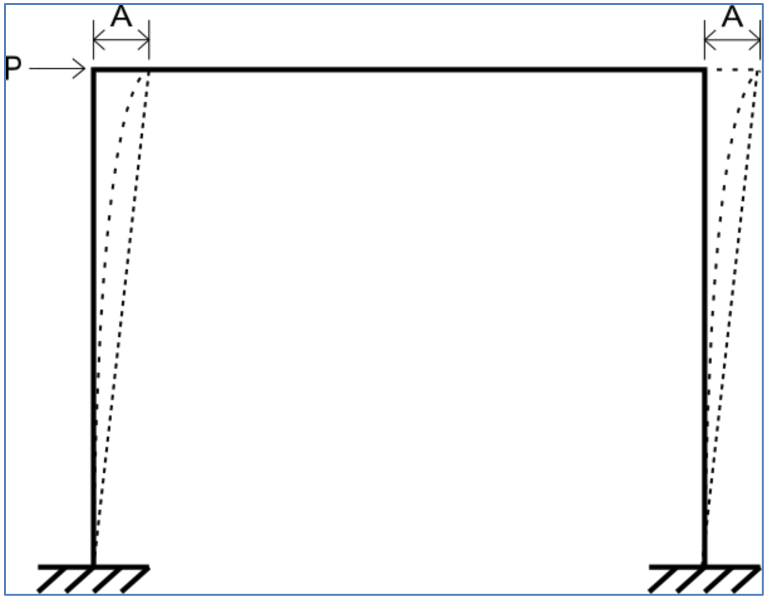
Structural Analysis

Chapter 10

Displacement Method of Analysis

Slope-Deflection Equations



Civil Engineering Department
Third Year (2023-2024)
Structural Analysis
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Displacement Method of Analysis

- Two main methods of analyzing indeterminate structure
  - Force method
    - The method of consistent deformations & the equation of three moment
    - The primary unknowns are forces or moments
  - Displacement method
    - The slope-deflection method & the moment distribution method
    - The primary unknown is displacements (rotation & deflection)
    - It is particularly useful for the analysis of highly statically indeterminate structures
    - Easily programmed on a computer & used to analyze a wide range of indeterminate structures

Displacement Method of Analysis

- Degree of Freedom
  - The number of possible joint rotations & independent joint translations in a structure is called the degree of freedom of the structure
  - In three dimensions each node on a frame can have at most three linear displacements & three rotational displacements.
  - In two dimension each node can have at most two linear displacements & one rotational displacement.

$$DOF = n\,J - R$$

- n number of possible joint's movements
- J number of joints
- R number of restrained movements

Displacement Method of Analysis

- Degree of Freedom

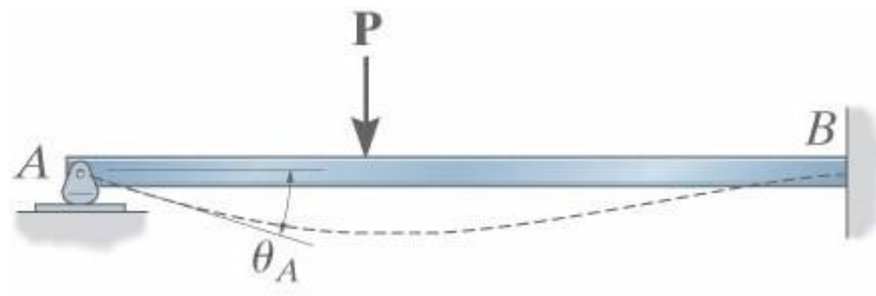
DOF = n J– R

  - n number of possible joint's movements
    - *n = 2 for two dimensional truss structures*
    - *n = 3 for three dimensional truss structures*
    - *n= 3 for two dimensional frame structures*
    - *n= 6 for three dimensional frame structures*
  - J number of joints
  - R number of restrained movements
- Neglecting Axial deformation

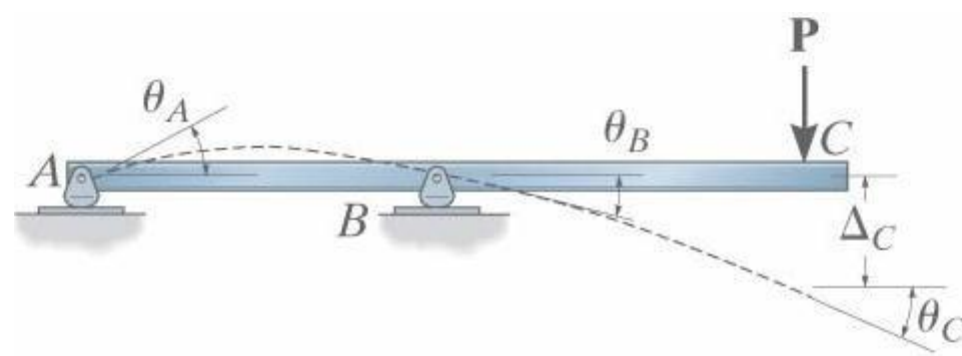
DOF = n J– R – m

  - m number of members

Displacement Method of Analysis



(a)



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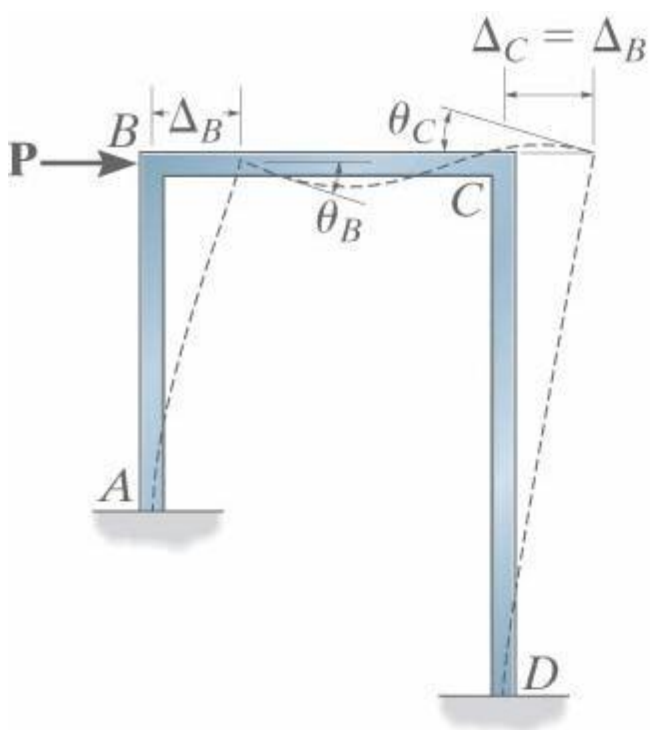
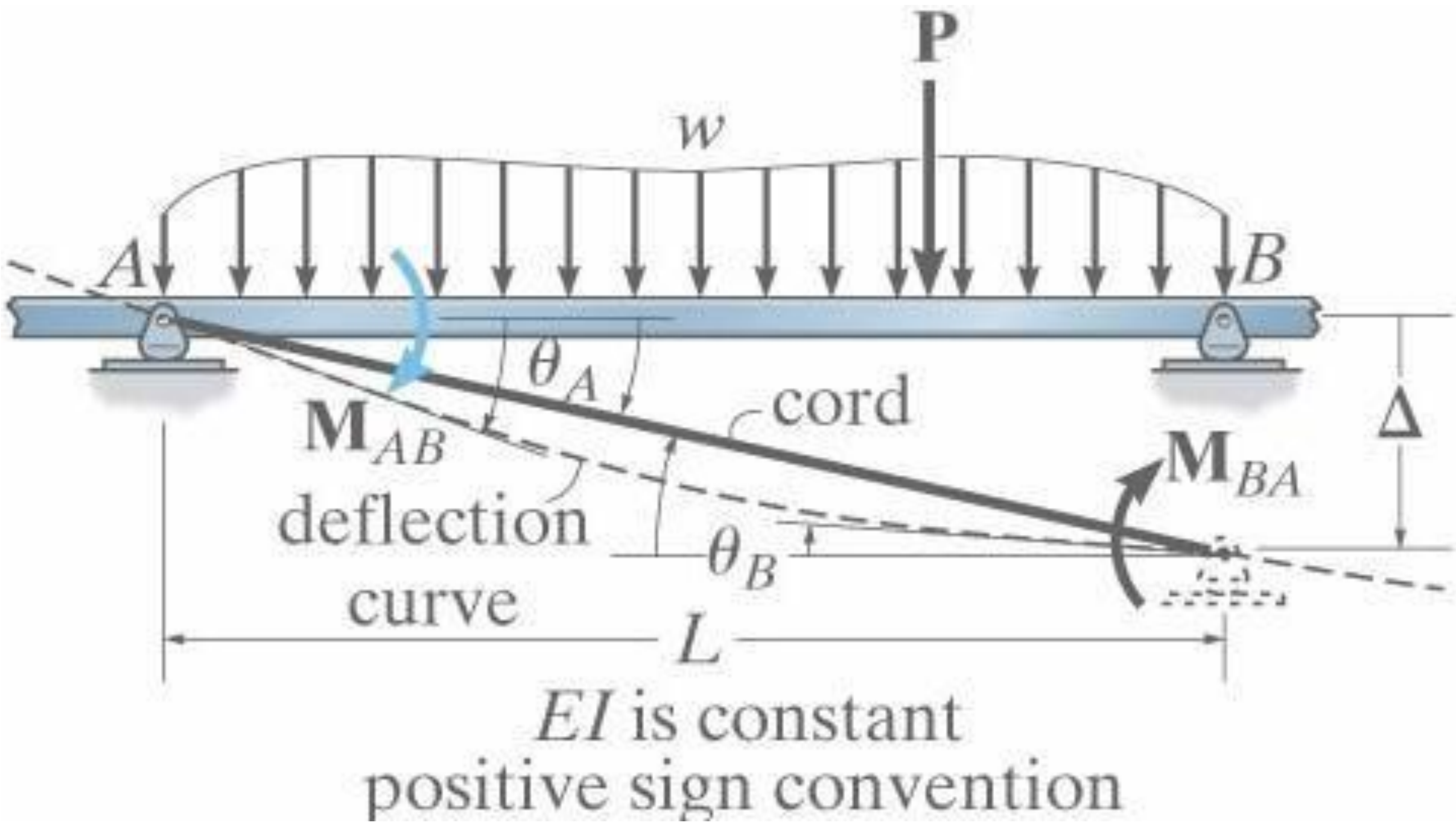


(c)

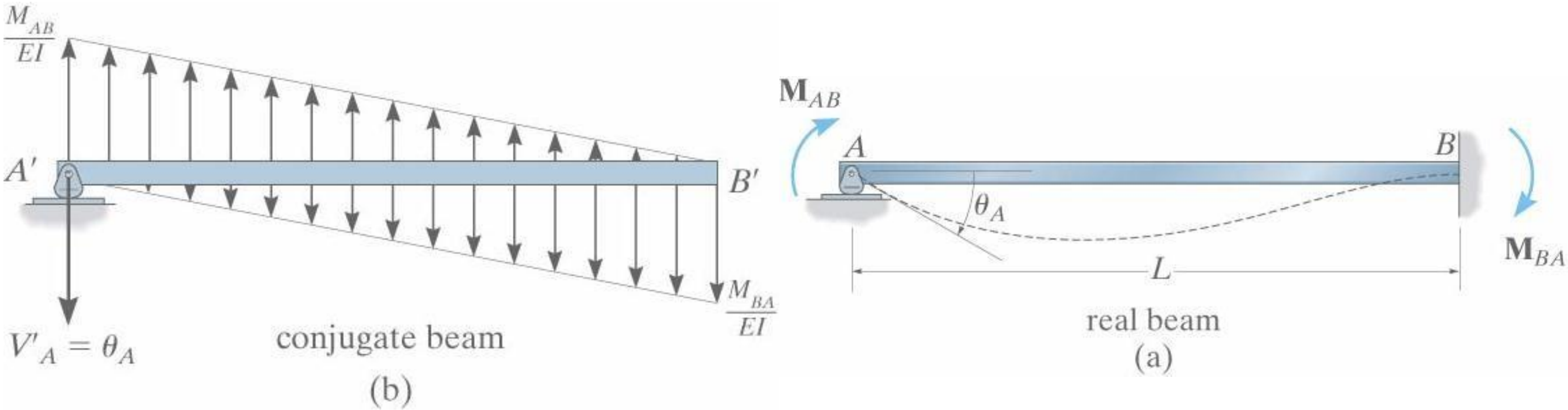
Slope-Deflection Equations

Derivation



Slope-Deflection Equations

- Angular Displacement at A, θ_A



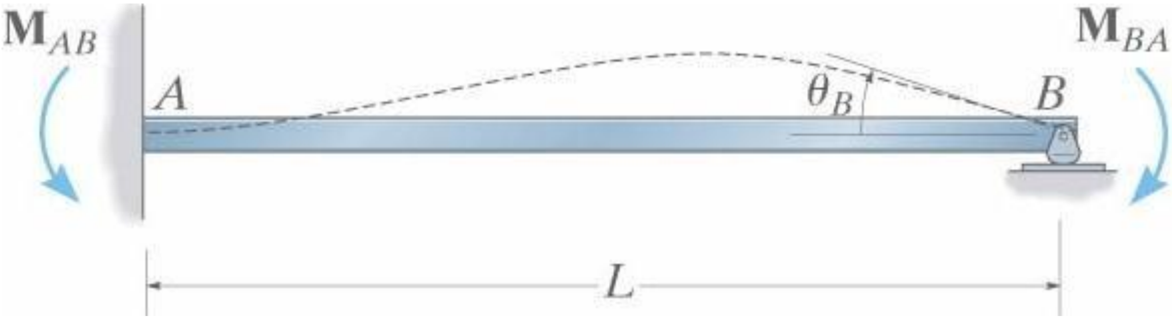
$$\sum M_{A'} = 0 \quad \Rightarrow \quad \left[\frac{1}{2}\left(\frac{M_{AB}}{EI}\right)L\right]\frac{L}{3} - \left[\frac{1}{2}\left(\frac{M_{BA}}{EI}\right)L\right]\frac{2L}{3} = 0$$

$$\sum M_{B'} = 0 \quad \Rightarrow \quad \left[\frac{1}{2}\left(\frac{M_{BA}}{EI}\right)L\right]\frac{L}{3} - \left[\frac{1}{2}\left(\frac{M_{AB}}{EI}\right)L\right]\frac{2L}{3} + \theta_A L = 0$$

