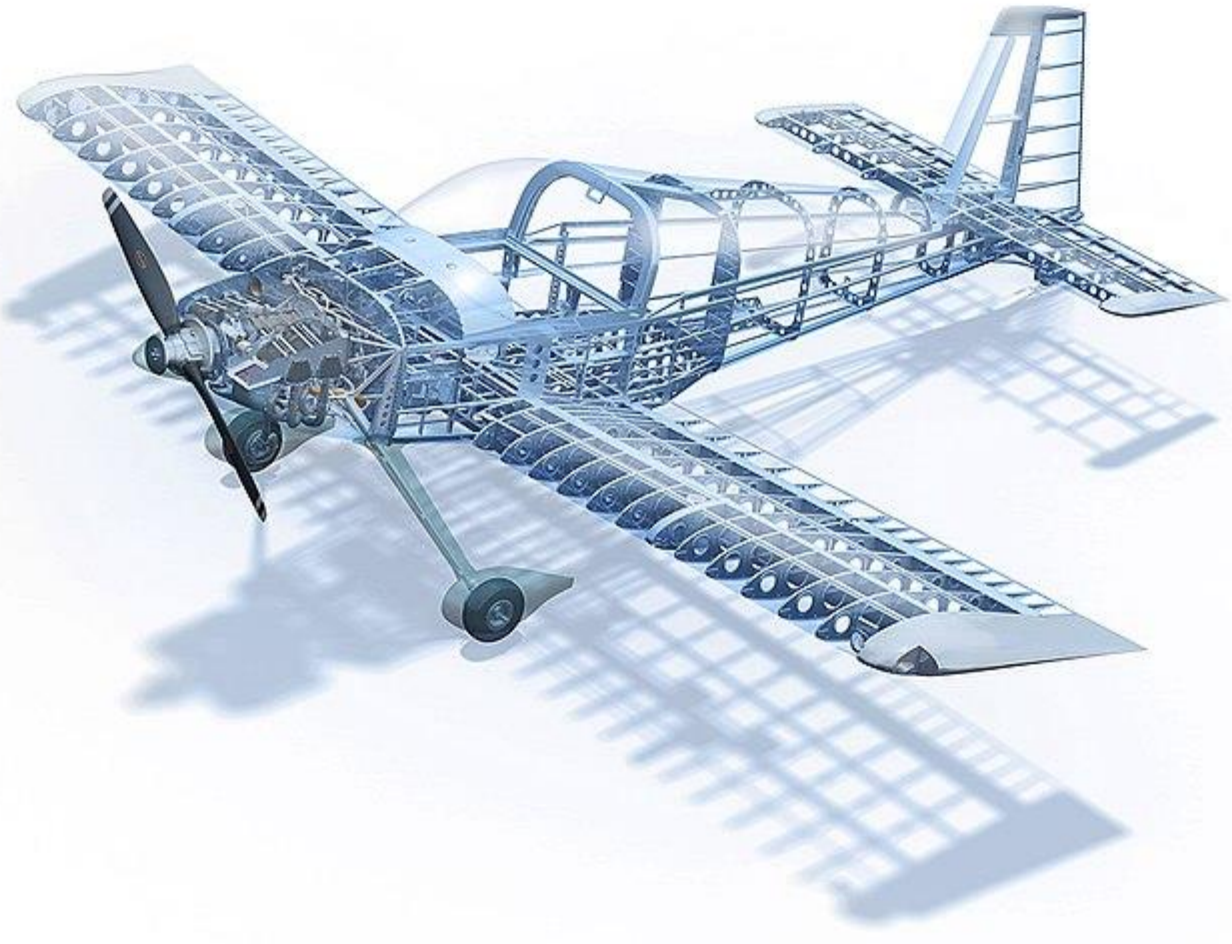


Aviation Department
First Grade- Spring Semester

Statics- Structural analysis (Lecture 8)

Lecturer: Ms. Jwan Khaleel M.

Lecture content:

- Structural analysis
  - Method of joint

Learning Outcomes:

At the end of the lecture the students will be able:

- To determine the forces in the members of a truss.
- To evaluate the method of joints and the method of sections.
- To analyze the forces acting on the members of frames and machines composed of pin-connected members.
- Solving related problems.

What Is Equilibrium?

Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero.

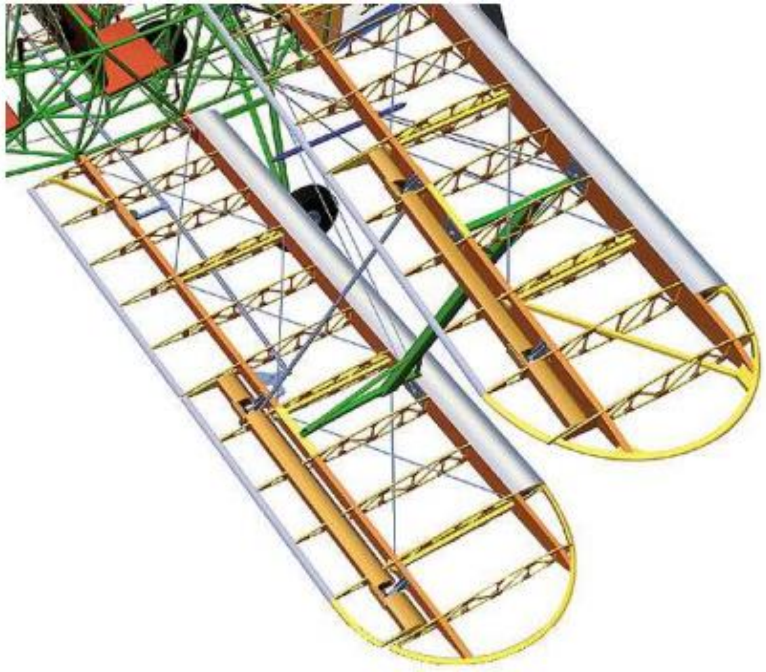
Previously we studied the equilibrium of a single rigid body, or a system of connected members treated as a single rigid body.

We first drew a free-body diagram of the body showing all forces external to the isolated body and then applied the force and moment equations of equilibrium.

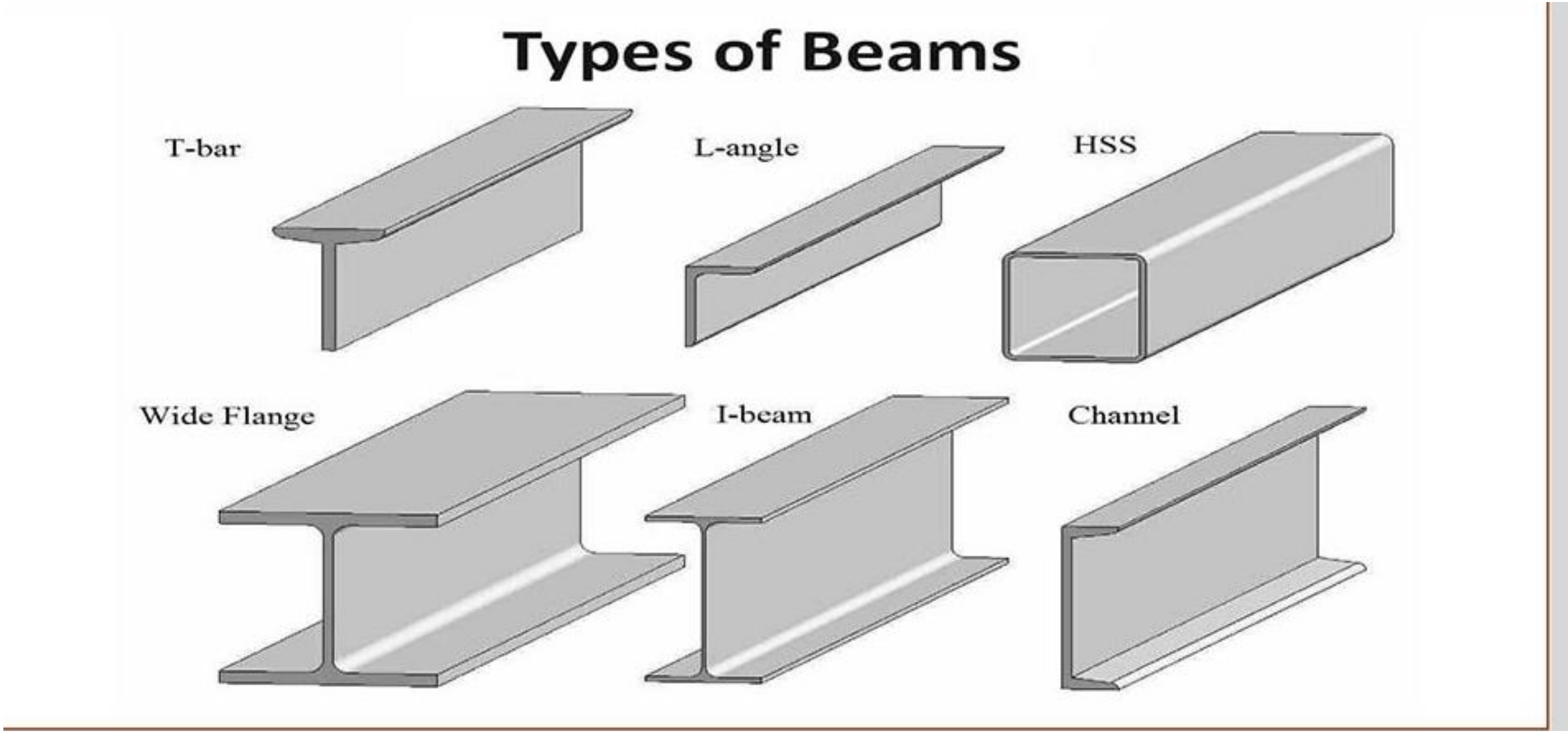
Now we focus on the determination of the forces internal to a structure that is, forces of action and reaction between the connected members.

