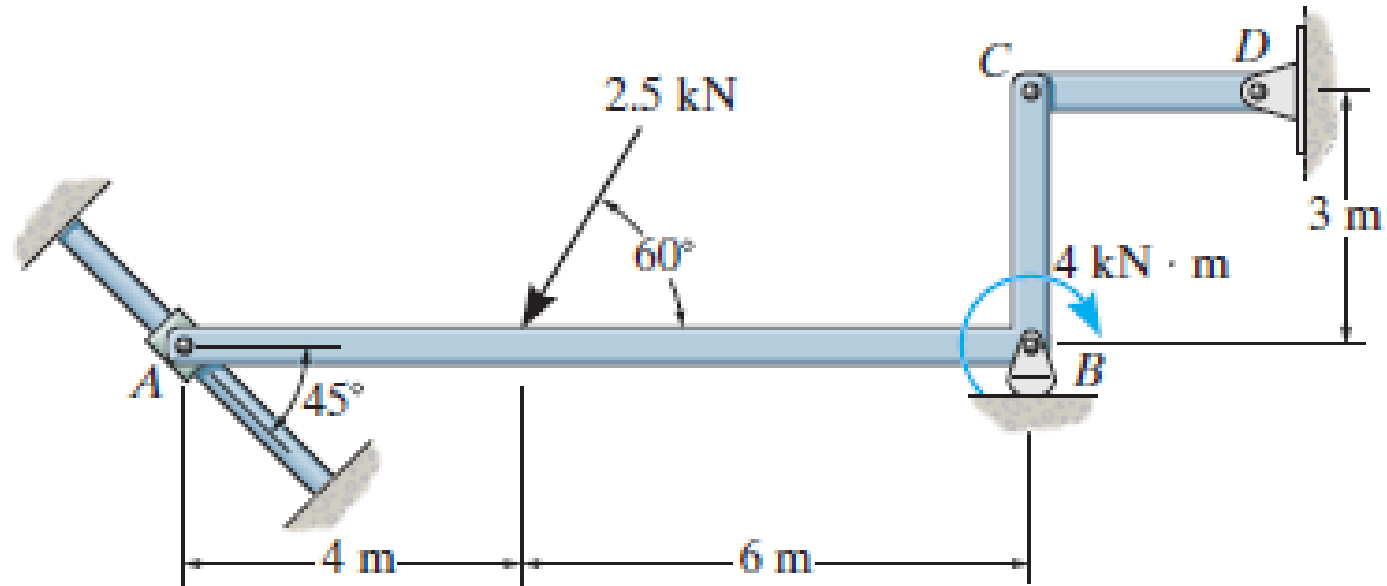
The joint of a space frame is subjected to four member forces. Member OA lies in the x – y plane and member OB lies in the y – z plane. Determine the forces acting in each of the members required for equilibrium of the joint.



Assignment 3

(Solve this problems then submit your answer)

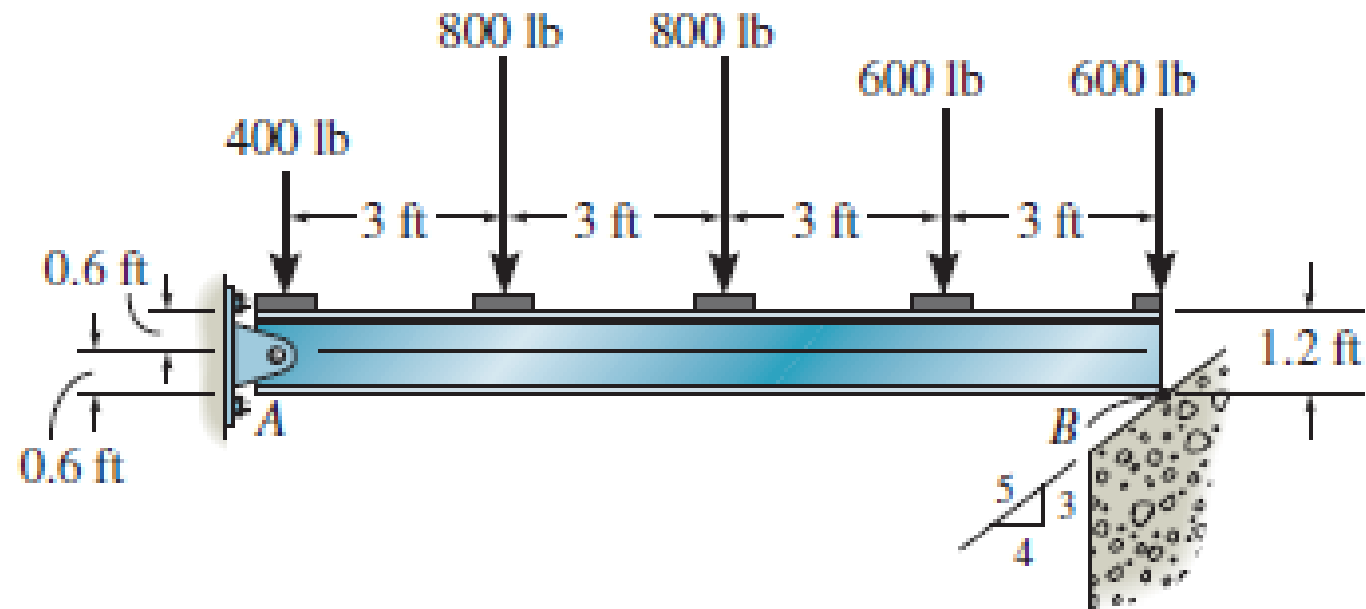
Draw the free-body diagram of member ABC which is supported by a smooth collar at A , rocker at B , and short link CD . Explain the significance of each force acting on the diagram.