$$M_{AB} = \frac{4EI}{L}\theta_A \quad \& \quad M_{BA} = \frac{2EI}{L}\theta_A$$

Slope-Deflection Equations

- Angular Displacement at B, θ_B

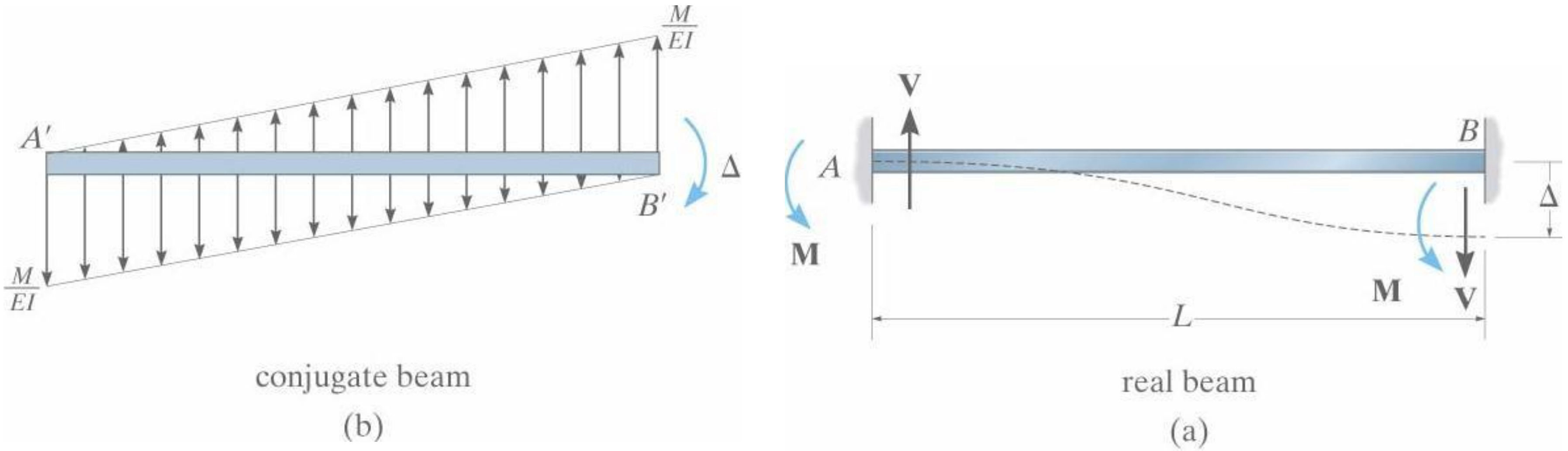


$$M_{BA} = \frac{4EI}{L}\theta_B$$

$$M_{AB} = \frac{2EI}{L}\theta_B$$

Slope-Deflection Equations

- Relative Linear Displacement Δ

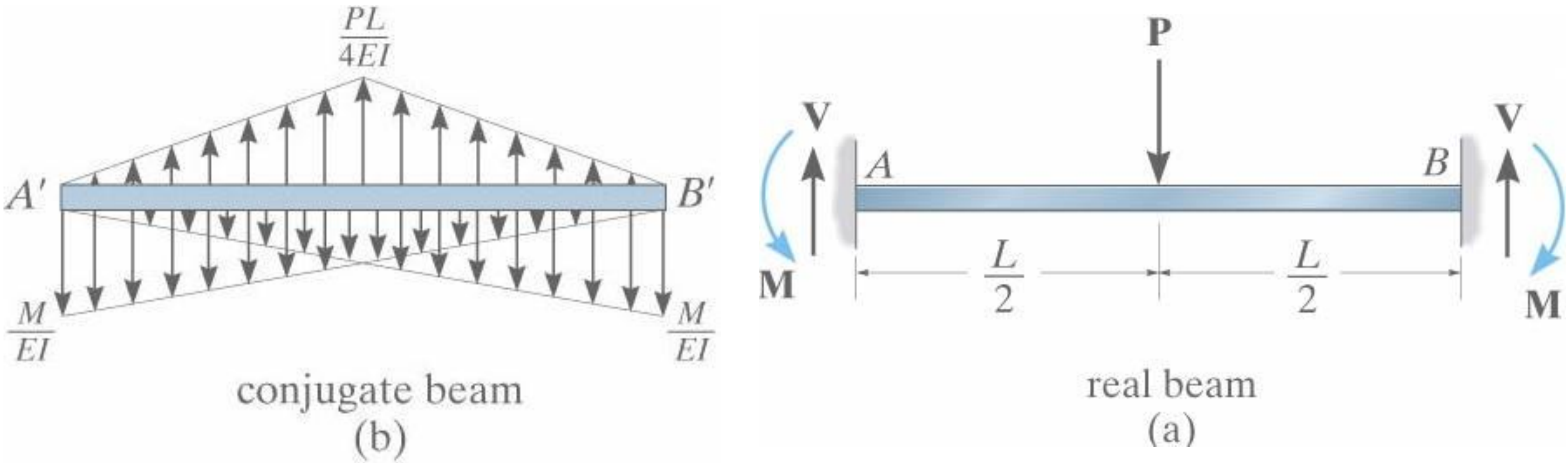


$$\sum M_{B'} = 0. \qquad \left[\frac{1}{2}\left(\frac{M}{EI}\right)L\right]\frac{2L}{3} - \left[\frac{1}{2}\left(\frac{M}{EI}\right)L\right]\frac{L}{3} + \Delta = 0.$$

$$M_{AB} = M_{BA} = M = \frac{-6EI}{L^2}\Delta$$

Slope-Deflection Equations

- Fixed-End Moment

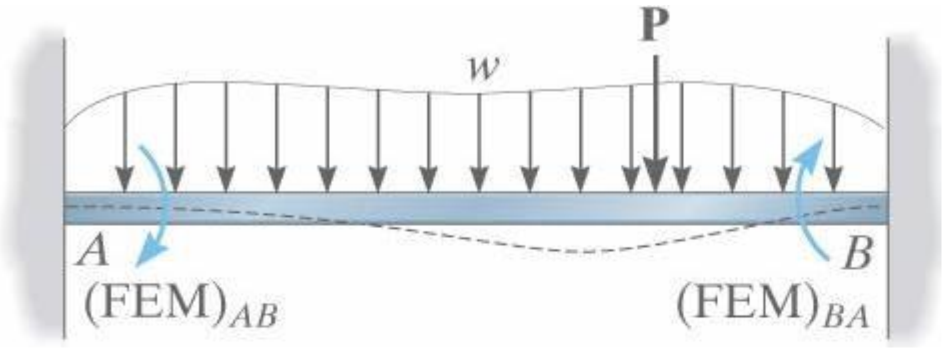


$$\sum F_y = 0. \qquad \left[\frac{1}{2}\left(\frac{PL}{EI}\right)L\right] - 2\left[\frac{1}{2}\left(\frac{M}{EI}\right)L\right] = 0.$$

Fixed-End moment FEM

$$M = \frac{PL}{8}$$

Slope-Deflection Equations



$$M_{AB} = (FEM)_{AB}$$
$$M_{BA} = (FEM)_{BA}$$

Slope-Deflection Equations

- Slope-deflection equations
  - The resultant moment (adding all equations together)

$$M_{AB} = 2E\left(\frac{I}{L}\right)\left[2\theta_A + \theta_B - 3\left(\frac{\Delta}{L}\right)\right] + (FEM)_{AB}$$

$$M_{BA} = 2E\left(\frac{I}{L}\right)\left[2\theta_B + \theta_A - 3\left(\frac{\Delta}{L}\right)\right] + (FEM)_{BA}$$

- Lets represent the member stiffness as k = I/L &
- The span ration due to displacement as $\psi = \Delta/L$
- Referring to one end of the span as near end (N) & the other end as the far end (F).

  - Rewrite the equations

$$M_N = 2EK\left[2\theta_N + \theta_F - 3\psi\right] + (FEM)_N$$

$$M_F = 2EK\left[2\theta_F + \theta_N - 3\psi\right] + (FEM)_F$$

Slope-Deflection Equations

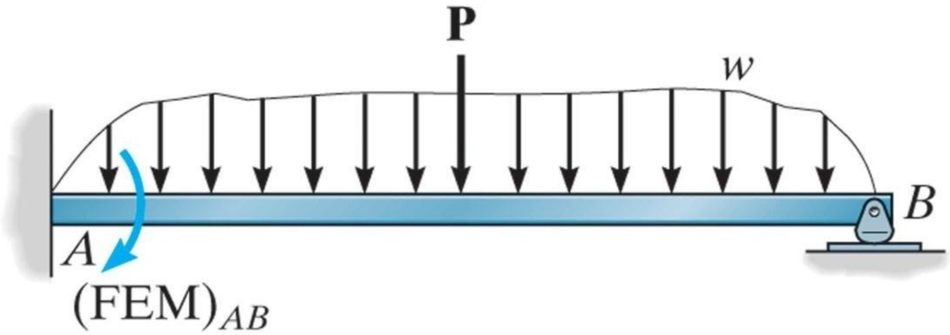
- Slope-deflection equations for pin supported End Span
  - If the far end is a pin or a roller support

$$M_N = 2EK[2\theta_N + \theta_F - 3\psi] + (FEM)'_N$$

$$0 = 2EK[2\theta_F + \theta_N - 3\psi] + 0$$

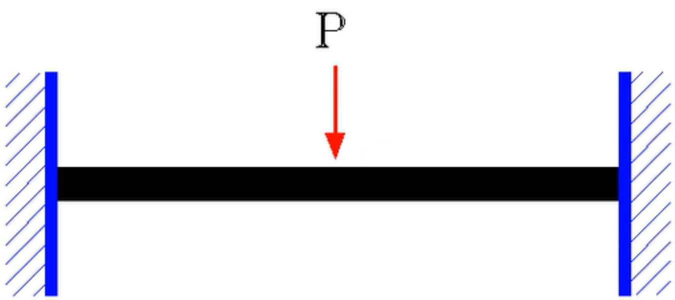
  - Multiply the first equation by 2 and subtracting the second equation from it

$$M_N = 3EK[\theta_N - \psi] + (FEM)'_N$$



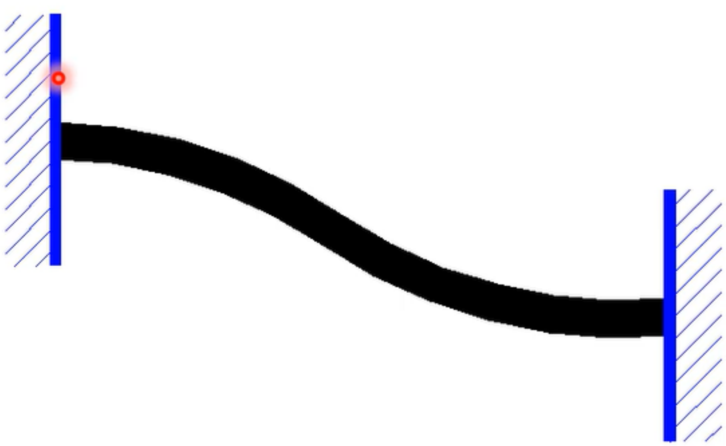
- The slope-deflection equations relate the momentes at the ends of a member to the rotations and displacements of its ends and the external loads applied to the member.

Source of Moments :



Load

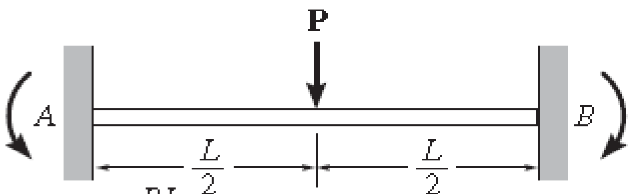
Left Rotation



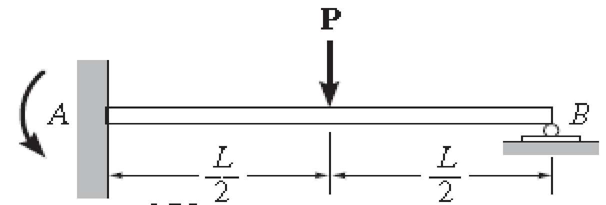
Settlement

Right Rotation

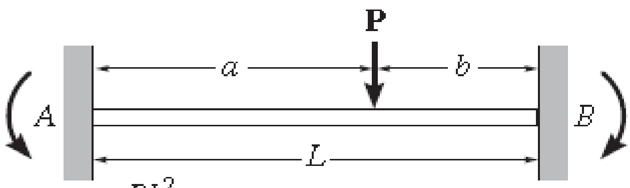
Fixed End Moment Table



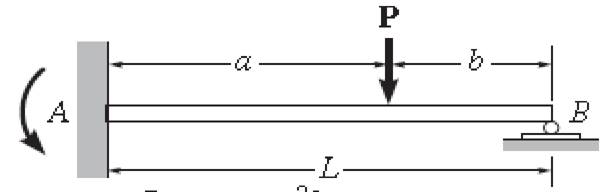
$(FEM)_{AB} = \frac{PL}{8}$ $(FEM)_{BA} = \frac{PL}{8}$



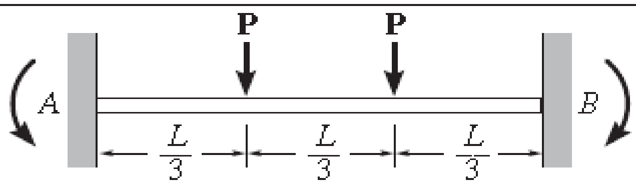
$(FEM)'_{AB} = \frac{3PL}{16}$



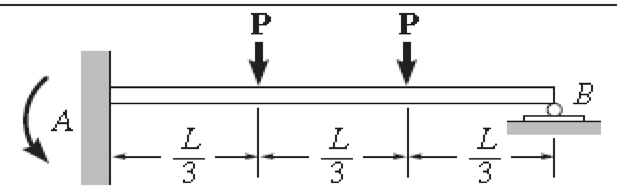
$(FEM)_{AB} = \frac{Pb^2a}{L^2}$ $(FEM)_{BA} = \frac{Pa^2b}{L^2}$



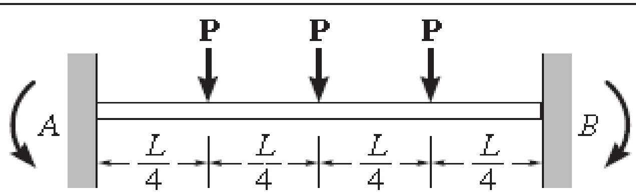
$(FEM)'_{AB} = (\frac{P}{L^2})(b^2a + \frac{a^2b}{2})$



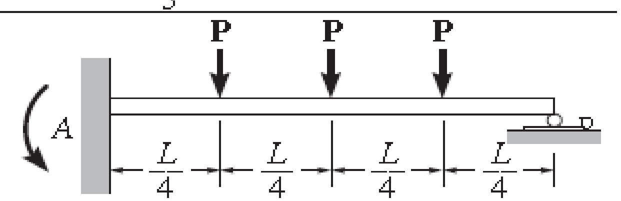
$(FEM)_{AB} = \frac{2PL}{9}$ $(FEM)_{BA} = \frac{2PL}{9}$



$(FEM)'_{AB} = \frac{PL}{3}$

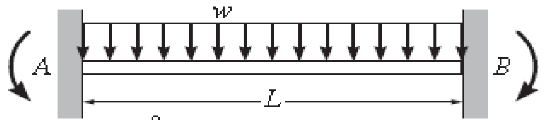


$(FEM)_{AB} = \frac{5PL}{16}$ $(FEM)_{BA} = \frac{5PL}{16}$

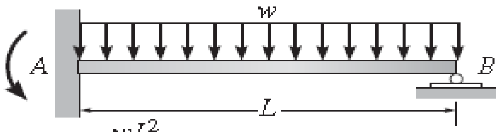


$(FEM)'_{AB} = \frac{45PL}{96}$

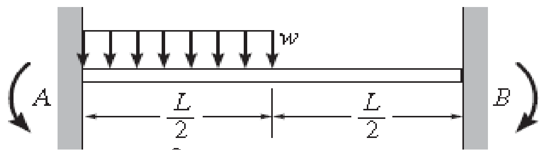
Fixed End Moment Table



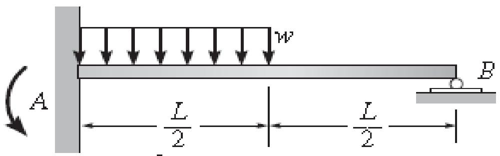
$(FEM)_{AB} = \frac{wL^2}{12}$ $\quad (FEM)_{BA} = \frac{wL^2}{12}$



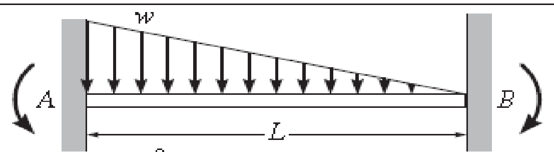
$(FEM)'_{AB} = \frac{wL^2}{8}$



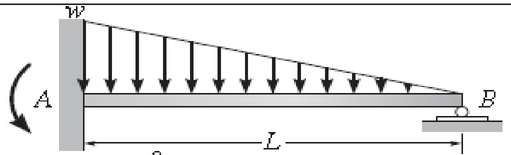
$(FEM)_{AB} = \frac{11wL^2}{192}$ $\quad (FEM)_{BA} = \frac{5wL^2}{192}$



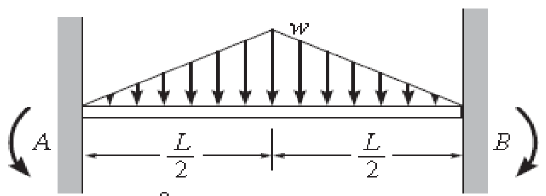
$(FEM)'_{AB} = \frac{9wL^2}{128}$



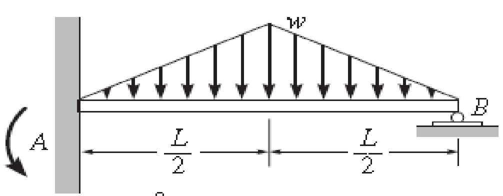
$(FEM)_{AB} = \frac{wL^2}{20}$ $\quad (FEM)_{BA} = \frac{wL^2}{30}$



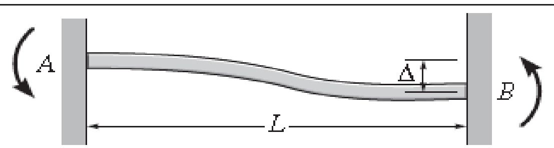
$(FEM)'_{AB} = \frac{wL^2}{15}$



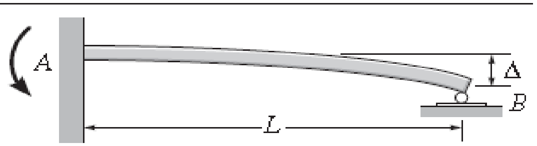
$(FEM)_{AB} = \frac{5wL^2}{96}$ $\quad (FEM)_{BA} = \frac{5wL^2}{96}$