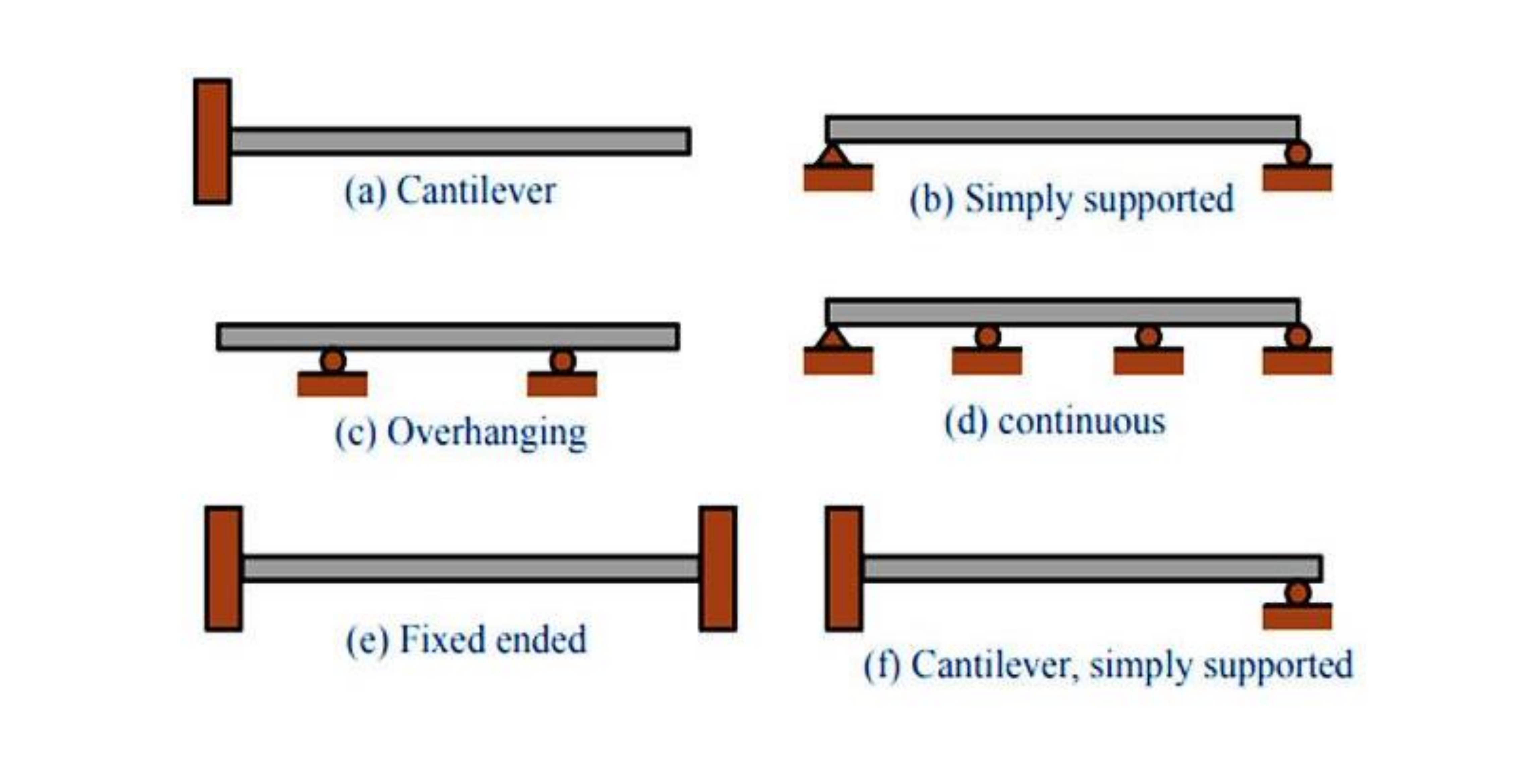
Engineering Structure

- An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.
- Static structural analysis is a branch of structural engineering that deals with the analysis of structures under static loading conditions. It involves studying the behavior of structures and components when subjected to various loads, such as forces, moments, and external pressures, while remaining in a state of static equilibrium. This type of analysis helps engineers determine the stresses, strains, and displacements in a structure to ensure that it can withstand the applied loads without failure.
- A lot of aerospace structures can be idealized as truss structures. This is clearly illustrated in picture 2.1 where it can be seen that the ribs in the wing are built up as a truss structure. Also in space applications, trusses are widely used because of their simplicity and light weightiness. Now the question rises: what is a truss?



Rib truss structure example


Types of Beams
T-bar
L-angle
HSS
Wide Flange
I-beam
Channel


(a) Cantilever
(b) Simply supported
(c) Overhanging
(d) continuous
(e) Fixed ended
(f) Cantilever, simply supported

PLANE TRUSSES

- **Statically Determinate Structures:**

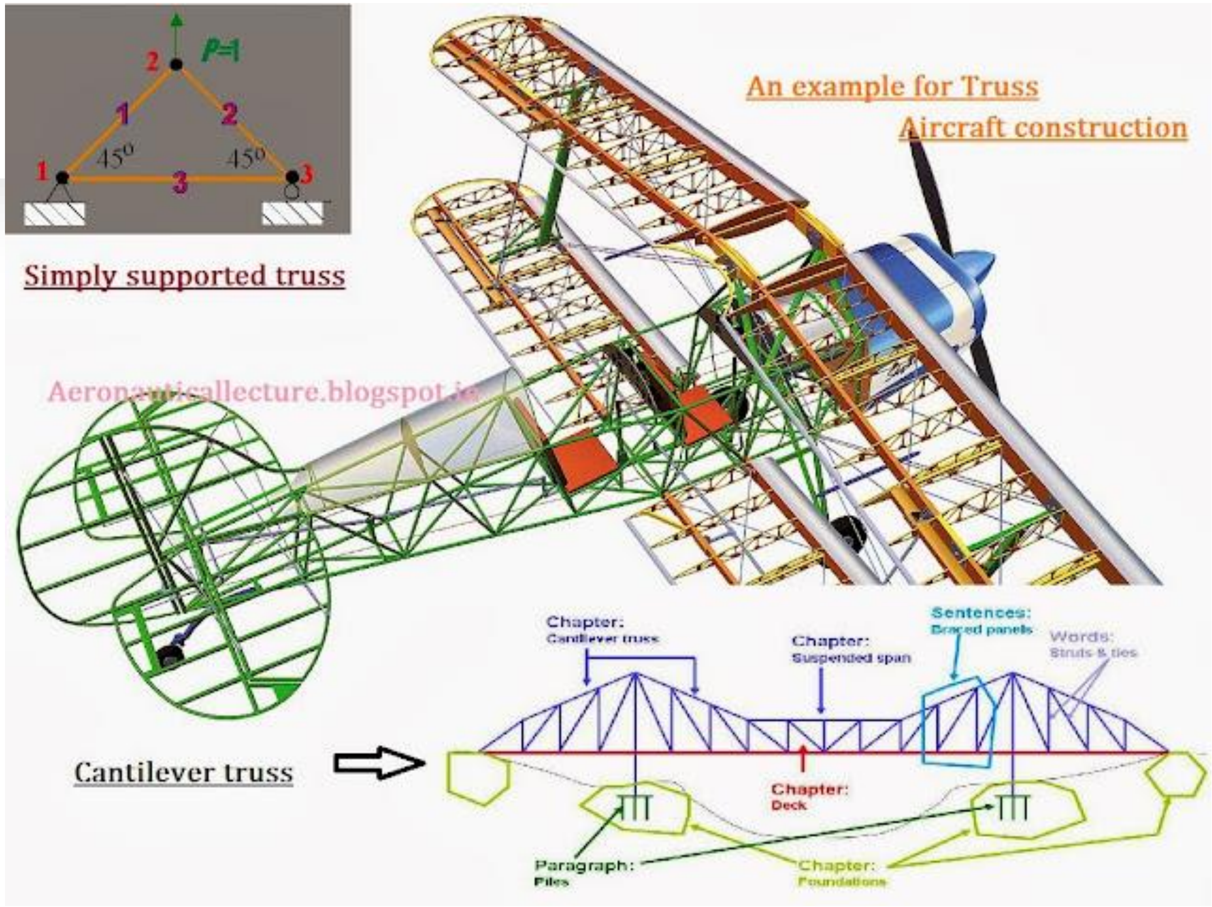
- To determine the internal forces in the structure, dismember the structure and analyze separate free body diagrams of individual members or combination of members. This analysis requires careful application of Newton's third law of motion.

- ***Truss***: A framework composed of members joined at their ends to form a rigid structure.

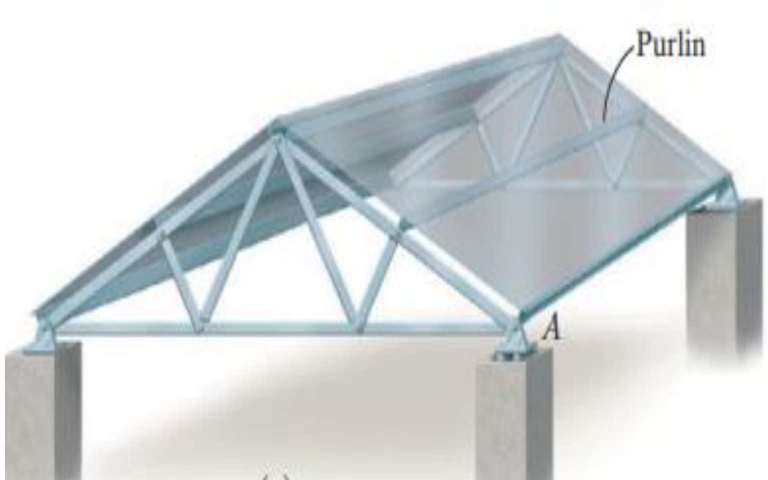
- Structural members commonly used are **I-**beams, angles, bars, and special shapes.

- Fastened together at their ends by **welding**, **riveted connections**, or large **bolts** or **pins**.

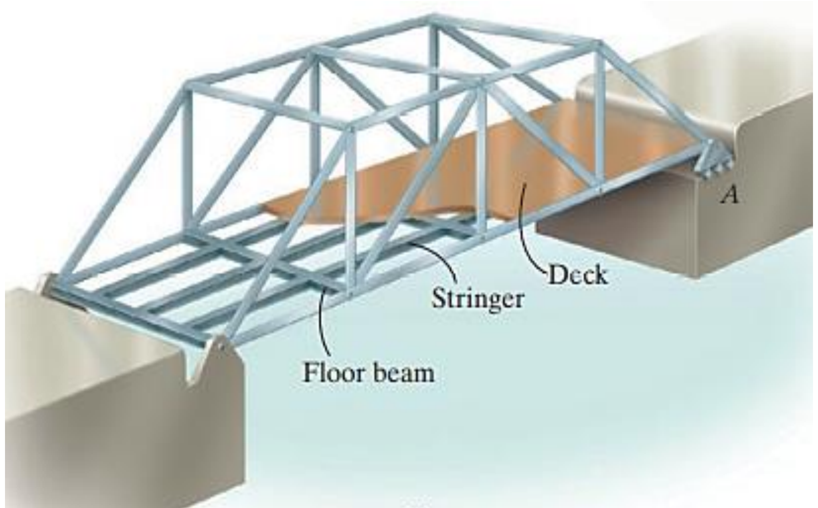
- When the members of the truss lie essentially in a single plane are often used to support roofs and bridges, the truss is called a ***plane truss*.**



- When bridge or roof trusses extend over large distances, a ***Rocker/Roller*** is commonly used for supporting one end, for example**, joint A**.
- This type of support allows freedom for expansion or contraction of the members due to a change in temperature or application of loads.