Assignment 4

(Solve this problems then submit your answer)

Draw the free-body diagram of the beam, which is pin supported at A and rests on the smooth incline at B .

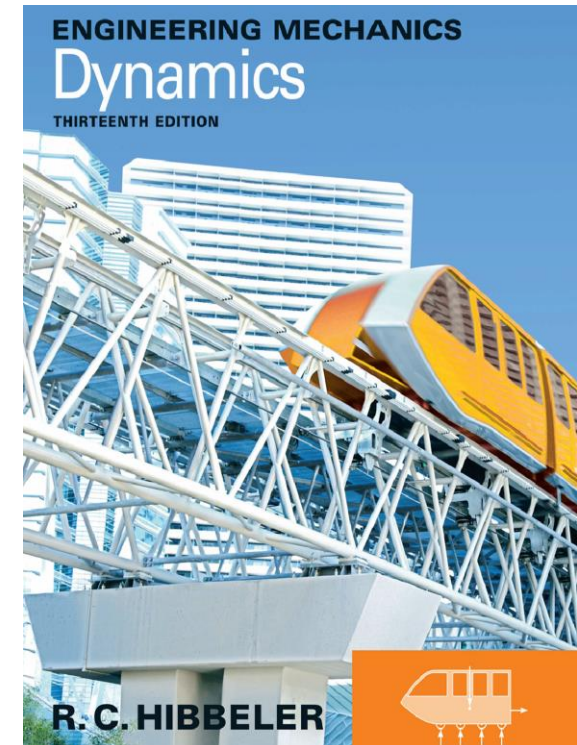
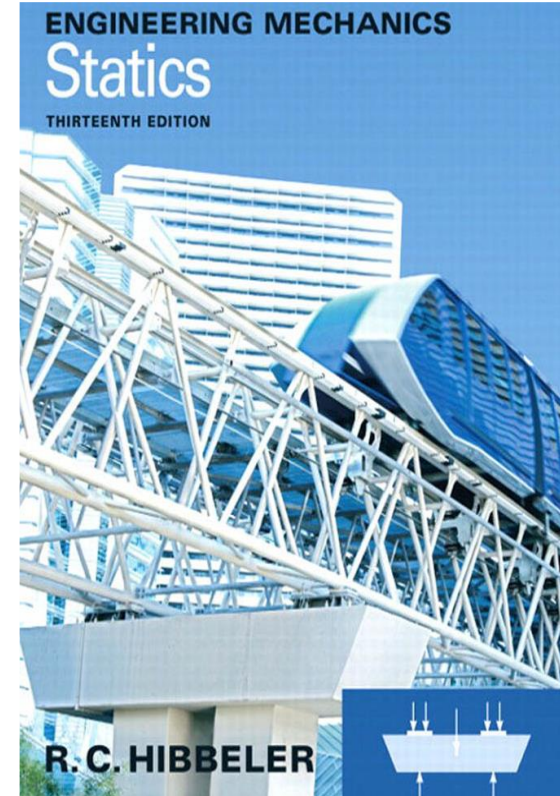


Next Lecture:

- Structural analysis

References:

Engineering Mechanics R.C.
Hibbeler 13th edition (Statics and
Dynamics).



The end of the lecture
Enjoy your time