$(FEM)'_{AB} = \frac{5wL^2}{64}$

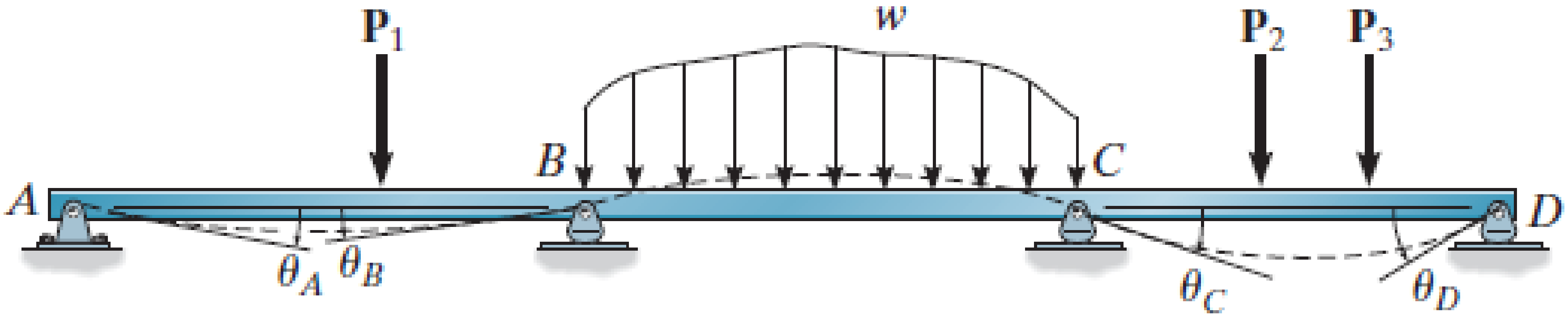


$(FEM)_{AB} = \frac{6EI\Delta}{L^2}$ $\quad (FEM)_{BA} = \frac{6EI\Delta}{L^2}$



$(FEM)'_{AB} = \frac{3EI\Delta}{L^2}$

Summary



$$M_N = 2EK\left[2\theta_N + \theta_F - 3\psi\right] + (FEM)_N$$

$$M_F = 2EK\left[2\theta_F + \theta_N - 3\psi\right] + (FEM)_F$$

$$M_N = 3EK\left[\theta_N - \psi\right] + (FEM)'_N$$

Slope-Deflection Equations

- M_N, M_F = the internal moment in the near & far end of the span.
  - Considered **positive** when acting in a **clockwise** direction
- E, k = modulus of elasticity of material & span stiffness k = I/L
- θ_N, θ_F = near & far end slope of the span at the supports in radians.
  - Considered **positive** when acting in a **clockwise** direction
- ψ = span ration due to a linear displacement Δ / L.
  - If the right end of a member sinks with respect the the left end the sign is positive

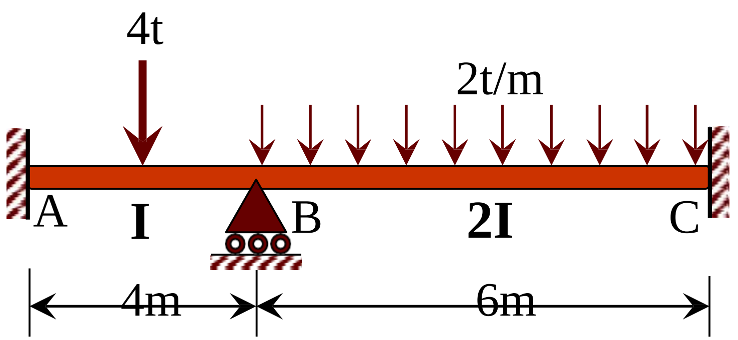
Slope-Deflection Equations

Steps to analyzing beams using this method

- Find the fixed end moments of each span (both ends left & right)
- Apply the slope deflection equation on each span & identify the unknowns
- Write down the joint equilibrium equations
- Solve the equilibrium equations to get the unknown rotation & deflections
- Determine the end moments and then treat each span as simply supported beam subjected to given load & end moments so you can workout the reactions & draw the bending moment & shear force diagram

Example 1

Draw the bending moment & shear force diagram.



Fixed End Moment

$$FEM_{AB} = -\frac{4\times 4}{8} = -2t.m \qquad FEM_{BA} = 2t.m$$

$$FEM_{BC} = -\frac{2\times 6^2}{12} = -6t.m \qquad FEM_{CB} = 6t.m$$

Slope Deflection Equations

$$M_{AB} = 2E\left(\frac{I}{4}\right)[2\theta_A + \theta_B - 0] - 2 \xrightarrow{\theta_A=0} M_{AB} = 0.5EI\theta_B - 2$$

$$M_{BA} = 2E\left(\frac{I}{4}\right)[\theta_A + 2\theta_B - 0] + 2 \xrightarrow{\theta_A=0} M_{BA} = EI\theta_B + 2$$

$$M_{BC} = 2E\left(\frac{2I}{6}\right)[2\theta_B + \theta_C - 0] - 6 \xrightarrow{\theta_C=0} M_{BC} = \frac{4}{3}EI\theta_B - 6$$

$$M_{CB} = 2E\left(\frac{2I}{6}\right)[\theta_B + 2\theta_C - 0] + 6 \xrightarrow{\theta_C=0} M_{CB} = \frac{2}{3}EI\theta_B + 6$$

Joint Equilibrium Equations

Joint B

$$M_{BA} + M_{BC} = 0$$

$$EI\theta_B + 2 + \frac{4}{3} EI\theta_B - 6 = 0. \quad \rightarrow \quad \theta_B = \frac{1.7}{EI}$$

Substituting in slope deflection equations

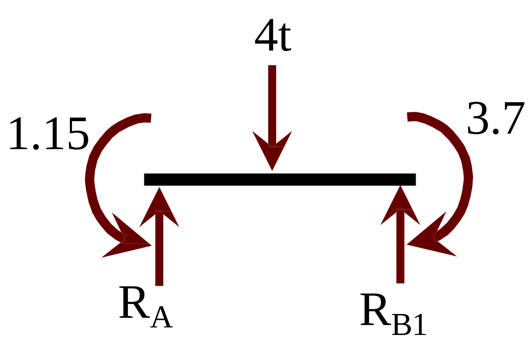
$$M_{AB} = 0.5EI\frac{1.7}{EI} - 2 = -1.15t.m$$

$$M_{BA} = EI\frac{1.7}{EI} + 2 = 3.7t.m$$

$$M_{BC} = -3.7t.m$$

$$M_{CB} = \frac{2}{3}EI\frac{1.7}{EI} + 6 = 7.1t.m$$

Computing The Reactions

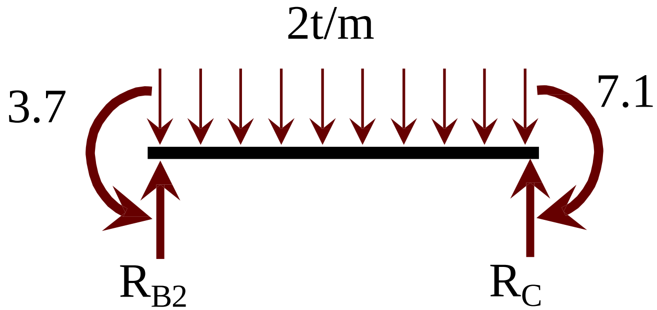


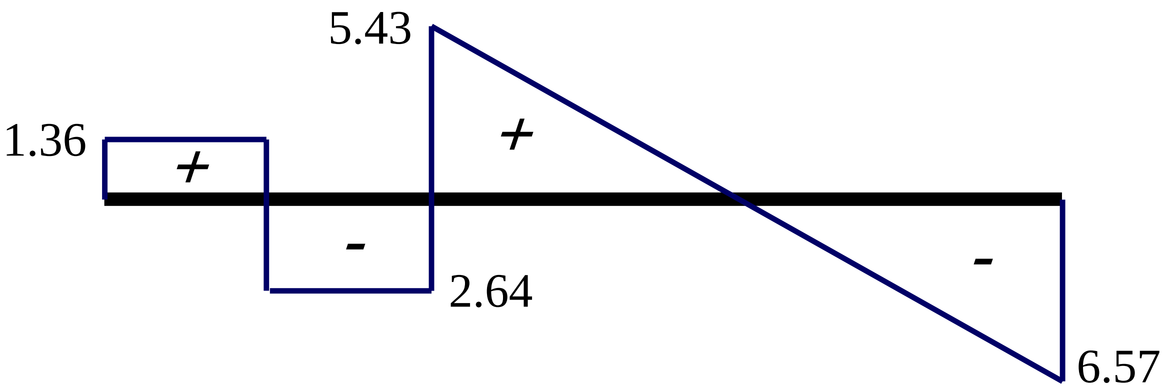
$$R_A = \frac{1.15 + 4 \times 2 - 3.7}{4} = 1.36t$$

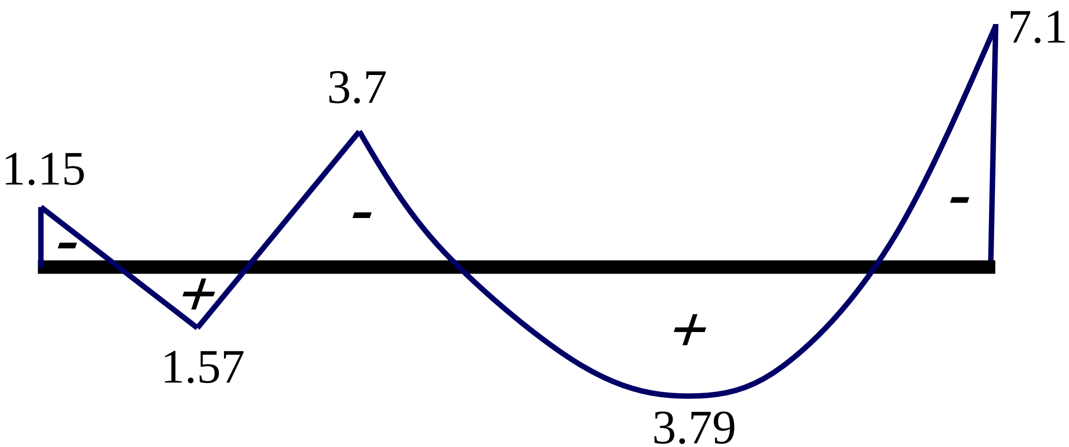
$$R_{B1} = 4 - 1.36 = 2.64t$$

$$R_{B2} = \frac{2\times6\times3+3.7-7.1}{6} = 5.43t$$

$$R_C = 2\times6-5.43 = 6.57t$$

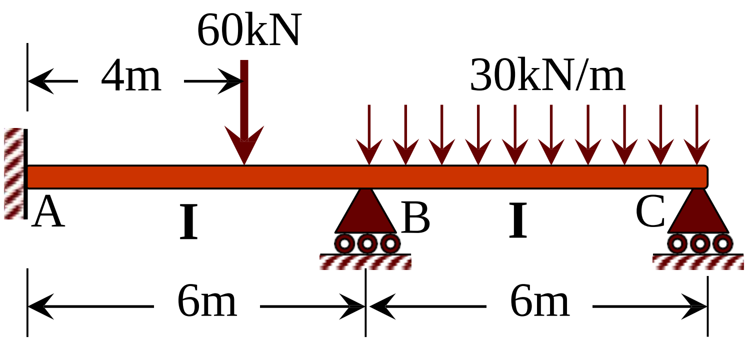






Example 2

- Determine the internal moments in the beam at the supports.



Fixed End Moment

$$FEM_{AB} = -\frac{60\times4\times2^2}{6^2} = -26.67kN.m$$

$$FEM_{BA} = \frac{60\times2\times4^2}{6^2} = 53.33kN.m$$

$$FEM_{BC} = -\frac{30\times6^2}{8} = -135kN.m$$

Slope Deflection Equations

$$M_{AB} = 2E\left(\frac{I}{6}\right)[2\theta_A + \theta_B - 0] - 26.67 \xrightarrow{\theta_A=0} M_{AB} = \frac{1}{3}EI\theta_B - 26.67$$

$$M_{BA} = 2E\left(\frac{I}{6}\right)[\theta_A + 2\theta_B - 0] + 53.33 \xrightarrow{\theta_A=0} M_{BA} = \frac{2}{3}EI\theta_B + 53.33$$

$$M_{BC} = 3E\left(\frac{I}{6}\right)[\theta_B - 0] - 135 \longrightarrow M_{BC} = \frac{1}{2}EI\theta_B - 135$$

Joint Equilibrium Equations

Joint B

$$M_{BA} + M_{BC} = 0$$

$$\frac{2}{3}EI\theta_B + 53.33 + \frac{1}{2}EI\theta_B - 135 = 0 \Rightarrow \quad \frac{7}{6}EI\theta_B = 81.67$$

$$EI\,\theta_B = 70$$

Substituting in slope deflection equations

$$M_{AB} = \frac{1}{3}(70) - 26.67 = -3.33\,kN.m$$

$$M_{BA} = \frac{2}{3}(70) + 53.33 = 100\,kN.m$$

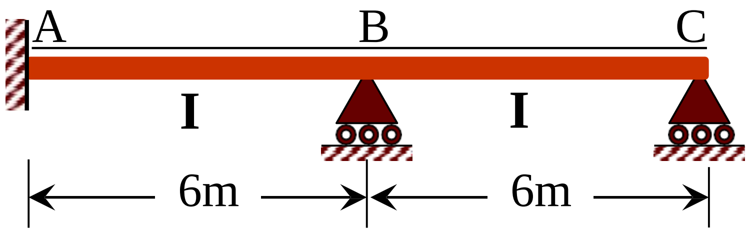
$$M_{BC} = \frac{1}{2}(70) - 135 = -100\,kN.m$$

Example 3

- Example 2: Determine the internal moments in the beam at the supports
  - Support A; downward movement of 0.3cm & clockwise rotation of 0.001 rad. Support B; downward movement of 1.2cm. Support C downward movement of 0.6cm. EI = 5000 t.m^2

Fixed End Moment

$$FEM_{AB} = FEM_{BA} = FEM_{BC} = FEM_{CB} = 0.$$



Displacements:

$$\theta_A = +0.001$$

$$\psi_{AB} = \frac{1.2 - 0.3}{600} = 0.0015$$

$$\psi_{BC} = \frac{0.6 - 1.2}{600} = -0.001$$

Slope Deflection Equations

$$M_{AB} = \left(\frac{2 \times 5000}{6}\right)\left[2 \times 0.001 + \theta_B - 3 \times 0.0015\right]$$

$$M_{BA} = \left(\frac{2 \times 5000}{6}\right)\left[2\theta_B + 0.001 - 3 \times 0.0015\right]$$

$$M_{BC} = \left(\frac{3 \times 5000}{6}\right)\left[\theta_B + 0.001\right]$$

Joint Equilibrium Equations

Joint B

$$M_{BA} + M_{BC} = 0$$

$$7\theta_B - 0.004 = 0$$

$$\theta_B = 0.00057\,rad$$

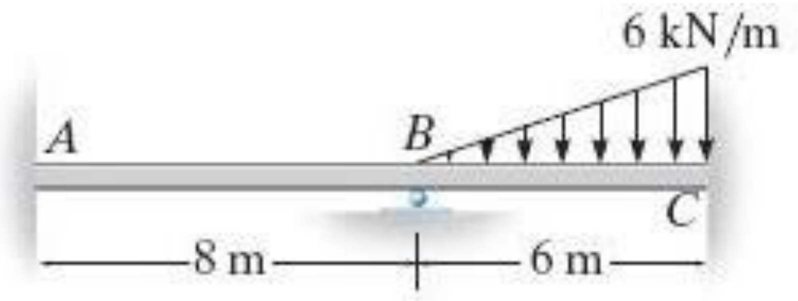
Substituting in slope deflection equations

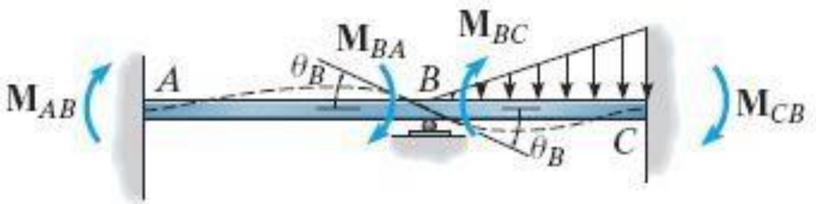
$$M_{AB} = \left(\frac{2 \times 5000}{6}\right)\left[2 \times 0.001 + 0.00057 - 3 \times 0.0015\right] = -3.22\,t.m$$

$$M_{BA} = \left(\frac{2 \times 5000}{6}\right)\left[2 \times 0.00057 + 0.001 - 3 \times 0.0015\right] = -3.93\,t.m$$

$$M_{BC} = \left(\frac{3 \times 5000}{6}\right)\left[0.00057 + 0.001\right] = 3.93\,t.m$$