(a)



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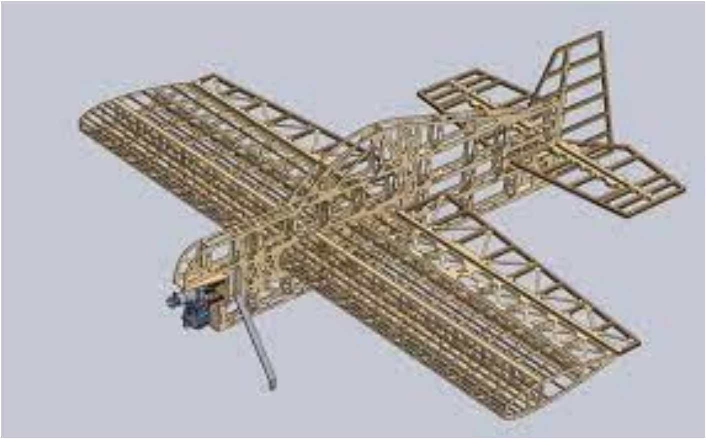
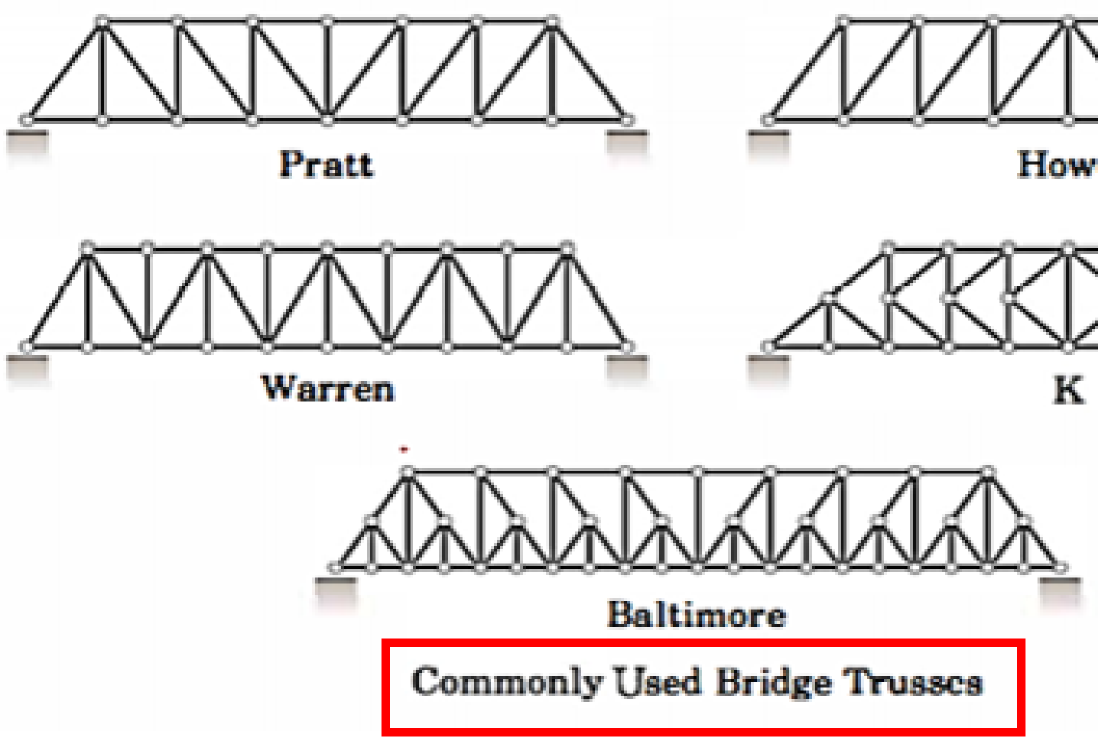
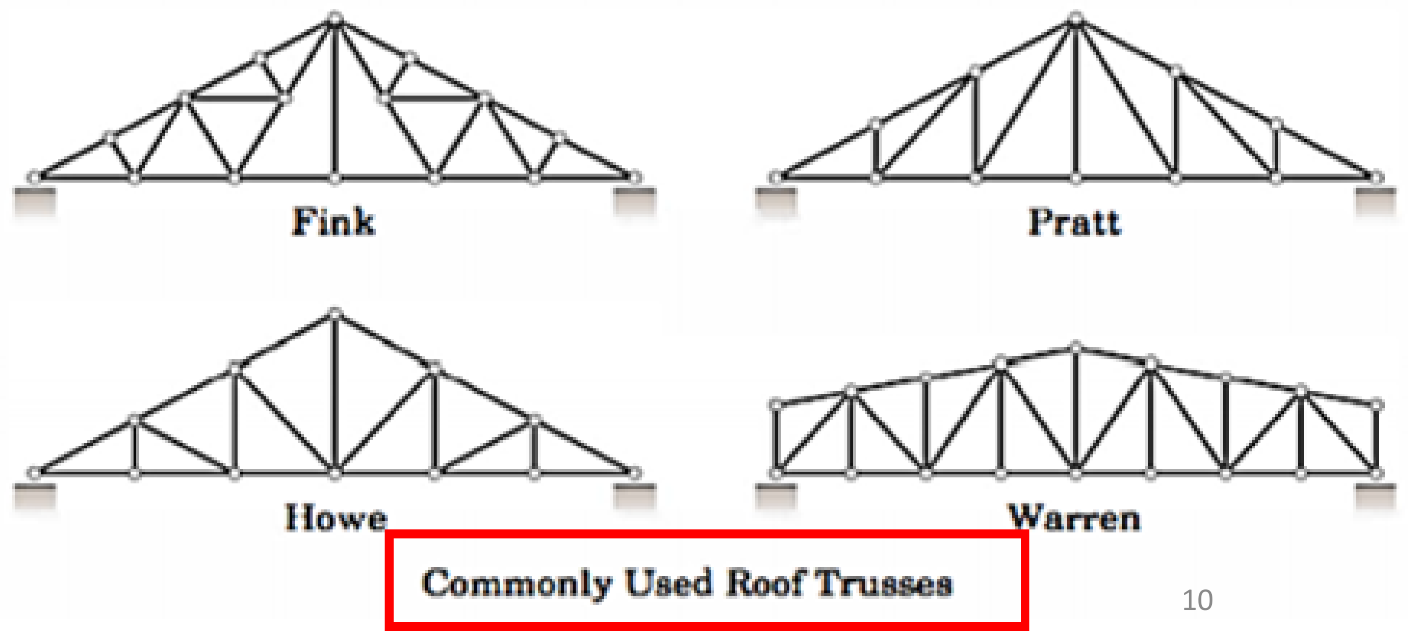


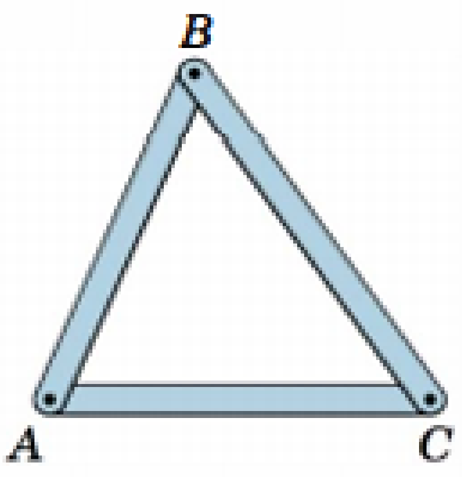
Plane trusses types



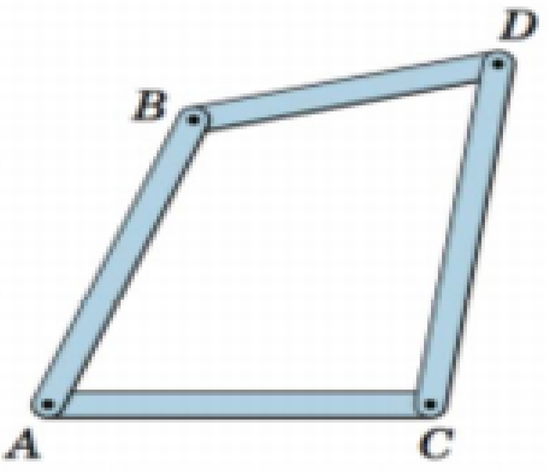


Simple truss

- Basic Element of a Plane Truss is the Triangle (Figure a)



(a)

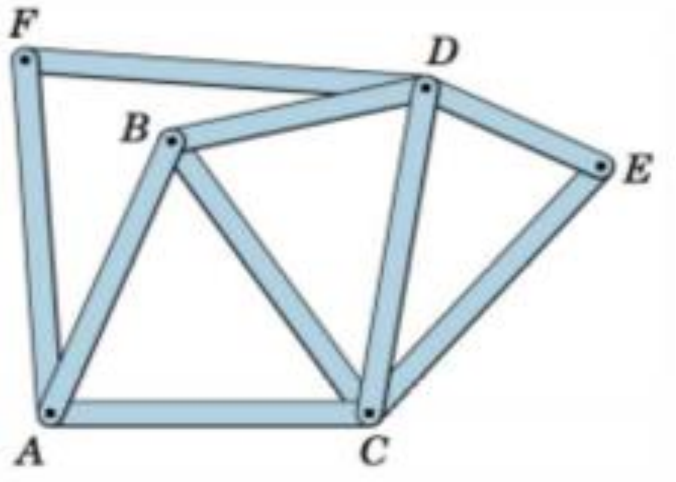


(b)

- Three bars joined by pins at their ends called **Rigid Frame**
  - Non-collapsible and deformation of members due to induced internal strains is negligible.

- Four or more bars polygon called **Non-Rigid Frame** (Figure b)

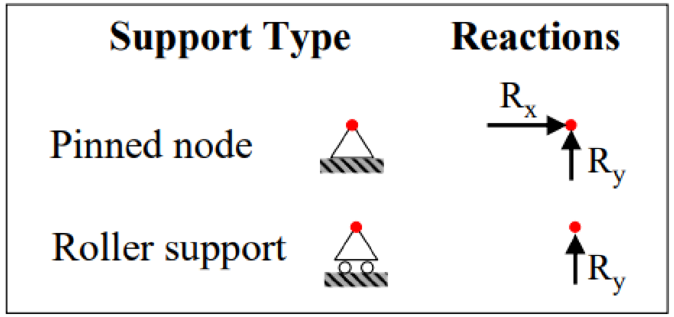
- For making it **rigid** or **stable**:
  - Structures built from basic triangles (by forming more triangles) (Figure c)



(c)

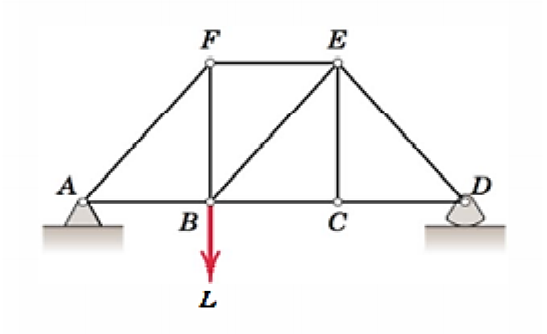
To design a truss, we must:

1. First determine the forces in the various member
2. Select appropriate sizes and structural shapes to withstand the forces.