Example 1b





$$FEM_{AB} = FEM_{BA} = 0.0$$

$$(\text{FEM})_{BC} = -\frac{wL^2}{30} = -\frac{6(6)^2}{30} = -7.2 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = \frac{wL^2}{20} = \frac{6(6)^2}{20} = 10.8 \text{ kN}\cdot\text{m}$$

$$\theta_A = \theta_C = 0.0$$

Slope Deflection Equations

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{8}\right)[2(0) + \theta_B - 3(0)] + 0 = \frac{EI}{4}\theta_B \qquad (1)$$

Now, considering B to be the near end and A to be the far end, we have

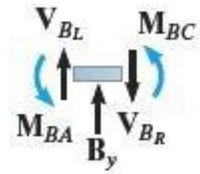
$$M_{BA} = 2E\left(\frac{I}{8}\right)[2\theta_B + 0 - 3(0)] + 0 = \frac{EI}{2}\theta_B \qquad (2)$$

In a similar manner, for span BC we have

$$M_{BC} = 2E\left(\frac{I}{6}\right)[2\theta_B + 0 - 3(0)] - 7.2 = \frac{2EI}{3}\theta_B - 7.2 \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{6}\right)[2(0) + \theta_B - 3(0)] + 10.8 = \frac{EI}{3}\theta_B + 10.8 \qquad (4)$$

Equilibrium Equations



$$\circlearrowleft + \Sigma M_B = 0; \qquad M_{BA} + M_{BC} = 0$$

To solve, substitute Eqs. (2) and (3) into Eq. (5), which yields

$$\theta_B = \frac{6.17}{EI}$$

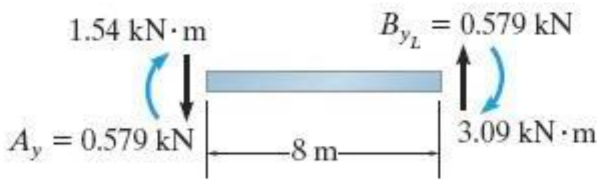
Resubstituting this value into Eqs. (1)–(4) yields

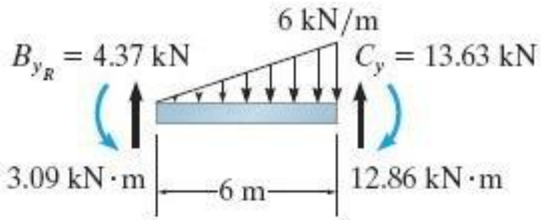
$$M_{AB} = 1.54 \text{ kN}\cdot\text{m}$$

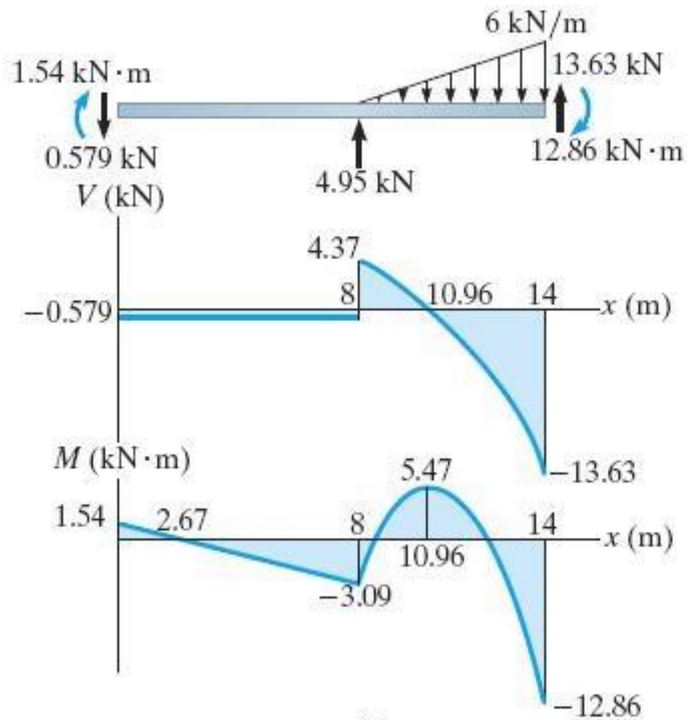
$$M_{BA} = 3.09 \text{ kN}\cdot\text{m}$$

$$M_{BC} = -3.09 \text{ kN}\cdot\text{m}$$

$$M_{CB} = 12.86 \text{ kN}\cdot\text{m}$$

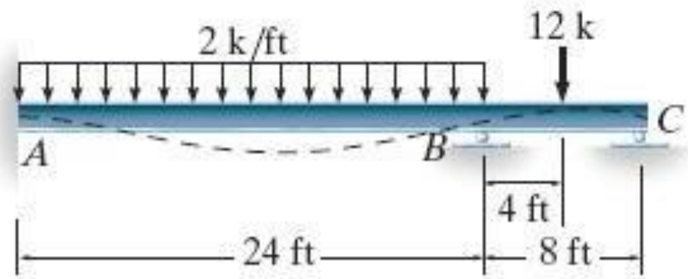






Example 2b

Draw the shear and moment diagrams for the beam shown in Fig. 11–11*a*. *EI* is constant.



$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{1}{12}(2)(24)^2 = -96\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{1}{12}(2)(24)^2 = 96\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = -\frac{3PL}{16} = -\frac{3(12)(8)}{16} = -18\ \text{k}\cdot\text{ft}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{24}\right)[2(0) + \theta_B - 3(0)] - 96$$

$$M_{AB} = 0.08333EI\theta_B - 96 \quad (1)$$

$$M_{BA} = 2E\left(\frac{I}{24}\right)[2\theta_B + 0 - 3(0)] + 96$$

$$M_{BA} = 0.1667EI\theta_B + 96 \quad (2)$$

Applying Eq. 11–10 with B as the near end and C as the far end, we have

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BC} = 3E\left(\frac{I}{8}\right)(\theta_B - 0) - 18$$

$$M_{BC} = 0.375EI\theta_B - 18 \quad (3)$$

Remember that Eq. 11–10 is *not* applied from C (near end) to B (far end).

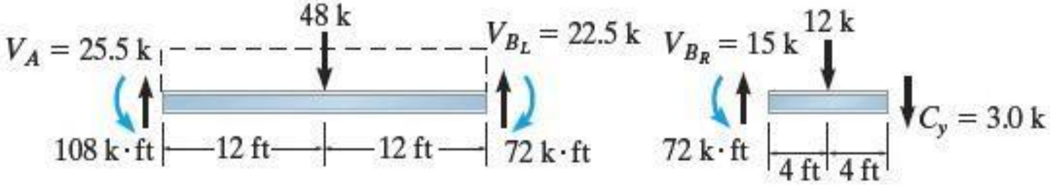
$\circlearrowleft + \Sigma M_B = 0; \qquad M_{BA} + M_{BC} = 0 \qquad (4)$

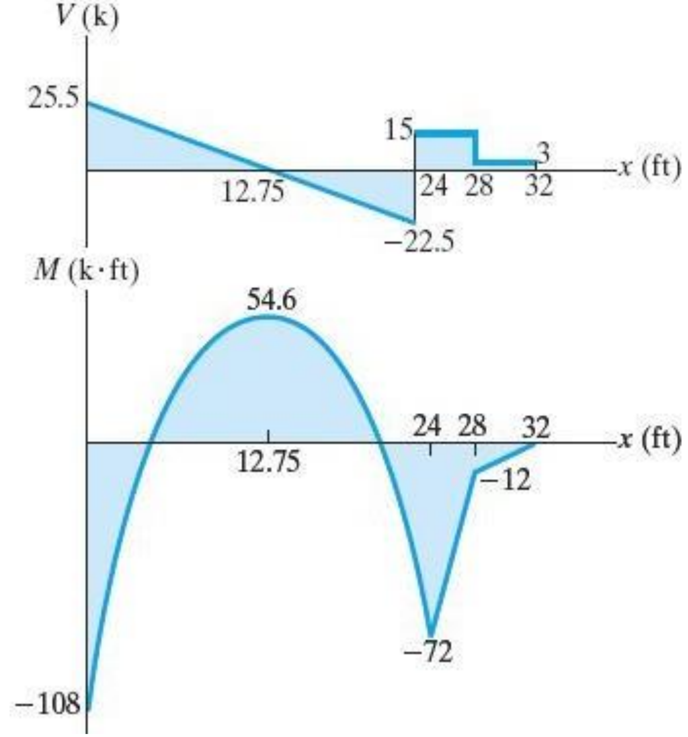
To solve, substitute Eqs. (2) and (3) into Eq. (4), which yields

$$\theta_B = -\frac{144.0}{EI}$$

Since θ_B is negative (counterclockwise) the elastic curve for the beam has been correctly drawn in Fig. 11–11*a*. Substituting θ_B into Eqs. (1)–(3), we get

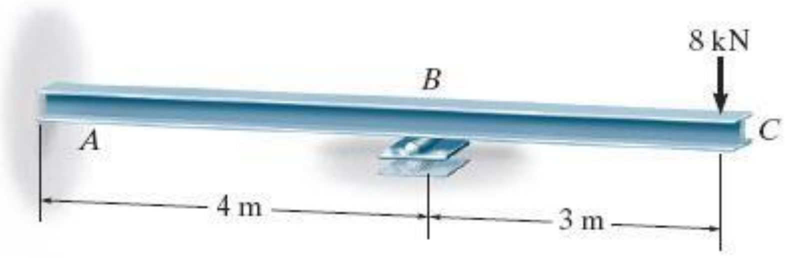
$$M_{AB} = -108.0\ \text{k}\cdot\text{ft}$$
$$M_{BA} = 72.0\ \text{k}\cdot\text{ft}$$
$$M_{BC} = -72.0\ \text{k}\cdot\text{ft}$$





Example 3b

Determine the moment at A and B for the beam shown in Fig. 11–12*a*. The support at B is displaced (settles) 80 mm. Take $E = 200$ GPa, $I = 5(10^6)$ mm^4.

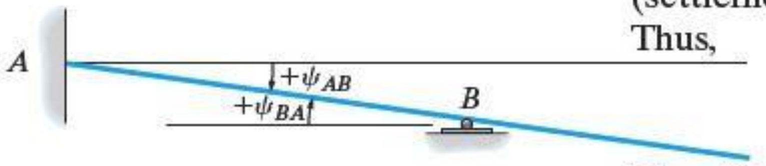


(a)

Fig. 11–12

Solution

Slope-Deflection Equations. Only one span (AB) must be considered in this problem since the moment $\mathbf{M}_{BC}$ due to the overhang can be calculated from statics. Since there is no loading on span AB, the FEMs are zero. As shown in Fig. 11–12*b*, the downward displacement (settlement) of B causes the cord for span AB to rotate clockwise. Thus,



(b)

$$\psi_{AB} = \psi_{BA} = \frac{0.08 \text{ m}}{4} = 0.02 \text{ rad}$$

The stiffness for AB is

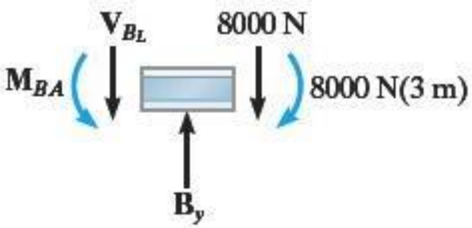
$$k = \frac{I}{L} = \frac{5(10^6)\text{ mm}^4(10^{-12})\text{ m}^4/\text{mm}^4}{4 \text{ m}} = 1.25(10^{-6})\text{ m}^3$$

Applying the slope-deflection equation, Eq. 11–8, to span AB, with $\theta_A = 0$, we have

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2(200(10^9)\ \text{N/m}^2)[1.25(10^{-6})\ \text{m}^3][2(0) + \theta_B - 3(0.02)] + 0 \qquad (1)$$

$$M_{BA} = 2(200(10^9)\ \text{N/m}^2)[1.25(10^{-6})\ \text{m}^3][2\theta_B + 0 - 3(0.02)] + 0 \qquad (2)$$



(c)

Equilibrium Equations. The free-body diagram of the beam at support B is shown in Fig. 11–12c. Moment equilibrium requires

$$\circlearrowleft + \Sigma M_B = 0; \qquad M_{BA} - 8000\ \text{N}(3\ \text{m}) = 0$$

Substituting Eq. (2) into this equation yields

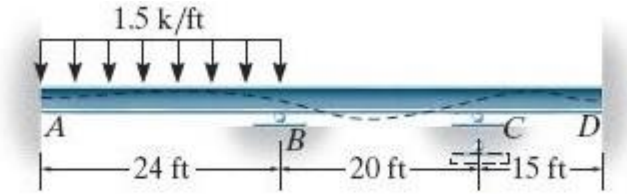
$$1(10^6)\theta_B - 30(10^3) = 24(10^3)$$
$$\theta_B = 0.054\ \text{rad}$$

Thus, from Eqs. (1) and (2),

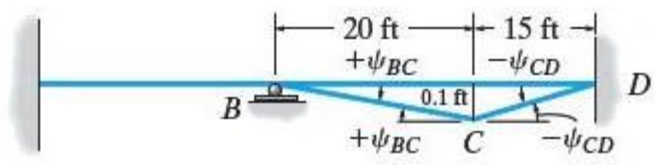
$$M_{AB} = -3.00\ \text{kN}\cdot\text{m}$$
$$M_{BA} = 24.0\ \text{kN}\cdot\text{m}$$

Example 4

Determine the internal moments at the supports of the beam shown in Fig. 11–13*a*. The support at *C* is displaced (settles) 0.1 ft. Take $E = 29(10^3)$ ksi, $I = 1500$ in^4.



(a)



(b)

$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{1}{12}(1.5)(24)^2 = -72.0\ \text{k}\cdot\text{ft}$$

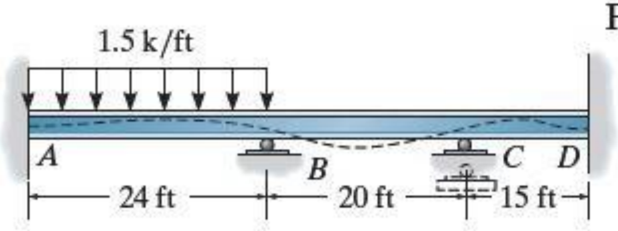
$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{1}{12}(1.5)(24)^2 = 72.0\ \text{k}\cdot\text{ft}$$

$$\psi_{BC} = \frac{0.1\ \text{ft}}{20\ \text{ft}} = 0.005\ \text{rad} \qquad \psi_{CD} = -\frac{0.1\ \text{ft}}{15\ \text{ft}} = -0.00667\ \text{rad}$$

Also, expressing the units for the stiffness in feet, we have

$$k_{AB} = \frac{1500}{24(12)^4} = 0.003014\ \text{ft}^3 \qquad k_{BC} = \frac{1500}{20(12)^4} = 0.003617\ \text{ft}^3$$

$$k_{CD} = \frac{1500}{15(12)^4} = 0.004823\ \text{ft}^3$$



(a)

For span *AB*:

$$M_{AB} = 2[29(10^3)(12)^2](0.003014)[2(0) + \theta_B - 3(0)] - 72$$

$$M_{AB} = 25\,173.6\theta_B - 72 \qquad (1)$$

$$M_{BA} = 2[29(10^3)(12)^2](0.003014)[2\theta_B + 0 - 3(0)] + 72$$

$$M_{BA} = 50\,347.2\theta_B + 72 \qquad (2)$$

For span *BC*:

$$M_{BC} = 2[29(10^3)(12)^2](0.003617)[2\theta_B + \theta_C - 3(0.005)] + 0$$

$$M_{BC} = 60\,416.7\theta_B + 30\,208.3\theta_C - 453.1 \qquad (3)$$

$$M_{CB} = 2[29(10^3)(12)^2](0.003617)[2\theta_C + \theta_B - 3(0.005)] + 0$$

$$M_{CB} = 60\,416.7\theta_C + 30\,208.3\theta_B - 453.1 \qquad (4)$$

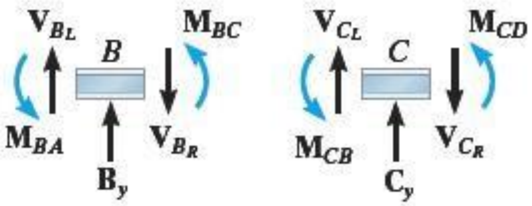
For span *CD*:

$$M_{CD} = 2[29(10^3)(12)^2](0.004823)[2\theta_C + 0 - 3(-0.00667)] + 0$$

$$M_{CD} = 80\,555.6\theta_C + 0 + 805.6 \qquad (5)$$

$$M_{DC} = 2[29(10^3)(12)^2](0.004823)[2(0) + \theta_C - 3(-0.00667)] + 0$$

$$M_{DC} = 40\,277.8\theta_C + 805.6 \qquad (6)$$



(c)

Equilibrium Equations. These six equations contain eight unknowns. Writing the moment equilibrium equations for the supports at B and C, Fig. 10–13c, we have

$$\circlearrowleft + \Sigma M_B = 0; \qquad M_{BA} + M_{BC} = 0 \qquad (7)$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad M_{CB} + M_{CD} = 0 \qquad (8)$$

In order to solve, substitute Eqs. (2) and (3) into Eq. (7), and Eqs. (4) and (5) into Eq. (8). This yields

$$\theta_C + 3.667\theta_B = 0.01262$$

$$-\theta_C - 0.214\theta_B = 0.00250$$

Thus,

$$\theta_B = 0.00438 \text{ rad} \qquad \theta_C = -0.00344 \text{ rad}$$

The negative value for θ_C indicates counterclockwise rotation of the tangent at C, Fig. 11–13a. Substituting these values into Eqs. (1)–(6) yields

$$M_{AB} = 38.2 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

$$M_{BA} = 292 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

$$M_{BC} = -292 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

$$M_{CB} = -529 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

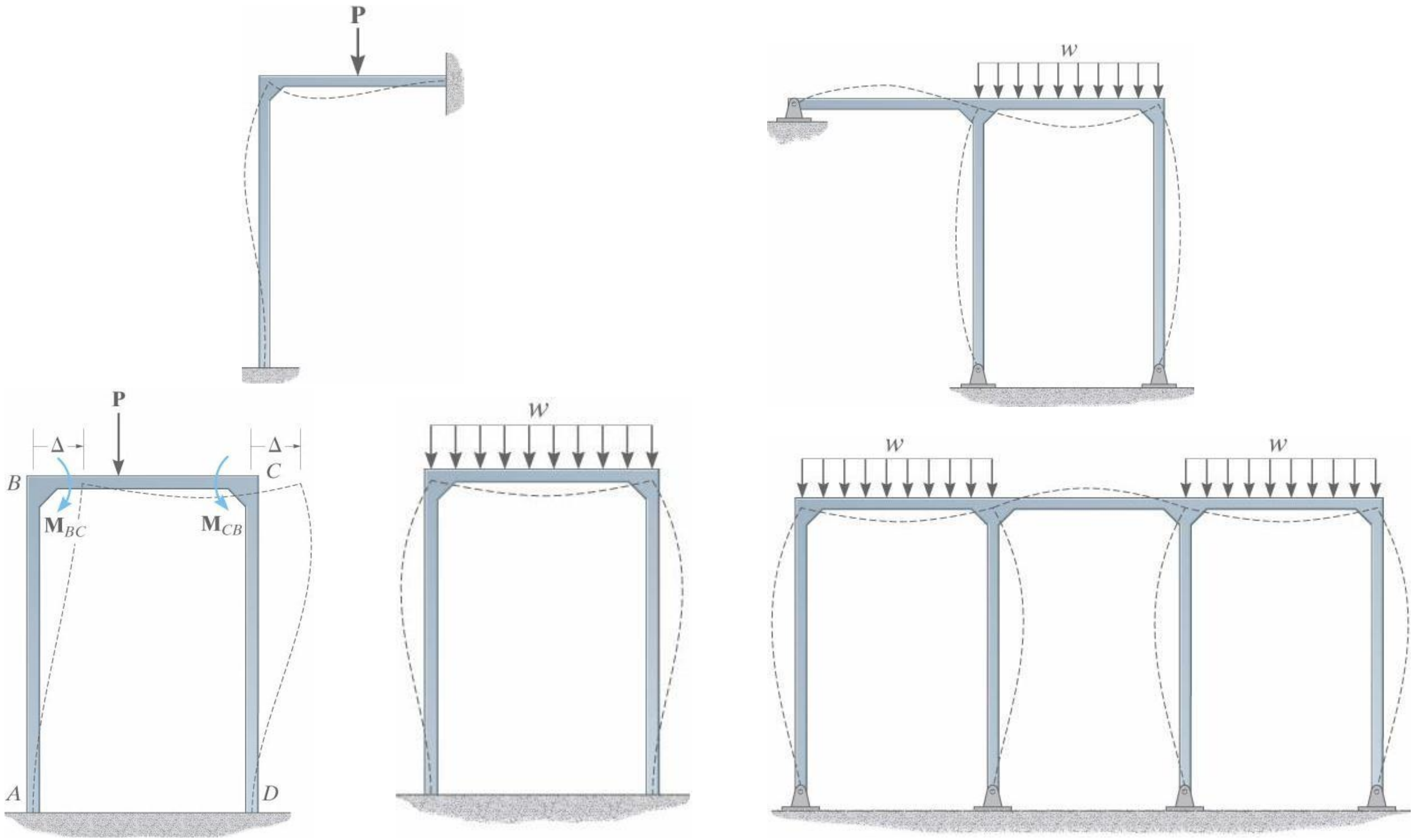
$$M_{CD} = 529 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

$$M_{DC} = 667 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

Exercises

- **Problems (Page 462-463): 1,2,3,4,5,6,3,8,9,10,11,12**

- Analysis of Frames Without Sway
  - The side movement of the end of a column in a frame is called SWAY.



Example 5

– Determine the moment at each joint of the frame. EI is constant

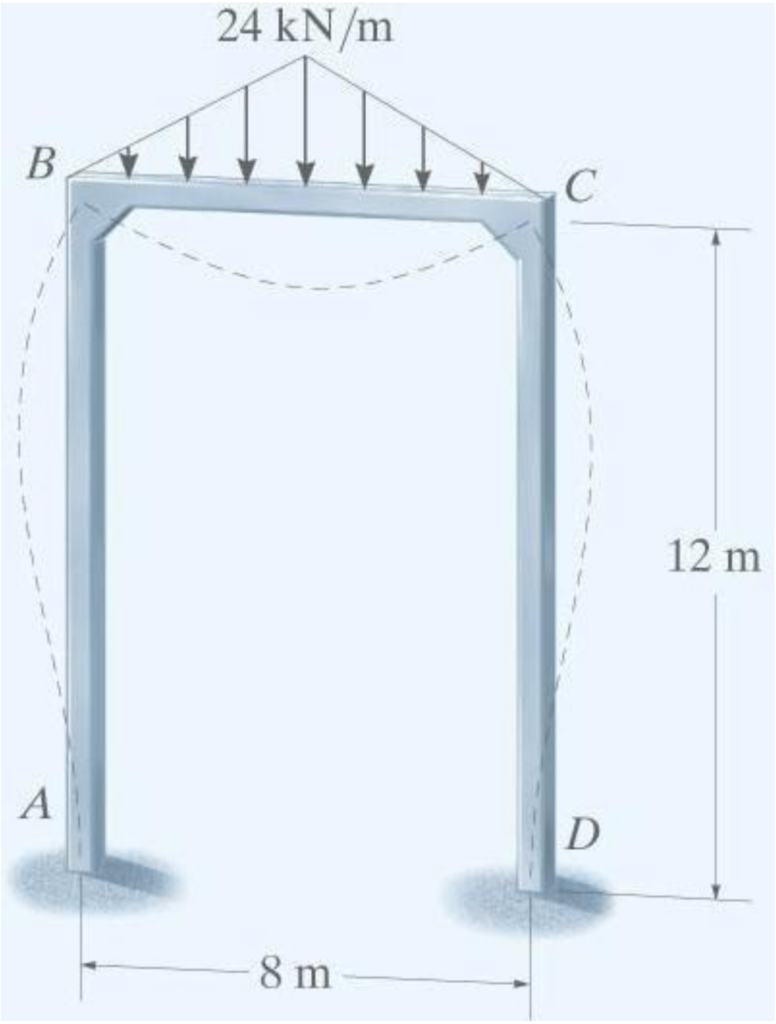
Fixed End Moment

$FEM_{AB} = FEM_{BA} = 0.$

$$FEM_{BC} = -\frac{5\times 24\times 8^2}{96} = -80\,kN.m$$

$$FEM_{CB} = \frac{5\times 24\times 8^2}{96} = 80\,kN.m$$

$FEM_{CD} = FEM_{DC} = 0.$



Slope Deflection Equations

$$M_{AB} = 2E\left(\frac{I}{12}\right)[0. + \theta_B - 0.] + 0$$

$\theta_A = 0.$

$$M_{AB} = 0.1667EI\theta_B$$

$$M_{BA} = 2E\left(\frac{I}{12}\right)[2\theta_B + 0 - 0.] + 0$$

$\theta_A = 0.$

$$M_{BA} = 0.333EI\theta_B$$

$$M_{BC} = 2E\left(\frac{I}{8}\right)[2\theta_B + \theta_C - 0] - 80 \longleftrightarrow M_{BC} = 0.5EI\theta_B + 0.25EI\theta_C - 80$$

$$M_{CB} = 2E\left(\frac{I}{8}\right)[2\theta_C + \theta_B - 0] + 80 \longleftrightarrow M_{CB} = 0.5EI\,\theta_C + 0.25EI\theta_B + 80$$

$$M_{CD} = 2E\left(\frac{I}{12}\right)[2\theta_C + 0 - 0.] + 0 \overset{\theta_D = 0.}{\longleftrightarrow} M_{CD} = 0.333EI\theta_C$$

$$M_{DC} = 2E\left(\frac{I}{12}\right)[0 + \theta_C - 0.] + 0 \overset{\theta_D = 0.}{\longleftrightarrow} M_{DC} = 0.1667EI\theta_C$$

Joint Equilibrium Equations

Joint B

$$M_{BA} + M_{BC} = 0$$

$$0.333EI\theta_B + 0.5EI\theta_B + 0.25EI\theta_C - 80 = 0. \longleftrightarrow 0.833EI\theta_B + 0.25EI\theta_C = 80$$

Joint C

$$M_{CB} + M_{CD} = 0$$

$$0.333EI\theta_C + 0.5EI\theta_C + 0.25EI\theta_B + 80 = 0. \longleftrightarrow 0.833EI\theta_C + 0.25EI\theta_B = -80$$

Two equation & two unknown

$$\theta_B = -\theta_C = \frac{137.1}{EI}$$

Substituting in slope deflection equations

$$M_{AB} = 22.9\,kN.m$$

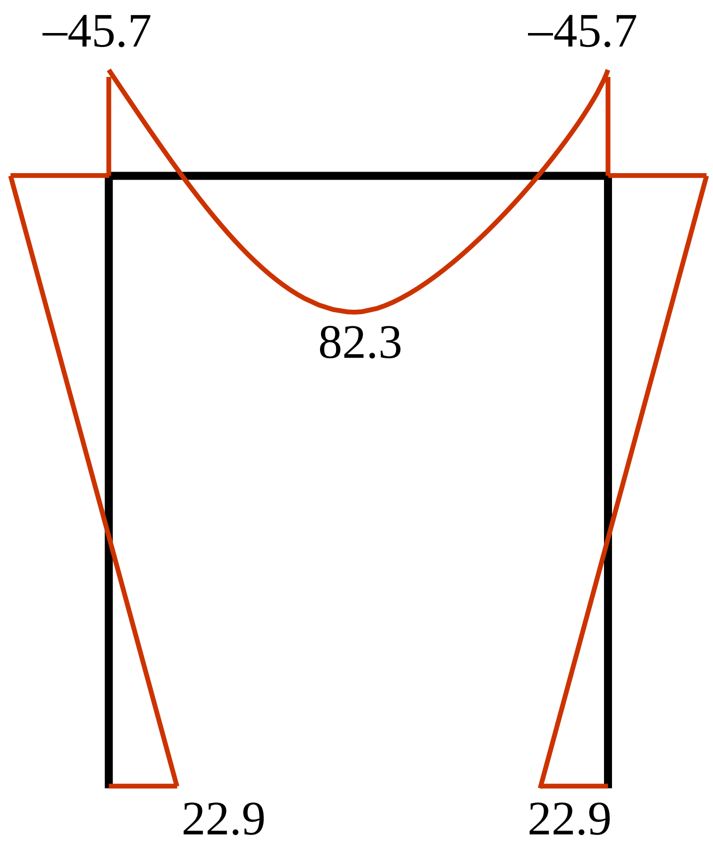
$$M_{BA} = 45.7\,kN.m$$

$$M_{BC} = -45.7\,kN.m$$

$$M_{CB} = 45.7\,kN.m$$

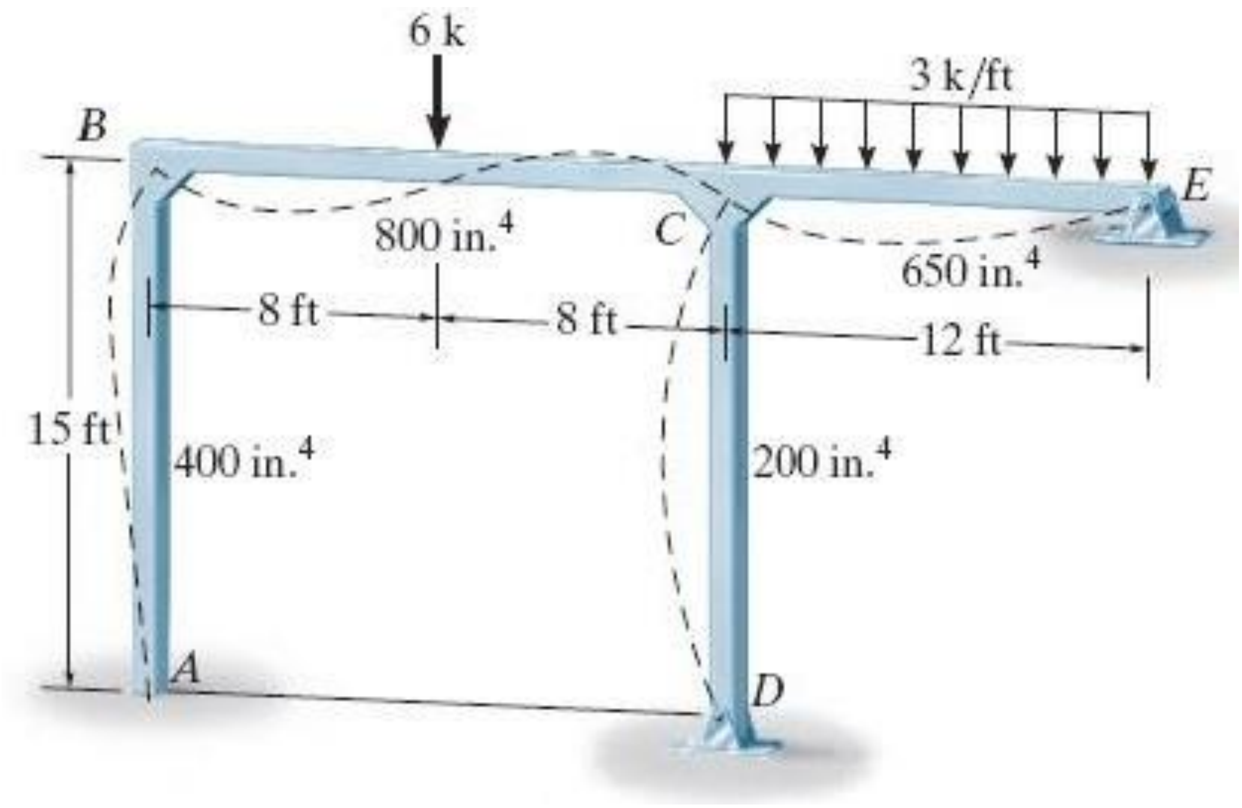
$$M_{CD} = -45.7\,kN.m$$

$$M_{DC} = -22.9\,kN.m$$



Example 6

Determine the internal moments at each joint of the frame shown in Fig. 11–17*a*. The moment of inertia for each member is given in the figure. Take $E = 29(10^3)$ ksi.



Slope-Deflection Equations. Four spans must be considered in this problem. Equation 11–8 applies to spans AB and BC, and Eq. 11–10 will be applied to CD and CE, because the ends at D and E are pinned.