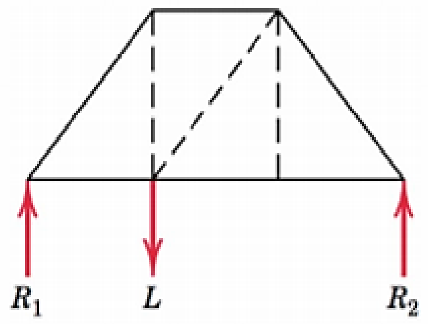


Several assumptions are made in the force analysis of simple trusses:

- **All loadings are applied at the joints/all members to be two-force members.**
  - Weight of the members is small compared with the force it supports (weight may be considered at joints)
  - No effect of bending on members even if weight is considered.
- **The members are joined together by smooth pins.**
  - External forces are applied at the pin connections
  - Welded or riveted connections
  - Pin Joint if the member centerlines are concurrent at the joint.

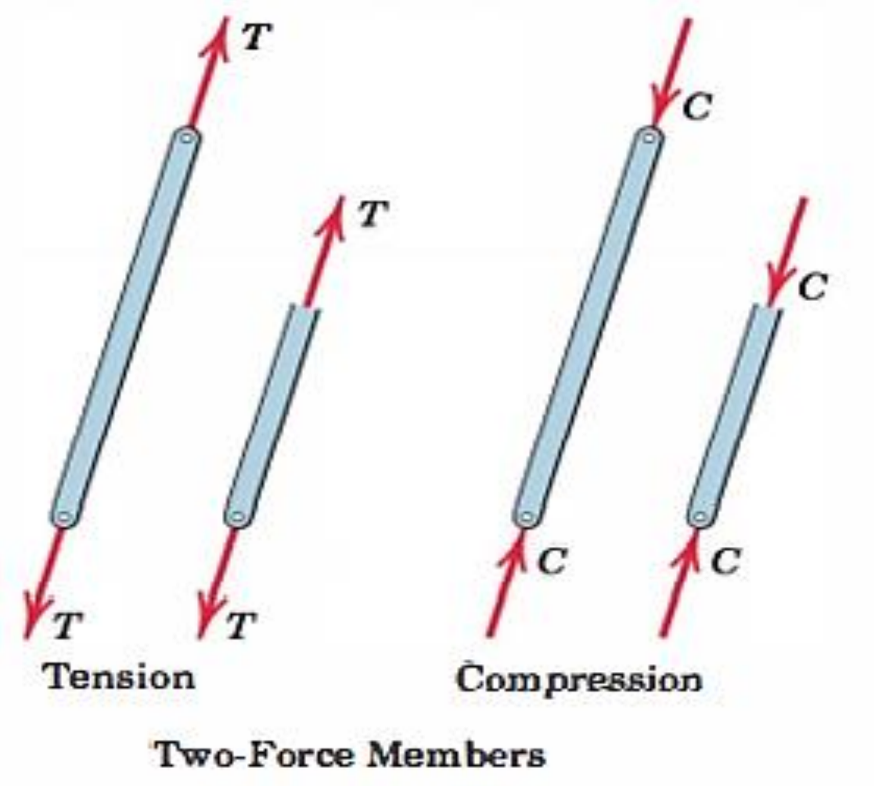


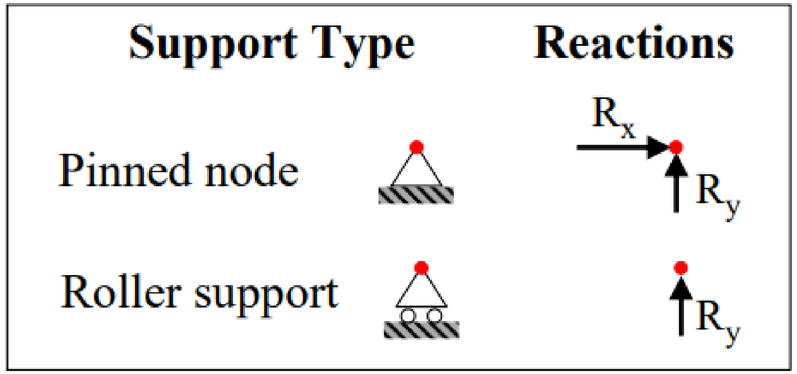
(a)



(b)

- The two forces are applied at the ends of the member and are necessarily equal, opposite, and collinear for equilibrium.
- The member may be in tension or compression, as shown in Fig. When we represent the equilibrium of a portion of a two-force member, the tension **T** or compression **C** acting on the cut section is the same.





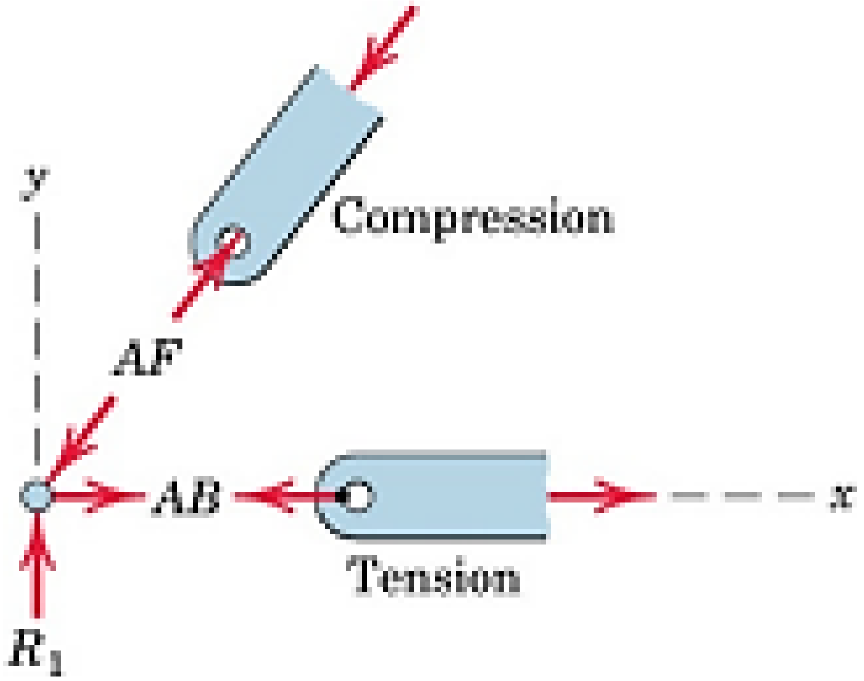
Truss analysis-Method of Joints:

Method of Joints: Conditions of equilibrium are satisfied for the forces at each joint.

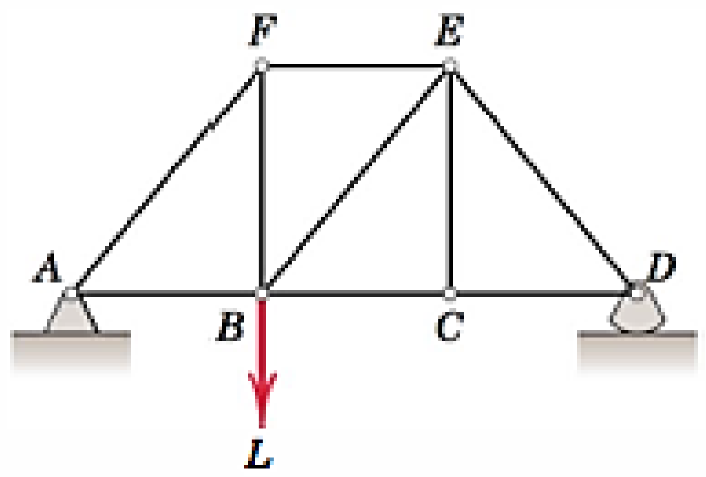
- Equilibrium of concurrent forces at each joint.
- Each joint is subjected to a force system that is coplanar and concurrent.
- Only two independent equilibrium equations are involved $\sum F_x = 0$, $\sum F_y = 0$

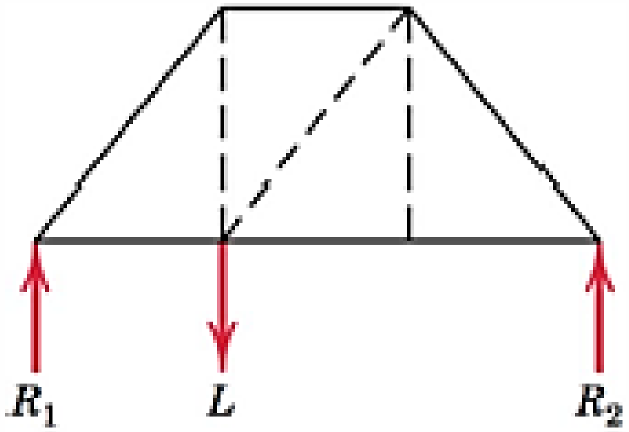
Steps of Analysis

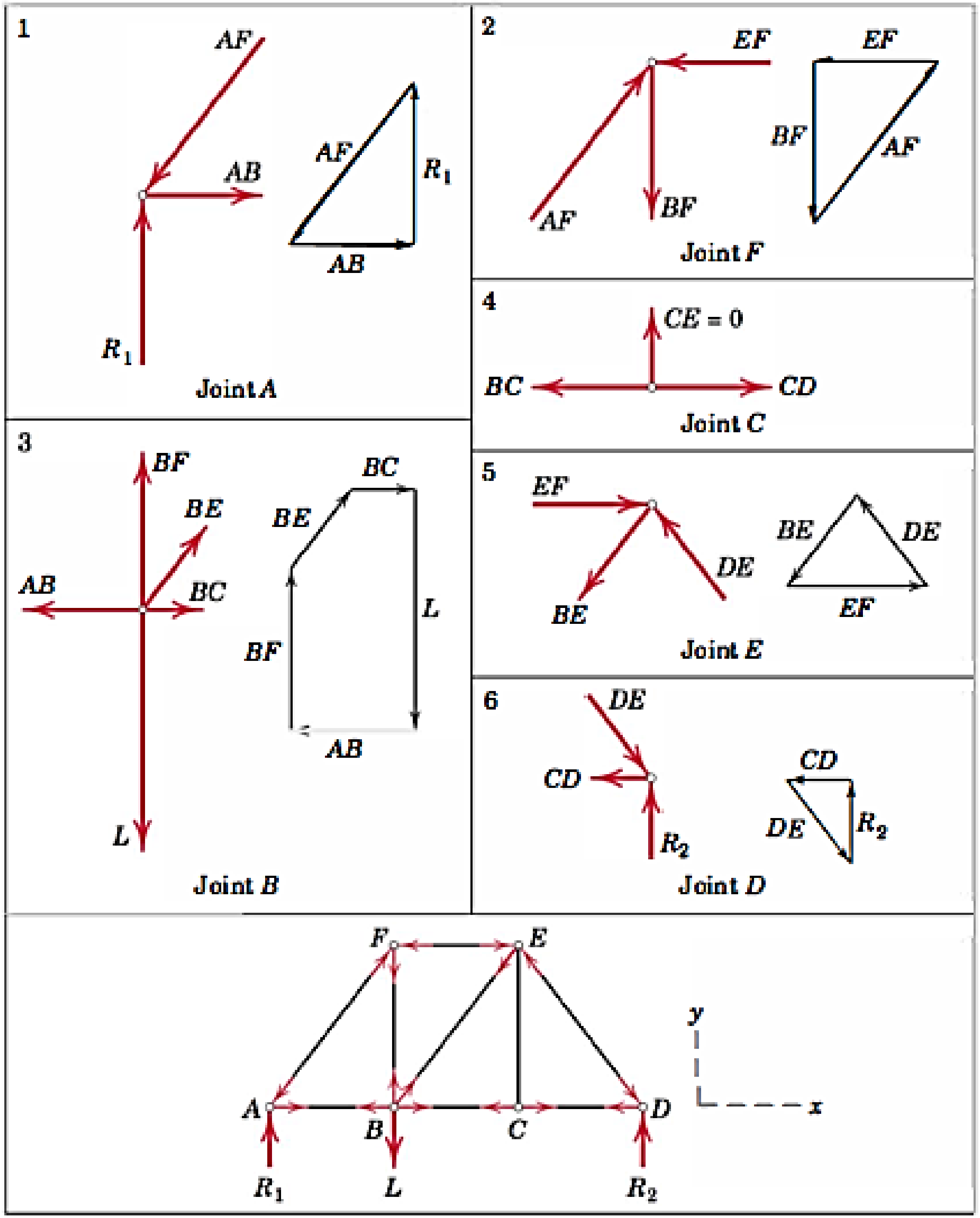
1. Draw Free Body Diagram of Truss
2. Determine external reactions by applying equilibrium equations to the whole truss.
3. Perform the force analysis of the remainder of the truss by Method of Joints



Steps of Analysis







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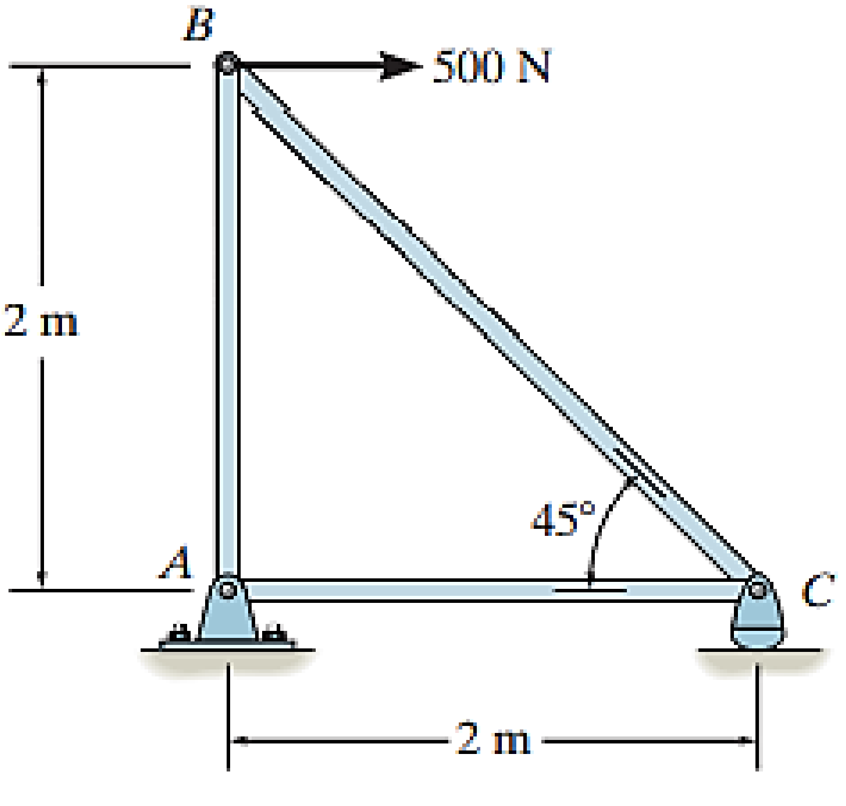
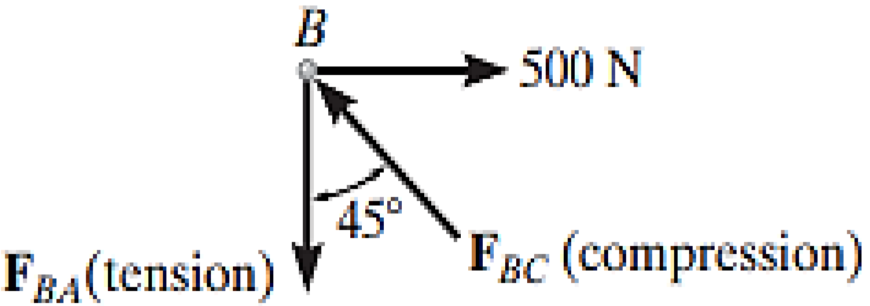
Steps of Analysis

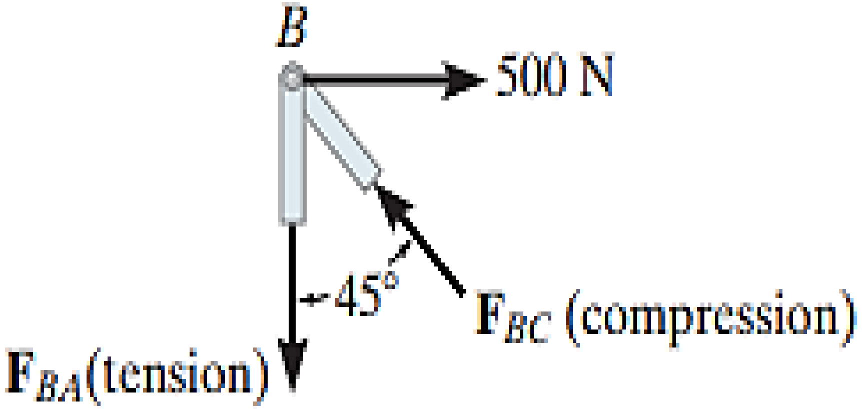
For example:



(a)



(b)

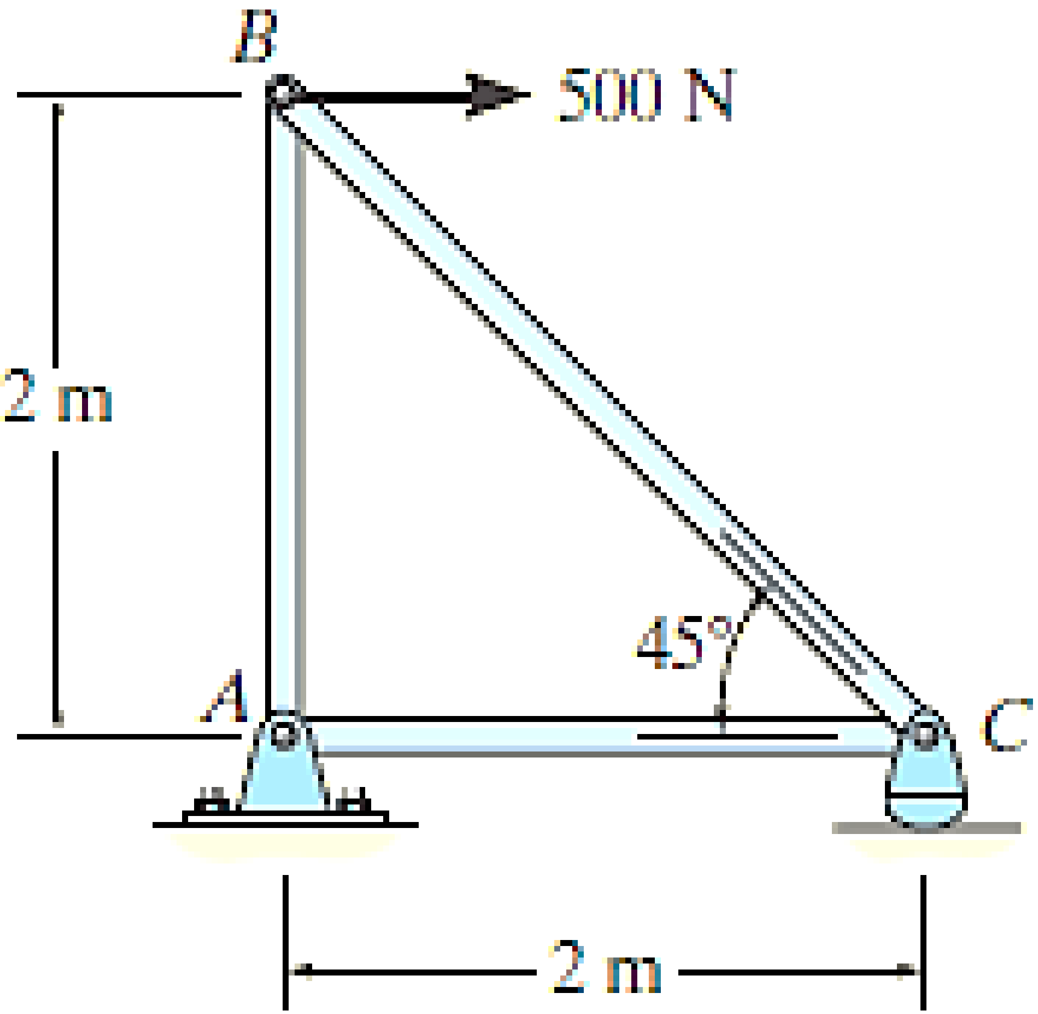


(c)

- $\mathbf{F_{BA}}$ is “pulling” on the pin, which means that member BA is in **tension**; whereas $\mathbf{F_{BC}}$ is “pushing” on the pin, and consequently member BC is in compression.
- When using the method of joints, always start at a joint having at least one known force and at most two unknown forces, $\sum F_x = 0$, $\sum F_y = 0$.
- “**by inspection”,** In more complicated cases, the sense of an unknown member force can be assumed.
- A **positive** answer indicates that the sense is correct, whereas a **negative** answer indicates that the sense shown on the free-body diagram must be reversed.

Sample Problem 1:

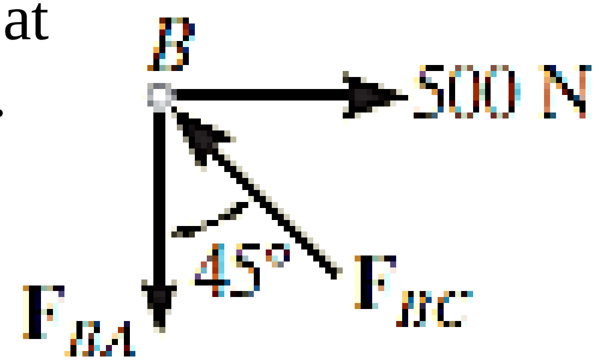
Determine the force in each member of the truss shown in Figure and indicate whether the members are in tension or compression.