Computing the member stiffnesses, we have

$$k_{AB} = \frac{400}{15(12)^4} = 0.001286\ \text{ft}^3 \qquad k_{CD} = \frac{200}{15(12)^4} = 0.000643\ \text{ft}^3$$

$$k_{BC} = \frac{800}{16(12)^4} = 0.002411\ \text{ft}^3 \qquad k_{CE} = \frac{650}{12(12)^4} = 0.002612\ \text{ft}^3$$

The FEMs due to the loadings are

$$(\text{FEM})_{BC} = -\frac{PL}{8} = -\frac{6(16)}{8} = -12\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = \frac{PL}{8} = \frac{6(16)}{8} = 12\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{CE} = -\frac{wL^2}{8} = -\frac{3(12)^2}{8} = -54\ \text{k}\cdot\text{ft}$$

Applying Eqs. 11–8 and 11–10 to the frame and noting that $\theta_A = 0$, $\psi_{AB} = \psi_{BC} = \psi_{CD} = \psi_{CE} = 0$ since no sidesway occurs, we have

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2[29(10^3)(12)^2](0.001286)[2(0) + \theta_B - 3(0)] + 0$$

$$M_{AB} = 10740.7\theta_B \qquad (1)$$

$$M_{BA} = 2[29(10^3)(12)^2](0.001286)[2\theta_B + 0 - 3(0)] + 0$$

$$M_{BA} = 21\,481.5\theta_B \quad (2)$$

$$M_{BC} = 2[29(10^3)(12)^2](0.002411)[2\theta_B + \theta_C - 3(0)] - 12$$

$$M_{BC} = 40\,277.8\theta_B + 20\,138.9\theta_C - 12 \quad (3)$$

$$M_{CB} = 2[29(10^3)(12)^2](0.002411)[2\theta_C + \theta_B - 3(0)] + 12$$

$$M_{CB} = 20\,138.9\theta_B + 40\,277.8\theta_C + 12 \quad (4)$$

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{CD} = 3[29(10^3)(12)^2](0.000643)[\theta_C - 0] + 0 \quad (5)$$

$$M_{CD} = 8055.6\theta_C$$

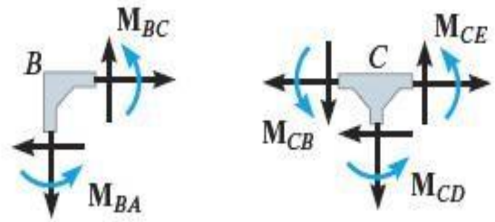
$$M_{CE} = 3[29(10^3)(12)^2](0.002612)[\theta_C - 0] - 54$$

$$M_{CE} = 32\,725.7\theta_C - 54 \quad (6)$$

Equations of Equilibrium. These six equations contain eight unknowns. Two moment equilibrium equations can be written for joints B and C, Fig. 11–17b. We have

$$M_{BA} + M_{BC} = 0 \quad (7)$$

$$M_{CB} + M_{CD} + M_{CE} = 0 \quad (8)$$



(b)

In order to solve, substitute Eqs. (2) and (3) into Eq. (7), and Eqs. (4)–(6) into Eq. (8). This gives

$$61\,759.3\theta_B + 20\,138.9\theta_C = 12$$

$$20\,138.9\theta_B + 81\,059.0\theta_C = 42$$

Solving these equations simultaneously yields

$$\theta_B = 2.758(10^{-5})\text{ rad} \qquad \theta_C = 5.113(10^{-4})\text{ rad}$$

These values, being clockwise, tend to distort the frame as shown in Fig. 11–17a. Substituting these values into Eqs. (1)–(6) and solving, we get

$$M_{AB} = 0.296\text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$

$$M_{BA} = 0.592\text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$

$$M_{BC} = -0.592\text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$

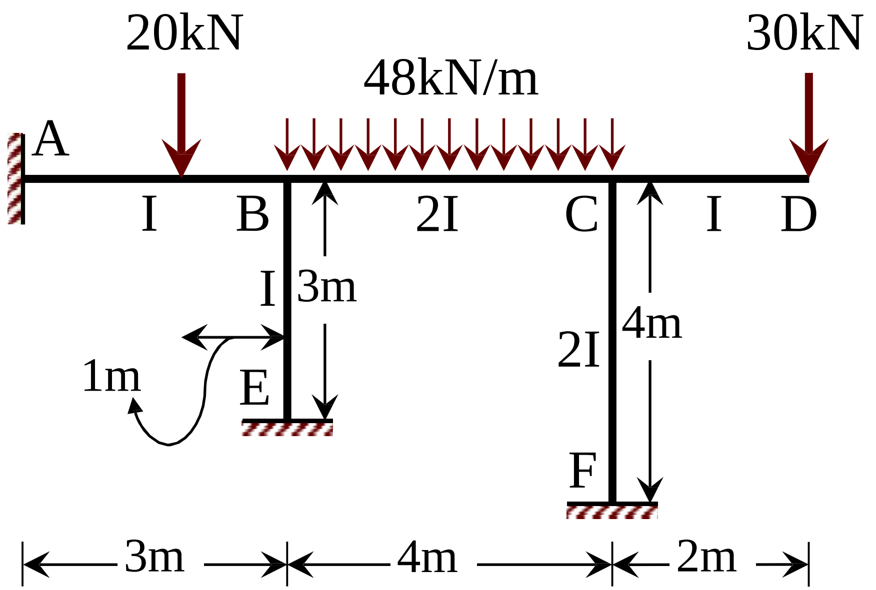
$$M_{CB} = 33.1\text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$

$$M_{CD} = 4.12\text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$

$$M_{CE} = -37.3\text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$

Example 6b

– Draw the bending moment diagram



Fixed End Moment

$$FEM_{AB} = -\frac{20\times 2\times 1^2}{3^2} = -4.44 kN.m$$

$$FEM_{BA} = \frac{20\times 1\times 2^2}{3^2} = 8.89 kN.m$$

$$FEM_{BC} = -\frac{48\times 4^2}{12} = -64 kN.m$$

$$FEM_{CB} = 64 kN.m$$

$$FEM_{BE} = FEM_{EB} = FEM_{CF} = FEM_{FC} = 0$$

$$M_{CD} = -30\times 2 = -60 kN.m$$

Slope Deflection Equations

$$M_{AB} = 2E\left(\frac{I}{3}\right)[2\theta_A + \theta_B - 0] - 4.44 \xrightarrow{\theta_A = 0.} M_{AB} = \frac{2}{3}EI\theta_B - 4.44$$

$$M_{BA} = 2E\left(\frac{I}{3}\right)[\theta_A + 2\theta_B - 0] + 8.89 \xrightarrow{\theta_A = 0.} M_{BA} = \frac{4}{3}EI\theta_B + 8.89$$

$$M_{BC} = 2E\left(\frac{2I}{4}\right)[2\theta_B + \theta_C - 0] - 64 \longrightarrow M_{BC} = 2EI\theta_B + EI\theta_C - 64$$

$$M_{CB} = 2E\left(\frac{2I}{4}\right)[\theta_B + 2\theta_C - 0] + 64 \longrightarrow M_{CB} = EI\theta_B + 2EI\theta_C + 64$$

$$M_{BE} = 2E\left(\frac{I}{3}\right)[2\theta_B + \theta_E - 0] + 0 \xrightarrow{\theta_E = 0.} M_{BE} = \frac{4}{3}EI\theta_B$$

$$M_{EB} = 2E\left(\frac{I}{3}\right)[2\theta_E + \theta_B - 0] + 0 \xrightarrow{\theta_E = 0.} M_{EB} = \frac{2}{3}EI\theta_B$$

$$M_{CF} = 2E\left(\frac{2I}{4}\right)[2\theta_C + \theta_F - 0] + 0 \xrightarrow{\theta_F = 0.} M_{CF} = 2EI\theta_C$$

$$M_{FC} = 2E\left(\frac{2I}{4}\right)[2\theta_F + \theta_C - 0] + 0 \xrightarrow{\theta_F = 0.} M_{FC} = EI\theta_C$$

Equilibrium Equations

Joint B

$$M_{BA} + M_{BC} + M_{BE} = 0$$

$$\frac{4}{3}EI\theta_B + 8.89 + 2EI\theta_B + EI\theta_C - 64 + \frac{4}{3}EI\theta_B = 0 \longleftrightarrow 4.67EI\theta_B + EI\theta_C = 55.11$$

Joint C

$$M_{CB} + M_{CF} + M_{CD} = 0$$

$$EI\theta_B + 2EI\theta_C + 64 + 2EI\theta_C - 60 = 0 \longleftrightarrow EI\theta_B + 4EI\theta_C = -4$$

Two equation & two unknown

$$EI\,\theta_B = 12.70 \qquad EI\,\theta_C = -4.18$$

Substituting in slope deflection equations

$$M_{AB} = \frac{2}{3}\times 12.7 - 4.44 = 4.03\,kN.m \qquad M_{BC} = 2\times 12.7 - 4.18 - 64 = -42.77\,kN.m$$

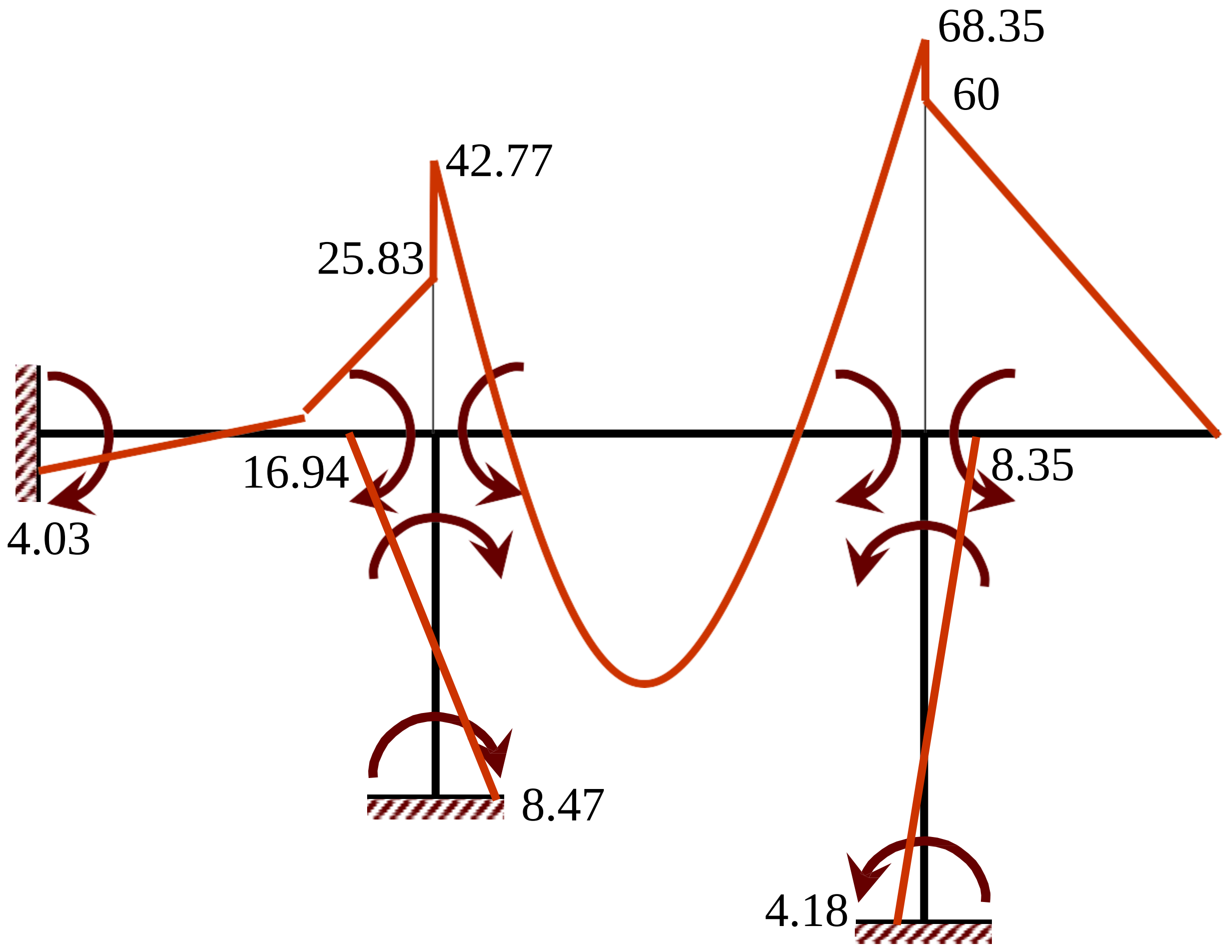
$$M_{BA} = \frac{4}{3}\times 12.70 + 8.89 = 25.83\,kNm \qquad M_{CB} = 12.7 + 2\times(-4.18) + 64 = 68.34\,kN.m$$

$$M_{BE} = \frac{4}{3}\times 12.70 = 16.94 kN.m$$

$$M_{EB} = \frac{2}{3}\times 12.70 = 8.47 kN.m$$

$$M_{CF} = 2EI(-4.18) = -8.35 kN.m$$

$$M_{FC} = -4.18 kN.m$$

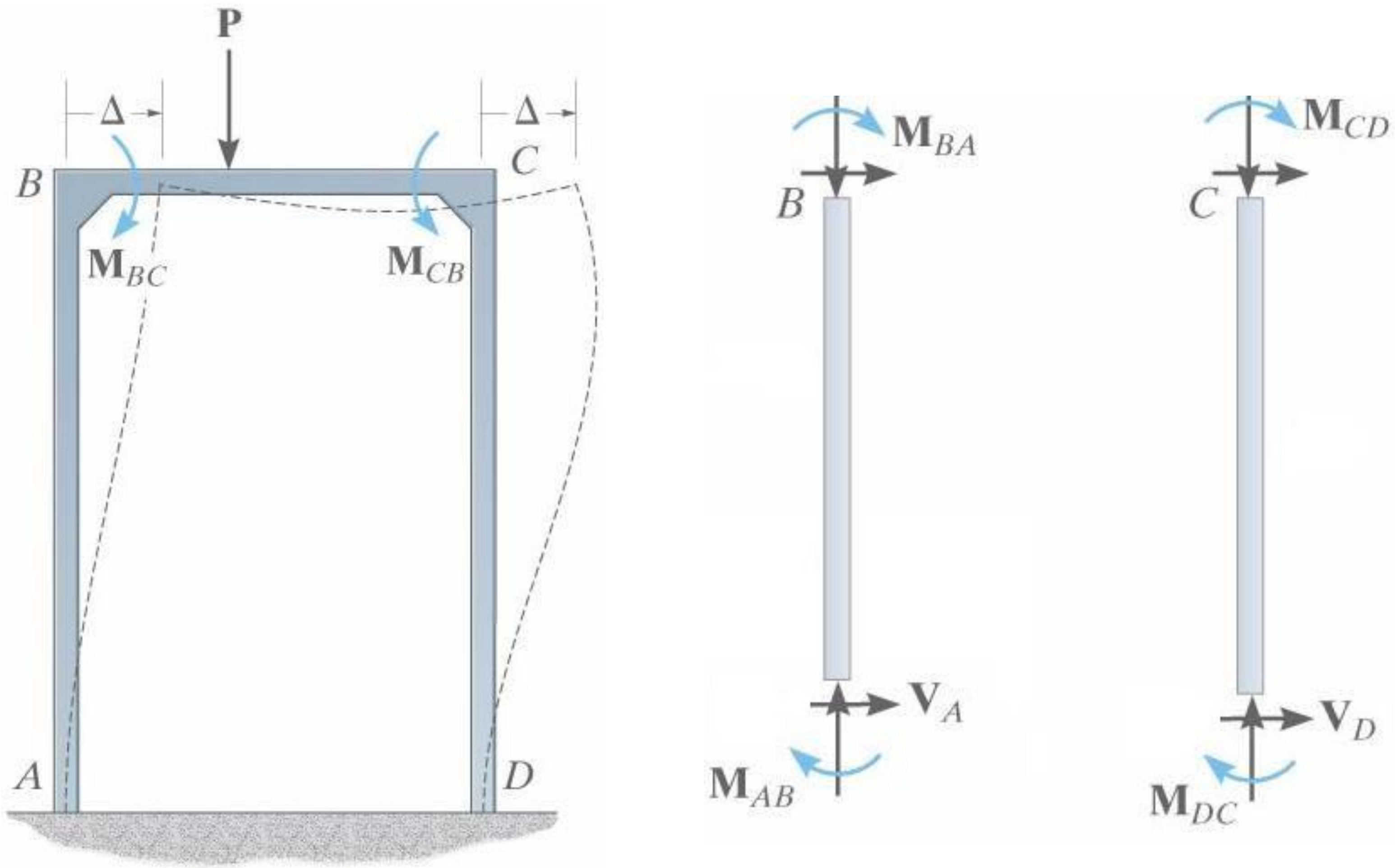


Exercises

- **Problems (Page 462-463): 13,14,15,17,18,19**

Slope Deflection (Frame with Sway)