(a)

Solution

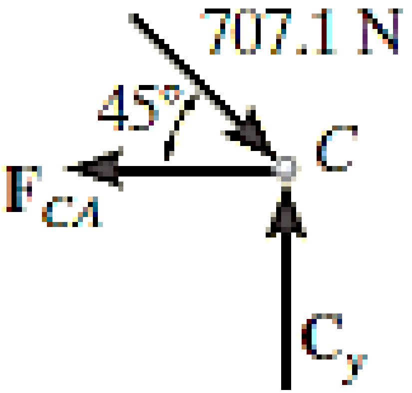
Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint *B* .



Joint *B*. The free-body diagram of the joint at *B* is shown in Figure *b* . Applying the equations of equilibrium, we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad 500 \text{ N} - F_{BC} \sin 45° = 0 \quad F_{BC} = 707.1 \text{ N (C)} \quad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} \cos 45° - F_{BA} = 0 \quad F_{BA} = 500 \text{ N (T)} \quad \textit{Ans.}$$

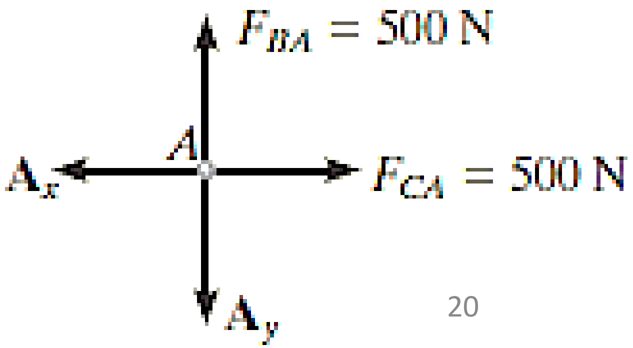


Joint *C*. From the free-body diagram of joint *C* , Fig. *c* , we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad -F_{CA} + 707.1 \cos 45° \text{ N} = 0 \quad F_{CA} = 500 \text{ N (T)} \quad \textit{Ans.}$$

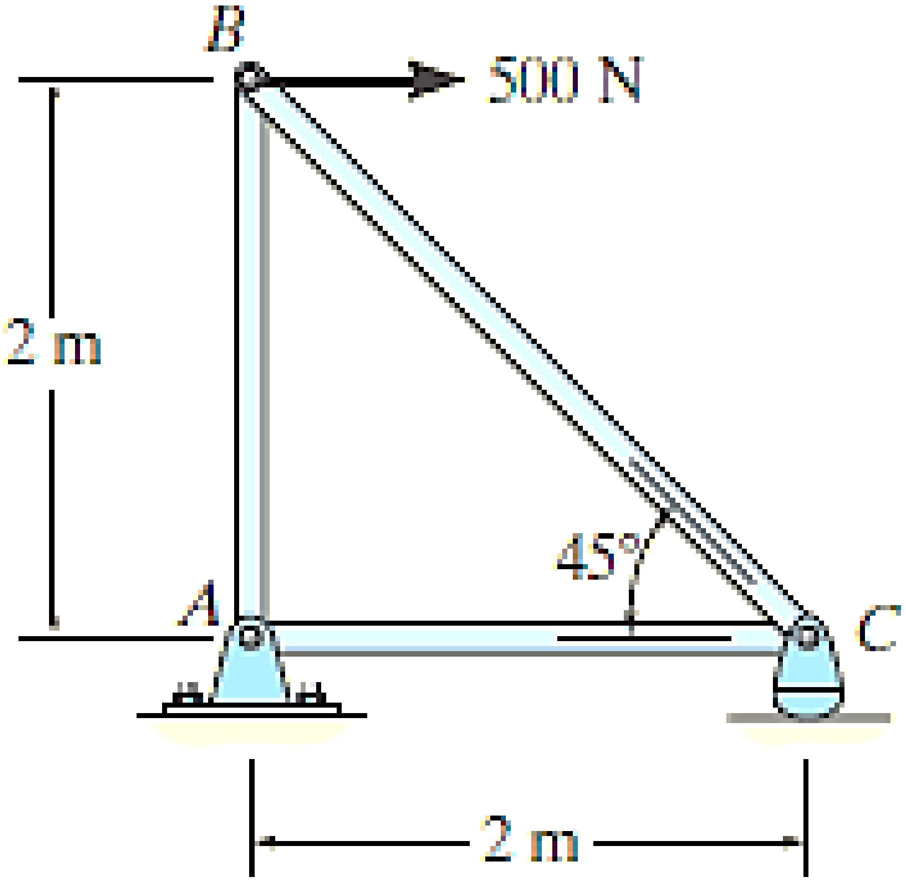
$$+\uparrow \Sigma F_y = 0; \quad C_y - 707.1 \sin 45° \text{ N} = 0 \quad C_y = 500 \text{ N} \quad \textit{Ans.}$$

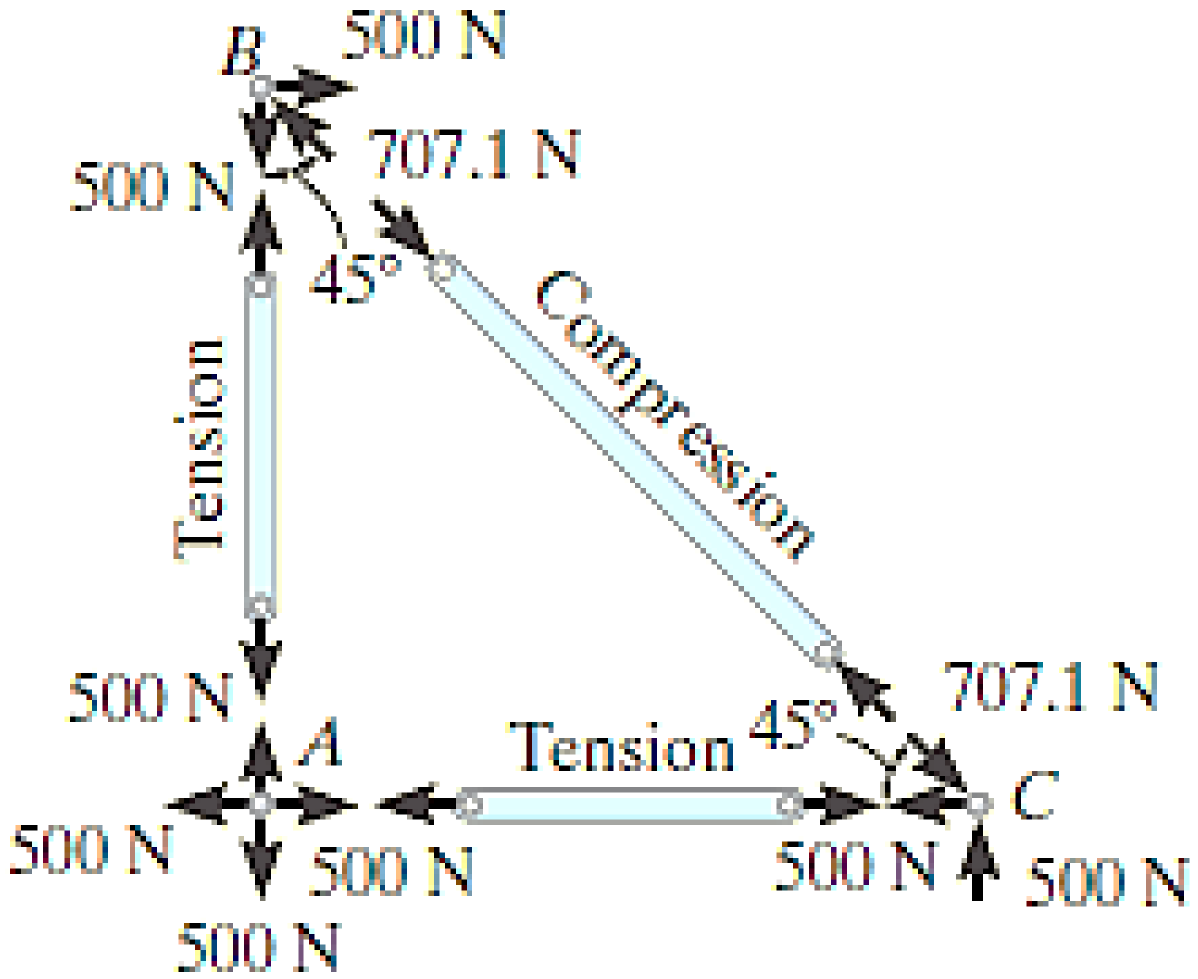
Joint A.



$$\xrightarrow{+} \Sigma F_x = 0; \quad 500 \text{ N} - A_x = 0 \quad A_x = 500 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 500 \text{ N} - A_y = 0 \quad A_y = 500 \text{ N}$$





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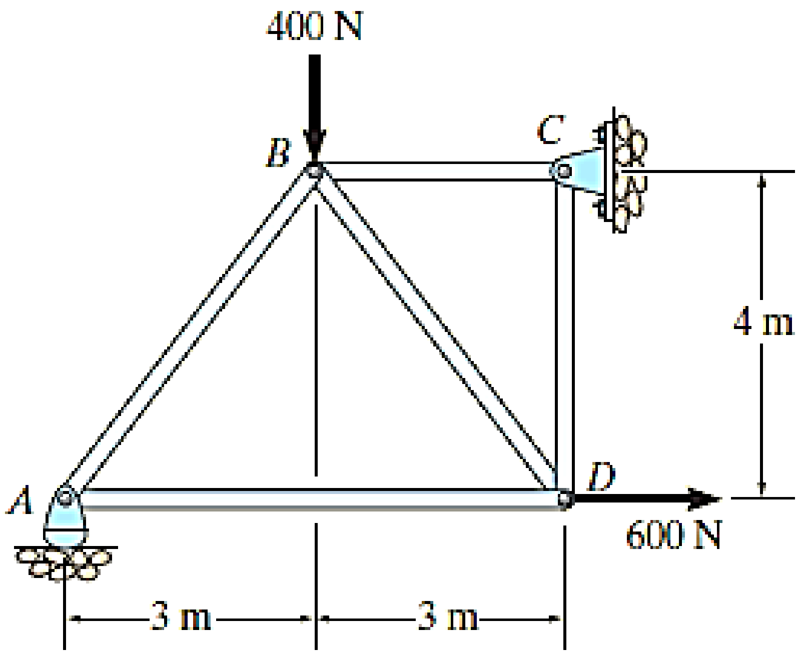
Sample Problem 2:

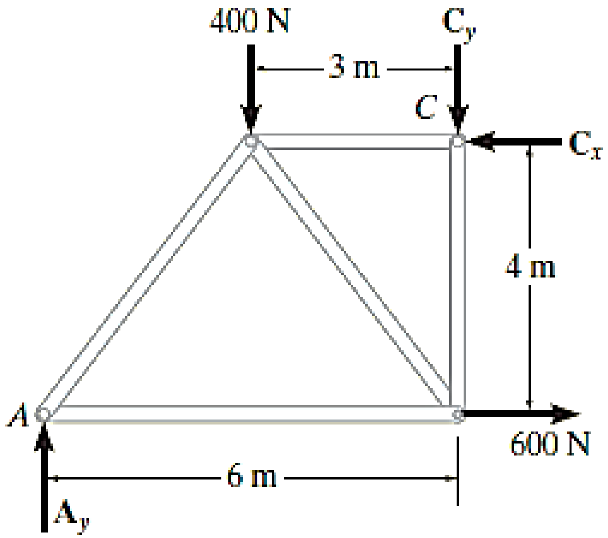
Determine the force in each member of the truss shown in Figure *a* . Indicate whether the members are in tension or compression.

Solution

Support Reactions. No joint can be analyzed until the support reactions are determined, because each joint has at least three unknown forces acting on it. A free-body diagram of the entire truss is given in Fig. *b* . Applying the equations of equilibrium, we have:



(a)



(b)

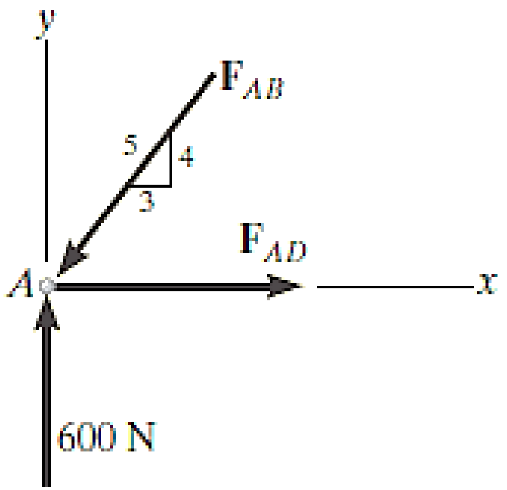
Support Forces

$$\xrightarrow{+} \Sigma F_x = 0; \quad 600\text{ N} - C_x = 0 \quad C_x = 600\text{ N}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad -A_y(6\text{ m}) + 400\text{ N}(3\text{ m}) + 600\text{ N}(4\text{ m}) = 0$$

$$A_y = 600\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 600\text{ N} - 400\text{ N} - C_y = 0 \quad C_y = 200\text{ N}$$

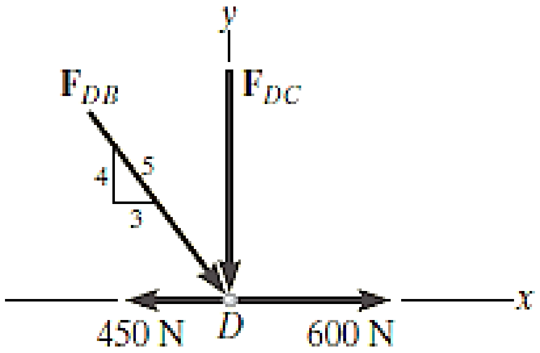


Join A

$$+\uparrow \Sigma F_y = 0; \quad 600\text{ N} - \tfrac{4}{5}F_{AB} = 0 \quad F_{AB} = 750\text{ N} \quad (\text{C})$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AD} - \tfrac{3}{5}(750\text{ N}) = 0 \quad F_{AD} = 450\text{ N} \quad (\text{T})$$

Join D



$$\xrightarrow{+} \Sigma F_x = 0; \quad -450\text{ N} + \tfrac{3}{5}F_{DB} + 600\text{ N} = 0 \quad F_{DB} = -250\text{ N}$$

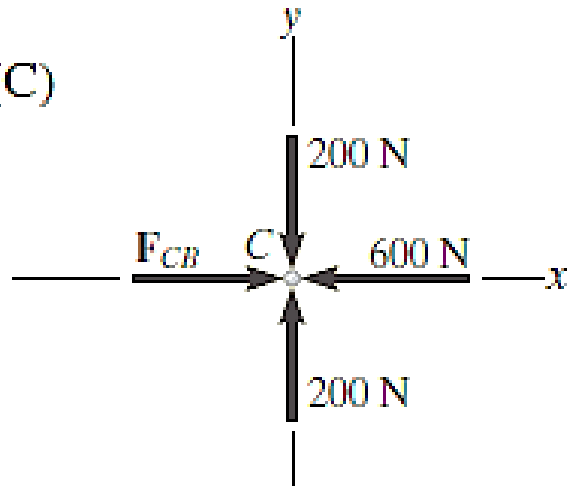
$$F_{DB} = 250\text{ N (T)}$$

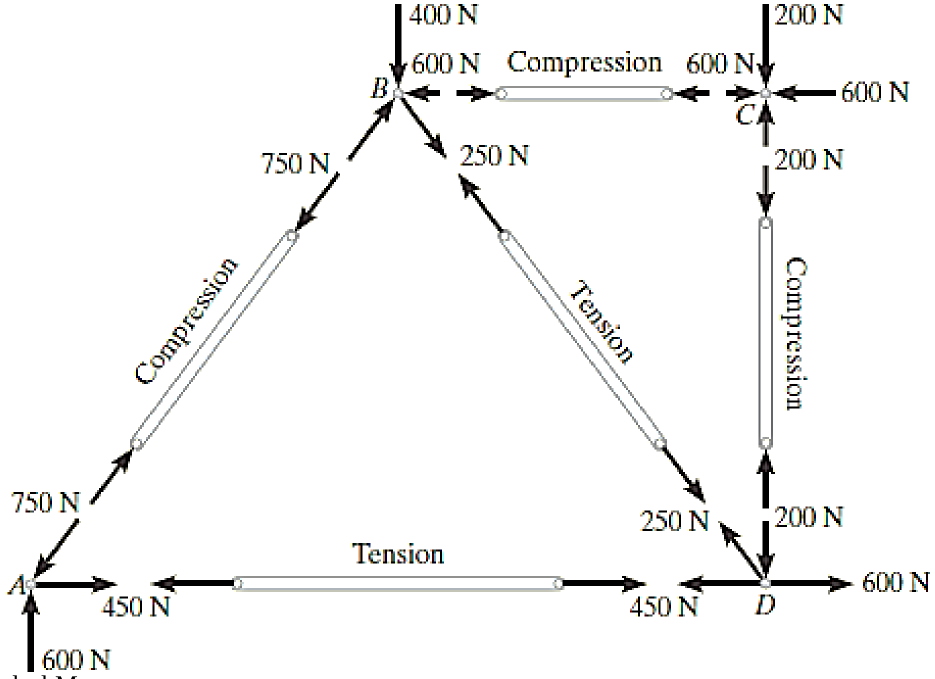
$$+\uparrow \Sigma F_y = 0; \quad -F_{DC} - \tfrac{4}{5}(-250\text{ N}) = 0 \quad F_{DC} = 200\text{ N} \quad (\text{C})$$

Join C

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CB} - 600\text{ N} = 0 \qquad F_{CB} = 600\text{ N} \quad (C)$$

$$+\uparrow \Sigma F_y = 0; \qquad 200\text{ N} - 200\text{ N} \equiv 0 \quad (\text{check})$$





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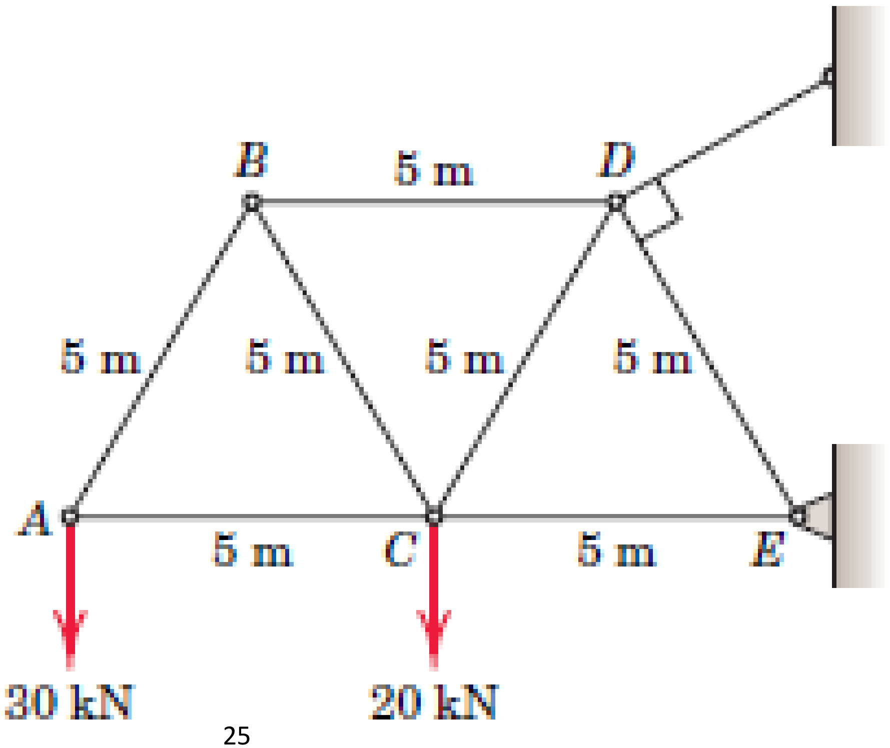
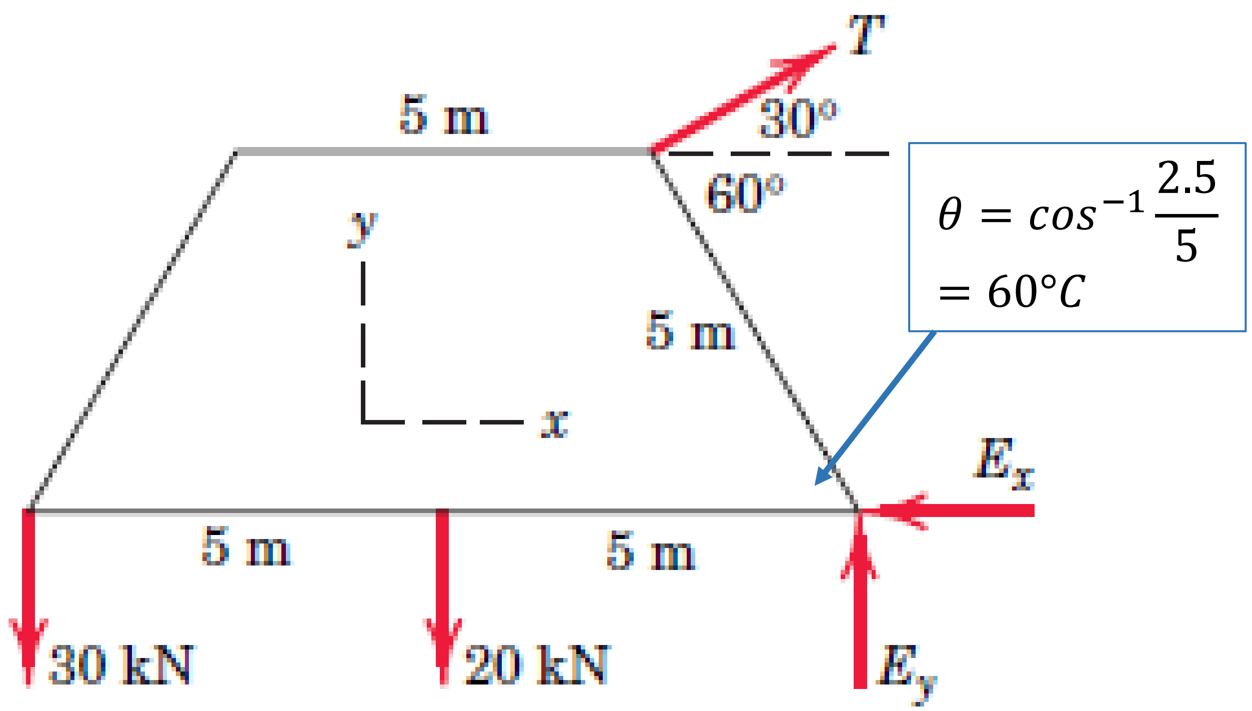
Extra Example:

Compute the force in each member of the loaded cantilever truss by the method of joints.