- Analysis of Frames with Sway



Example 7

– Draw the bending moment diagram. EI constant

Fixed End Moment

As there is no span loading in any of the member FEM for all the members is zero

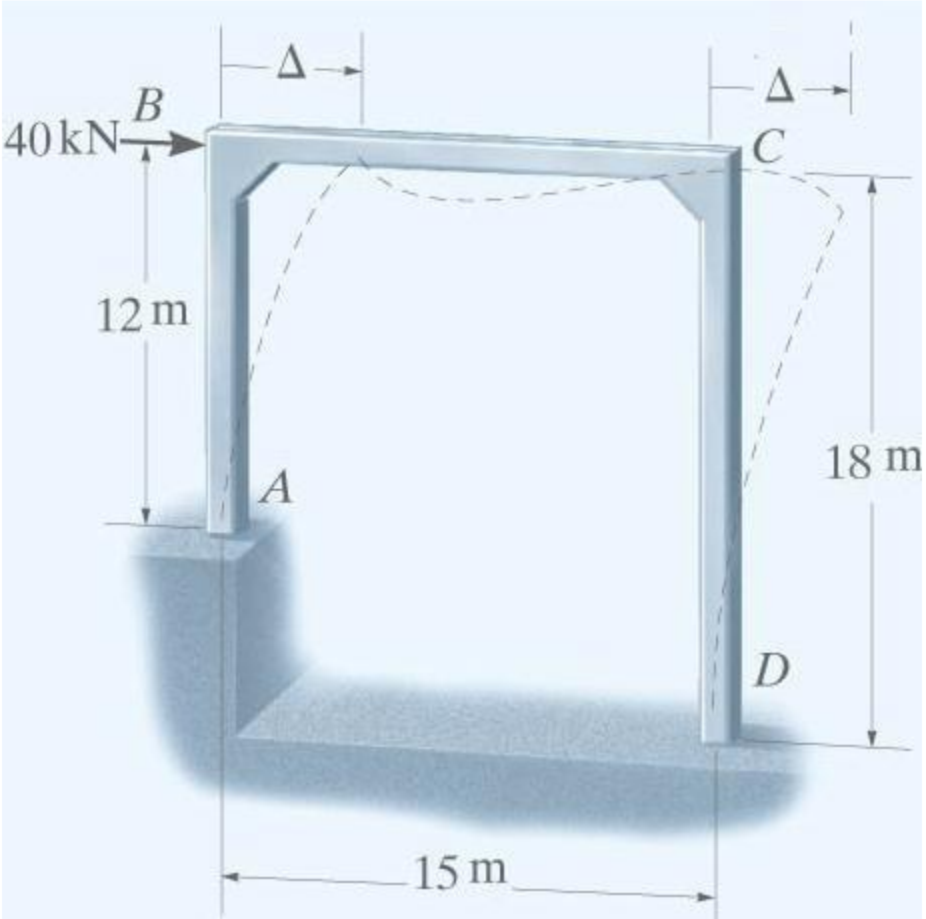
Slope Deflection Equations

$$M_{AB} = 2E\left(\frac{I}{12}\right)\left[0. + \theta_B - 3\left(\frac{\Delta}{12}\right)\right] + 0$$

$$= \frac{1}{6}EI\theta_B - \frac{1}{24}EI\Delta$$

$$M_{BA} = 2E\left(\frac{I}{12}\right)\left[2\theta_B + 0. - 3\left(\frac{\Delta}{12}\right)\right] + 0$$

$$= \frac{1}{3}EI\theta_B - \frac{1}{24}EI\Delta$$



$$M_{BC} = 2E\left(\frac{I}{15}\right)\left[2\theta_B + \theta_C - 3\left(\frac{0.}{15}\right)\right] + 0 \qquad = \frac{4}{15}EI\theta_B + \frac{2}{15}EI\theta_C$$

$$M_{CB} = 2E\left(\frac{I}{15}\right)[2\theta_C + \theta_B - 0] + 0 \qquad = \frac{2}{15}EI\theta_B + \frac{4}{15}EI\theta_C$$

$$M_{CD} = 2E\left(\frac{I}{18}\right)\left[2\theta_C + 0 - 3\left(\frac{\Delta}{18}\right)\right] + 0. \qquad = \frac{2}{9}EI\theta_C - \frac{1}{54}EI\Delta$$

$$M_{DC} = 2E\left(\frac{I}{18}\right)\left[0 + \theta_C - 3\left(\frac{\Delta}{18}\right)\right] + 0 \qquad = \frac{1}{9}EI\theta_C - \frac{1}{54}EI\Delta$$

Equilibrium Equations

Joint B $\Sigma M_B = 0$

$$M_{BA} + M_{BC} = 0$$

$$\frac{1}{3}EI\theta_B - \frac{1}{24}EI\Delta + \frac{4}{15}EI\theta_B + \frac{2}{15}EI\theta_C = 0.$$

$$-EI\Delta + 14.4EI\theta_B + 3.2EI\theta_C = 0 \qquad \text{----------- (1)}$$

Joint C $\Sigma M_C = 0$

$$M_{CB} + M_{CD} = 0$$

$$\frac{2}{15}EI\theta_B + \frac{4}{15}EI\theta_C + \frac{2}{9}EI\theta_C - \frac{1}{54}EI\Delta = 0.$$

$$-EI\Delta + 7.2EI\theta_B + 26.4EI\theta_C = 0 \quad \text{----------------} \; (2)$$

Three unknown & just two equations so we need another equilibrium equation. Let take $\Sigma F_x = 0$.

$$40 - H_A - H_D = 0.$$

$$H_A = -\frac{M_{BA} + M_{AB}}{12}$$

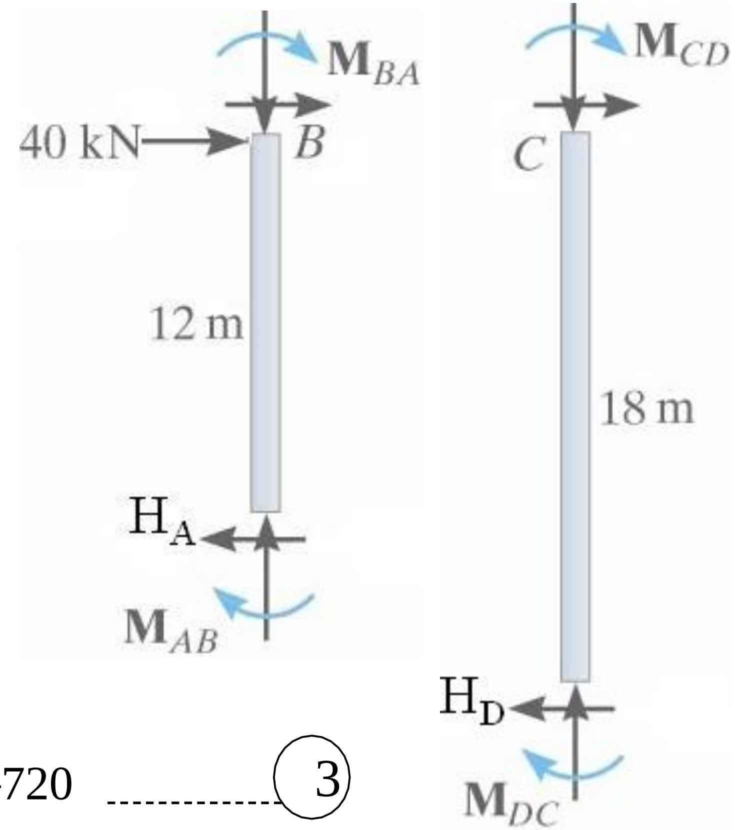
$$H_D = -\frac{M_{CD} + M_{DC}}{18}$$

$$40 \times 18 + 1.5(M_{BA} + M_{AB}) + (M_{CD} + M_{DC}) = 0.$$

$$720 + 1.5\left(\frac{1}{3}EI\theta_B - \frac{1}{24}EI\Delta + \frac{1}{6}EI\theta_B - \frac{1}{24}EI\Delta\right)$$

$$+\left(\frac{2}{9}EI\theta_C - \frac{1}{54}EI\Delta + \frac{1}{9}EI\theta_C - \frac{1}{54}EI\Delta\right) = 0.$$

$$-0.162EI\Delta + 0.75EI\theta_B + 0.333EI\theta_C = -720 \quad \text{----------------} \; (3)$$



Now solve the three equation

$$\begin{bmatrix} -1 & 14.4 & 3.2 \\ -1 & 7.2 & 26.4 \\ -0.162 & 0.75 & 0.333 \end{bmatrix} \begin{Bmatrix} EI\Delta \\ EI\theta_B \\ EI\theta_C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -720 \end{Bmatrix} \longrightarrow \begin{matrix} EI\theta_C = 136.2 \\ EI\theta_B = 438.9 \\ EI\Delta = 6756.6 \end{matrix}$$

Substituting in slope deflection equations

$$M_{AB} = -208 kN.m$$

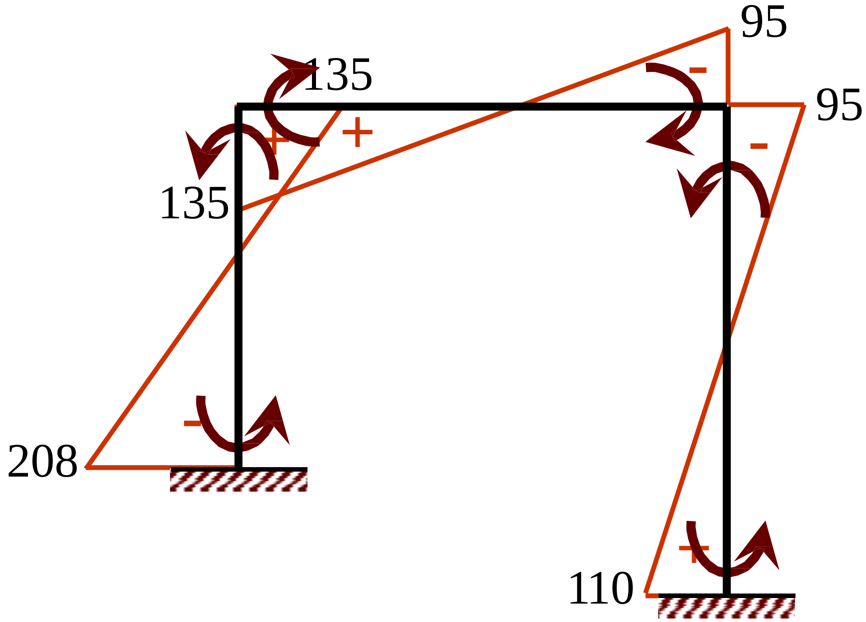
$$M_{BA} = -135 kN.m$$

$$M_{BC} = 135 kN.m$$

$$M_{CB} = 95 kN.m$$

$$M_{CD} = -95 kN.m$$

$$M_{DC} = -110 kN.m$$


95
135
+
-
95
+
-
135
-
208
+
110

Example 7

– Draw the bending moment diagram. EI constant

Fixed End Moment

$$FEM_{AB} = FEM_{BA} = 0.$$

$$FEM_{BC} = -\frac{20\times 15^2}{12} = -375 kN.m$$

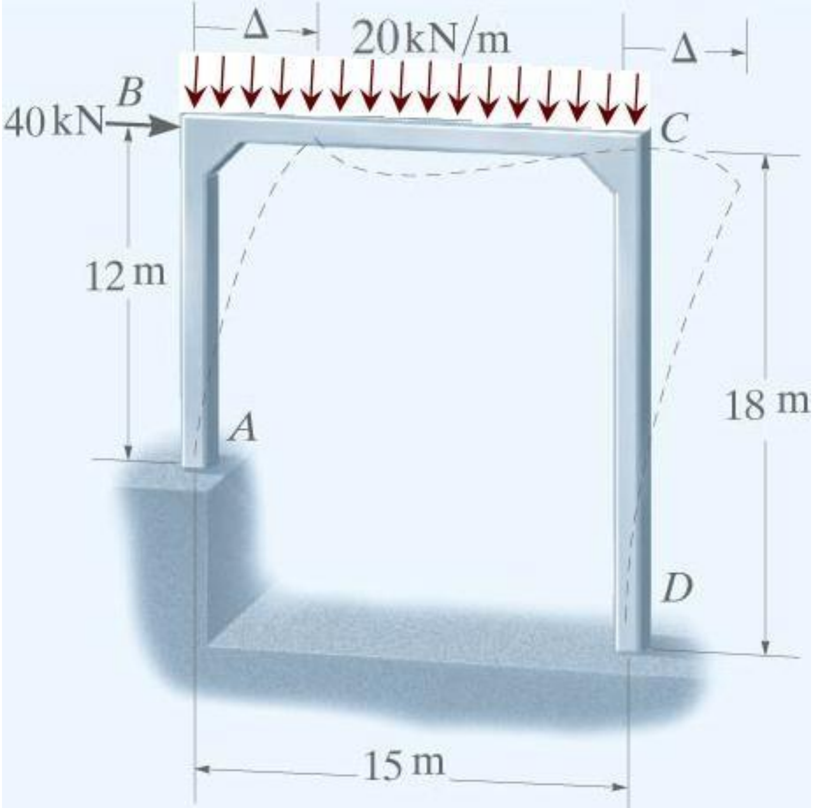
$$FEM_{CB} = \frac{20\times 15^2}{12} = 375 kN.m$$

$$FEM_{CD} = FEM_{DC} = 0.$$

Slope Deflection Equations

$$M_{AB} = 2E\left(\frac{I}{12}\right)\left[0. + \theta_B - 3\left(\frac{\Delta}{12}\right)\right] + 0$$

$$= \frac{1}{6} EI\theta_B - \frac{1}{24} EI\Delta$$



$$M_{BA} = 2E\left(\frac{I}{12}\right)\left[2\theta_B + 0. - 3\left(\frac{\Delta}{12}\right)\right] + 0 \qquad = \frac{1}{3}EI\theta_B - \frac{1}{24}EI\Delta$$

$$M_{BC} = 2E\left(\frac{I}{15}\right)\left[2\theta_B + \theta_C - 3\left(\frac{0.}{15}\right)\right] - 375 \qquad = \frac{4}{15}EI\theta_B + \frac{2}{15}EI\theta_C - 375$$

$$M_{CB} = 2E\left(\frac{I}{15}\right)\left[2\theta_C + \theta_B - 0\right] + 375 \qquad = \frac{2}{15}EI\theta_B + \frac{4}{15}EI\theta_C + 375$$

$$M_{CD} = 2E\left(\frac{I}{18}\right)\left[2\theta_C + 0 - 3\left(\frac{\Delta}{18}\right)\right] + 0. \qquad = \frac{2}{9}EI\theta_C - \frac{1}{54}EI\Delta$$

$$M_{DC} = 2E\left(\frac{I}{18}\right)\left[0 + \theta_C - 3\left(\frac{\Delta}{18}\right)\right] + 0 \qquad = \frac{1}{9}EI\theta_C - \frac{1}{54}EI\Delta$$

Equilibrium Equations

Joint B $\Sigma M_B = 0$

$$M_{BA} + M_{BC} = 0$$

$$\frac{1}{3}EI\theta_B - \frac{1}{24}EI\Delta + \frac{4}{15}EI\theta_B + \frac{2}{15}EI\theta_C - 375 = 0.$$

$$-EI\Delta + 14.4EI\theta_B + 3.2EI\theta_C = 9000 \qquad \text{-------------} \; (1)$$

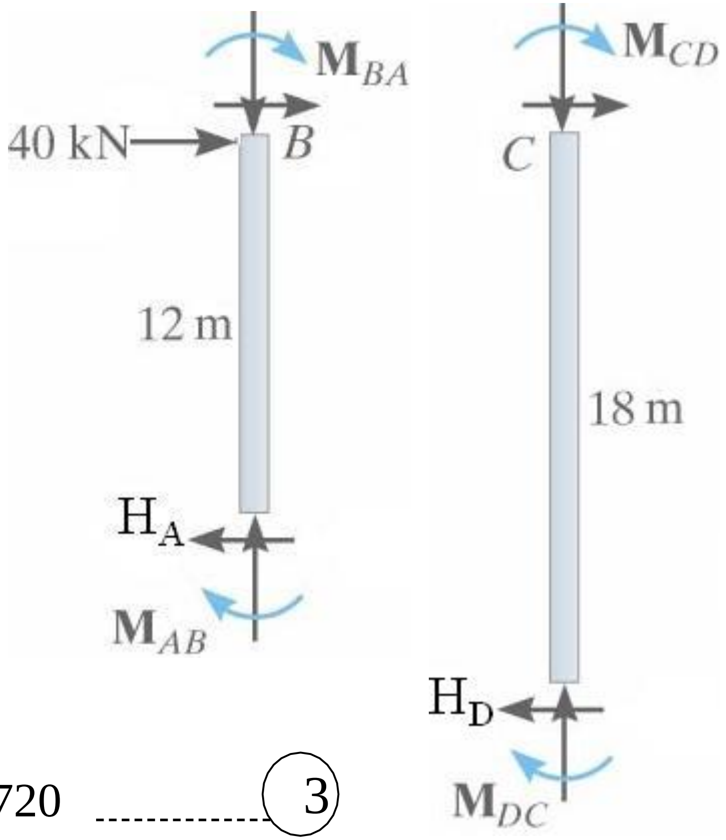
Joint C $\Sigma M_C = 0$

$$M_{CB} + M_{CD} = 0$$

$$\frac{2}{15}EI\theta_B + \frac{4}{15}EI\theta_C + 375 + \frac{2}{9}EI\theta_C - \frac{1}{54}EI\Delta = 0.$$

$$-EI\Delta + 7.2EI\theta_B + 26.4EI\theta_C = -20250 \quad \text{------------} \quad (2)$$