Solution

For finding the external forces at D and E as a first step. A free-body diagram of the entire truss is given in Figure shown. Applying the equations of equilibrium, we have:

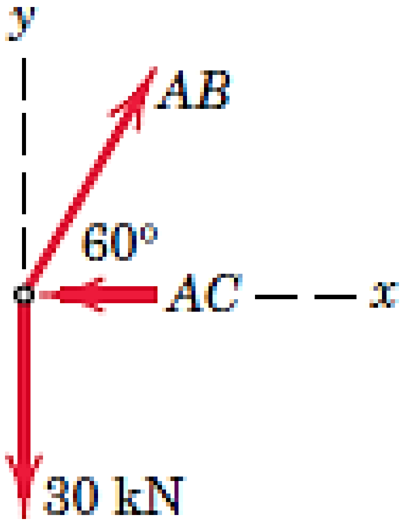


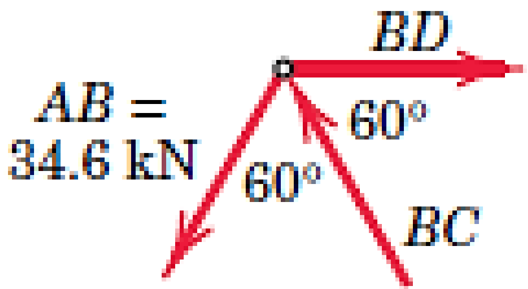
$[\Sigma M_E = 0]$	$5T - 20(5) - 30(10) = 0$	$T = 80$ kN
$[\Sigma F_x = 0]$	$80 \cos 30° - E_x = 0$	$E_x = 69.3$ kN
$[\Sigma F_y = 0]$	$80 \sin 30° + E_y - 20 - 30 = 0$	$E_y = 10$ kN

Join A

$[\Sigma F_y = 0]$ $\quad 0.866AB - 30 = 0 \quad AB = 34.6 \text{ kN } T$

$[\Sigma F_x = 0]$ $\quad AC - 0.5(34.6) = 0 \quad AC = 17.32 \text{ kN } C$





Join B

$[\Sigma F_y = 0]$ $\quad 0.866BC - 0.866(34.6) = 0 \quad BC = 34.6 \text{ kN } C$

$[\Sigma F_x = 0]$ $\quad BD - 2(0.5)(34.6) = 0 \quad BD = 34.6 \text{ kN } T$

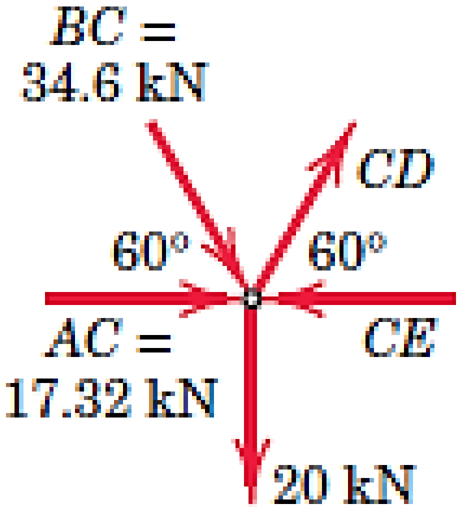
Join C

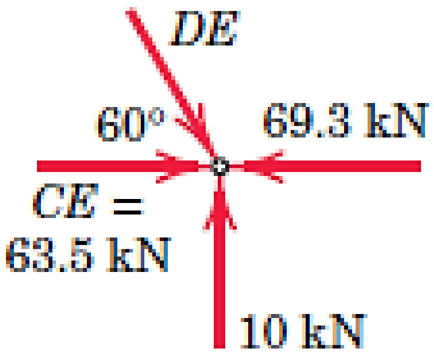
$[\Sigma F_y = 0]$ $\quad 0.866CD - 0.866(34.6) - 20 = 0$

$CD = 57.7 \text{ kN } T$

$[\Sigma F_x = 0]$ $\quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$CE = 63.5 \text{ kN } C$



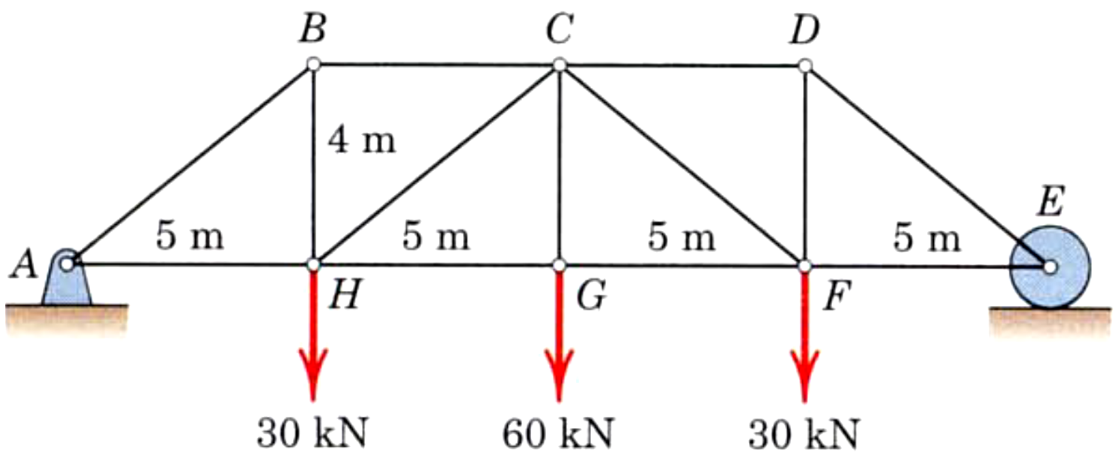


Join E

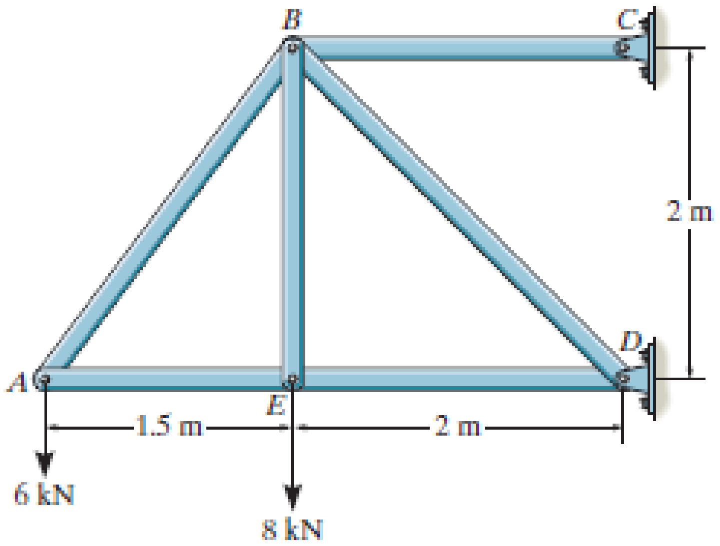
$[\Sigma F_y = 0]$ $\quad 0.866DE = 10 \quad DE = 11.55 \text{ kN } C$

Assignments

1. Determine the force in each member of the loaded truss. Make use of the symmetry of the truss and of the loading.

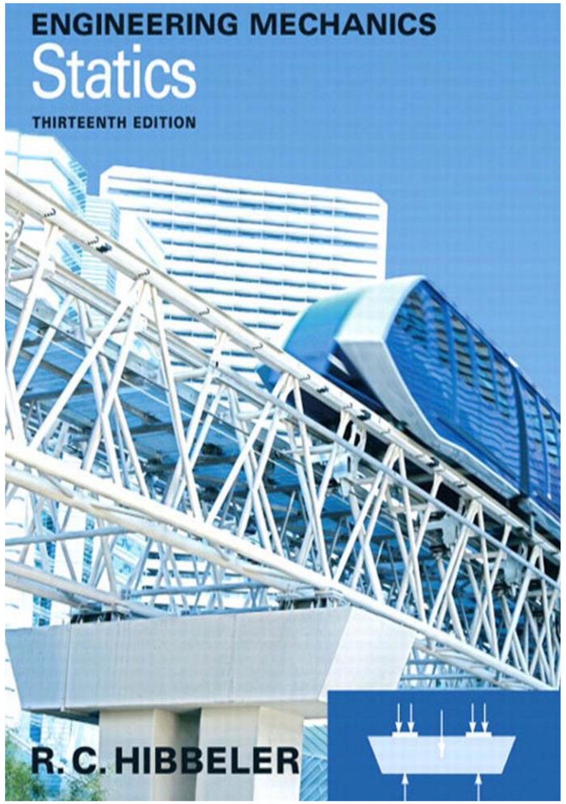


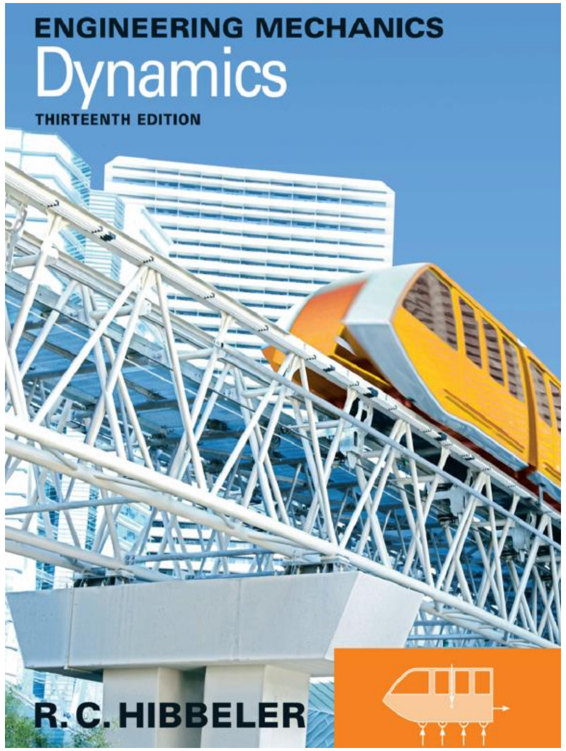
2. Determine the force in each member of the truss and state if the members are in tension or compression.



References:

Engineering Mechanics R.C. Hibbeler 13th edition (Statics and Dynamics).





The end of the lecture
Enjoy your time