Three unknown & just two equations so we need another equilibrium equation. Let take $\Sigma F_x = 0$.



$$40 - H_A - H_D = 0.$$

$$H_A = -\frac{M_{BA} + M_{AB}}{12}$$

$$H_D = -\frac{M_{CD} + M_{DC}}{18}$$

$$40 \times 18 + 1.5(M_{BA} + M_{AB}) + (M_{CD} + M_{DC}) = 0.$$

$$720 + 1.5\left(\frac{1}{3}EI\theta_B - \frac{1}{24}EI\Delta + \frac{1}{6}EI\theta_B - \frac{1}{24}EI\Delta\right)$$

$$+\left(\frac{2}{9}EI\theta_C - \frac{1}{54}EI\Delta + \frac{1}{9}EI\theta_C - \frac{1}{54}EI\Delta\right) = 0.$$

$$-0.162EI\Delta + 0.75EI\theta_B + 0.333EI\theta_C = -720 \quad \text{------------} \quad (3)$$

Now solve the three equation

$$\begin{bmatrix} -1 & 14.4 & 3.2 \\ -1 & 7.2 & 26.4 \\ -0.162 & 0.75 & 0.333 \end{bmatrix} \begin{Bmatrix} EI\Delta \\ EI\theta_B \\ EI\theta_C \end{Bmatrix} = \begin{Bmatrix} 9000 \\ -20250 \\ -720 \end{Bmatrix} \xrightarrow[0.162R1-R3]{R1-R2}$$

$$\begin{bmatrix} -1 & 14.4 & 3.2 \\ 0 & 7.2 & -23.2 \\ 0 & 1.58 & 0.185 \end{bmatrix} \begin{Bmatrix} EI\Delta \\ EI\theta_B \\ EI\theta_C \end{Bmatrix} = \begin{Bmatrix} 9000 \\ 29250 \\ 2178 \end{Bmatrix} \xrightarrow{0.22R2-R3}$$

$$\begin{bmatrix} -1 & 14.4 & 3.2 \\ 0. & 7.2 & -23.2 \\ 0. & 0. & -5.29 \end{bmatrix} \begin{Bmatrix} EI\Delta \\ EI\theta_B \\ EI\theta_C \end{Bmatrix} = \begin{Bmatrix} 9000 \\ 29250 \\ 4257 \end{Bmatrix} \longrightarrow$$

$$EI\theta_C = \frac{4257}{-5.29} = -804.7$$

$$EI\theta_B = 1469.6$$

$$EI\Delta = 9587.2$$

Substituting in slope deflection equations

$$M_{AB} = \frac{1}{6}1467.2 - \frac{1}{24}9587.2 = -155\,kN.m$$

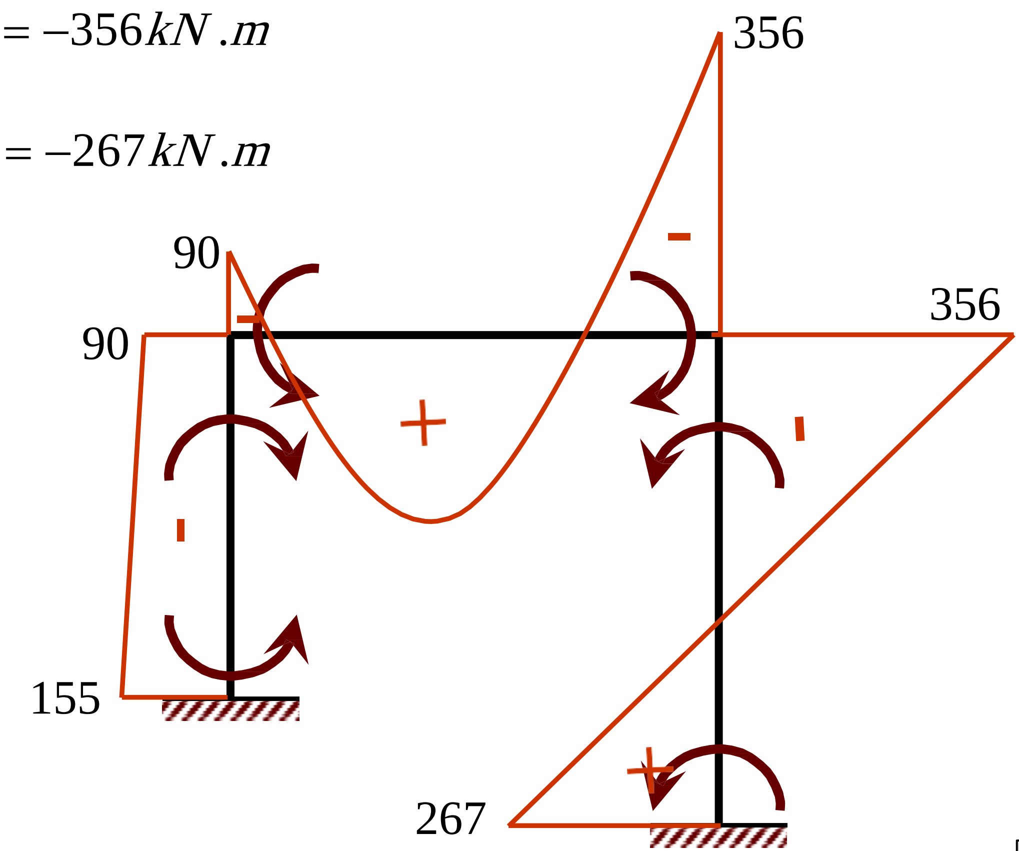
$$M_{BA} = \frac{1}{3}1469.6 - \frac{1}{24}9587.2 = 90\,kN.m$$

$$M_{BC} = \frac{4}{15}1469.6 + \frac{2}{15}(-804.7) - 375 = -90 kN.m$$

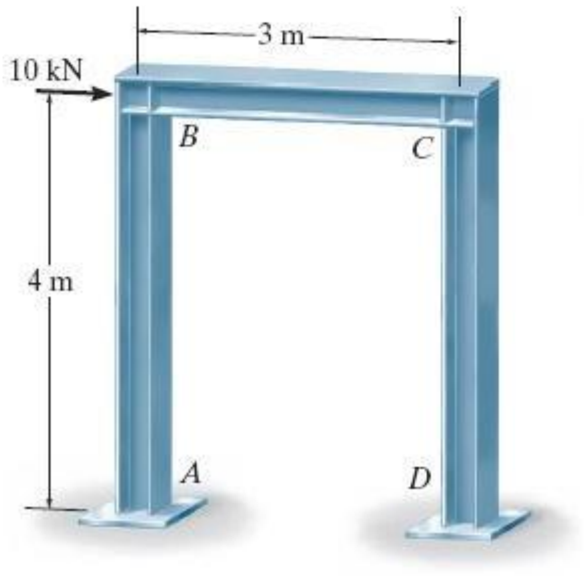
$$M_{CB} = \frac{2}{15}1469.6 + \frac{4}{15}(-804.7) + 375 = 356 kN.m$$

$$M_{CD} = \frac{2}{9}(-804.7) - \frac{1}{54}9587.2 = -356 kN.m$$

$$M_{DC} = \frac{1}{9}(-804.7) - \frac{1}{54}9587.2 = -267 kN.m$$



Example 8



(a)

Fig. 11–20

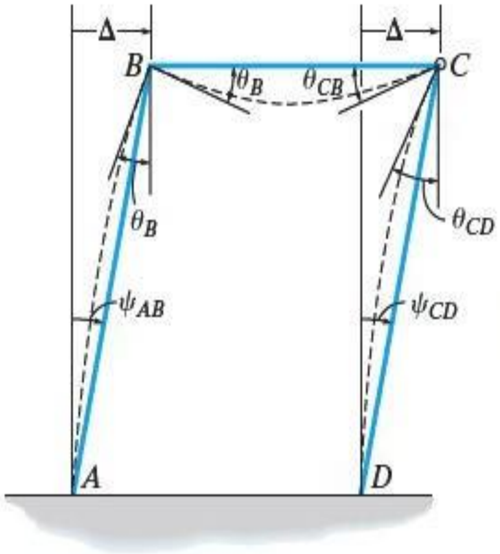
Determine the moments at each joint of the frame shown in Fig. 11–20*a*. The supports at *A* and *D* are fixed and joint *C* is assumed pin connected. *EI* is constant for each member.

Solution

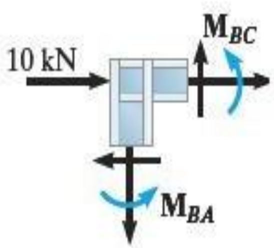
Slope-Deflection Equations. We will apply Eq. 11–8 to member *AB* since it is fixed connected at both ends. Equation 11–10 can be applied from *B* to *C* and from *D* to *C* since the pin at *C* supports zero moment. As shown by the deflection diagram, Fig. 11–20*b*, there is an unknown linear displacement Δ of the frame and unknown angular displacement θ_B at joint *B*.* Due to Δ, the cord members *AB* and *CD* rotate clockwise, $\psi = \psi_{AB} = \psi_{DC} = \Delta/4$. Realizing that $\theta_A = \theta_D = 0$ and that there are no FEMs for the members, we have

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{4}\right)[2(0) + \theta_B - 3\psi] + 0 \qquad (1)$$



(b)



(c)

$$M_{BA} = 2E\left(\frac{I}{4}\right)(2\theta_B + 0 - 3\psi) + 0 \qquad (2)$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BC} = 3E\left(\frac{I}{3}\right)(\theta_B - 0) + 0 \qquad (3)$$

$$M_{DC} = 3E\left(\frac{I}{4}\right)(0 - \psi) + 0 \qquad (4)$$

Equilibrium Equations. Moment equilibrium of joint B, Fig. 11–20c, requires

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

If forces are summed for the *entire frame* in the horizontal direction, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 10 - V_A - V_D = 0 \qquad (6)$$

As shown on the free-body diagram of each column, Fig. 11–20d, we have

$$\Sigma M_B = 0; \qquad V_A = -\frac{M_{AB} + M_{BA}}{4}$$

$$\Sigma M_C = 0; \qquad V_D = -\frac{M_{DC}}{4}$$

*The angular displacements θ_{CB} and θ_{CD} at joint C (pin) are not included in the analysis since Eq. 11–10 is to be used.

Thus, from Eq. (6),

$$10 + \frac{M_{AB} + M_{BA}}{4} + \frac{M_{DC}}{4} = 0 \qquad (7)$$

Substituting the slope-deflection equations into Eqs. (5) and (7) and simplifying yields

$$\theta_B = \frac{3}{4}\psi$$

$$10 + \frac{EI}{4}\left(\frac{3}{2}\theta_B - \frac{15}{4}\psi\right) = 0$$

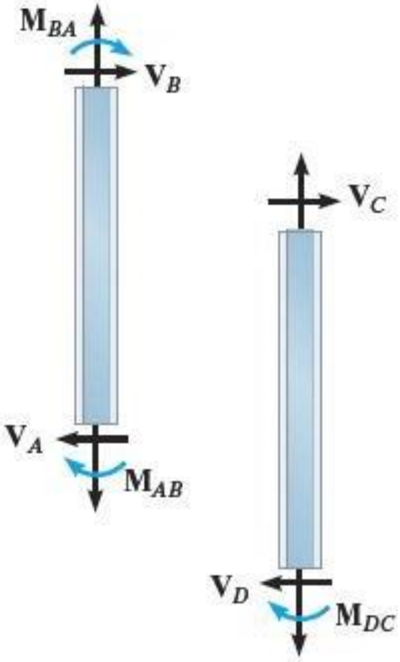
Thus,

$$\theta_B = \frac{240}{21EI} \qquad \psi = \frac{320}{21EI}$$

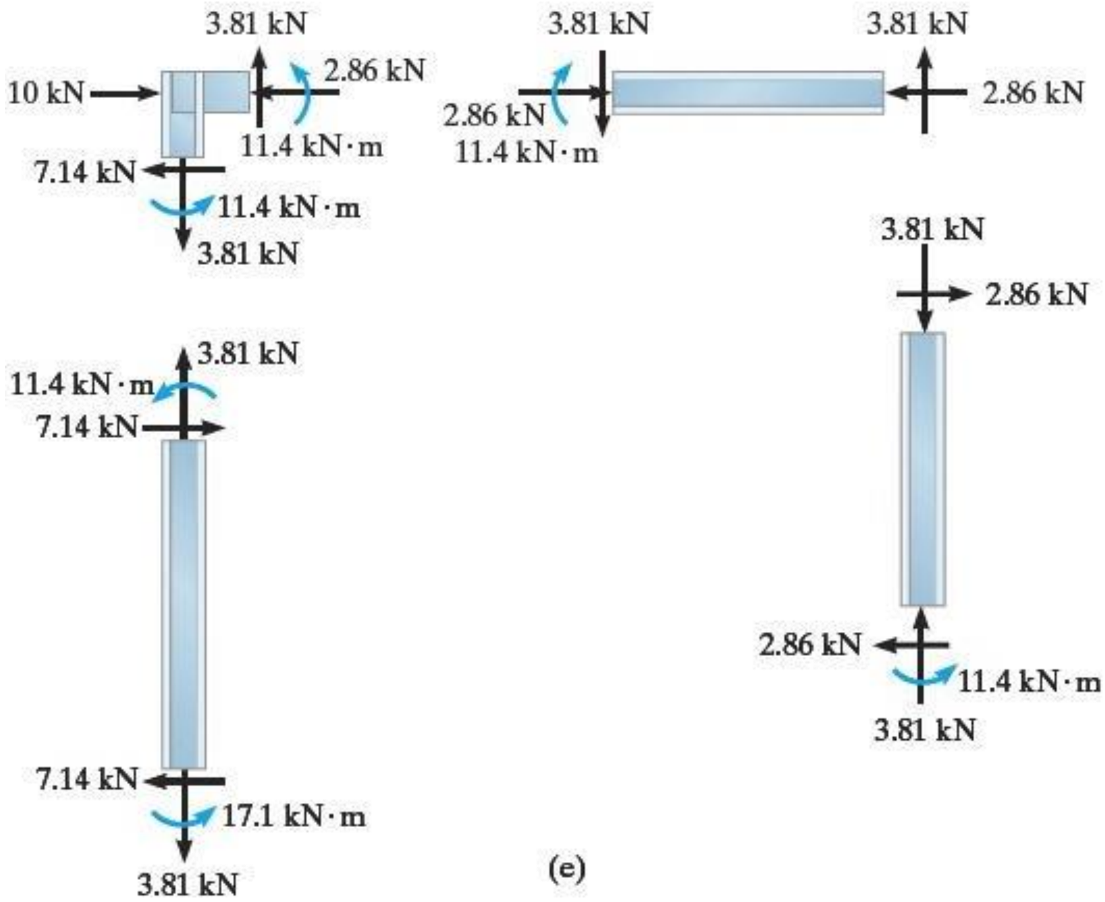
Substituting these values into Eqs. (1)–(4), we have

$M_{AB} = -17.1 \text{ kN}\cdot\text{m}$, $M_{BA} = -11.4 \text{ kN}\cdot\text{m}$ *Ans.*

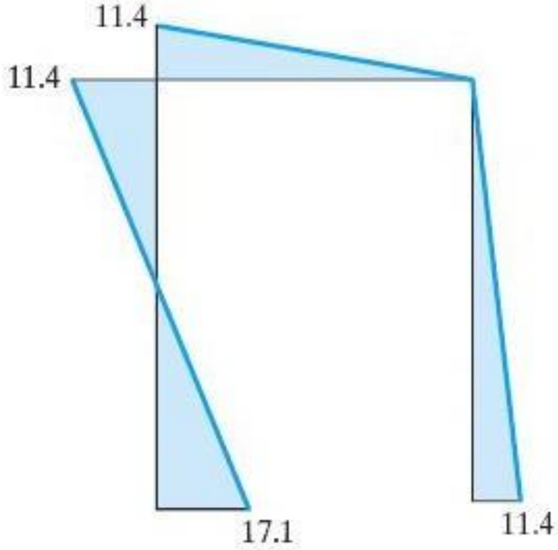
$M_{BC} = 11.4 \text{ kN}\cdot\text{m}$, $M_{DC} = -11.4 \text{ kN}\cdot\text{m}$ *Ans.*



(d)


3.81 kN
10 kN
2.86 kN
11.4 kN·m
7.14 kN
11.4 kN·m
3.81 kN
3.81 kN
3.81 kN
2.86 kN
2.86 kN
11.4 kN·m
3.81 kN
2.86 kN
3.81 kN
11.4 kN·m
7.14 kN
2.86 kN
11.4 kN·m
3.81 kN
7.14 kN
17.1 kN·m
3.81 kN

(e)


11.4
11.4
17.1
11.4

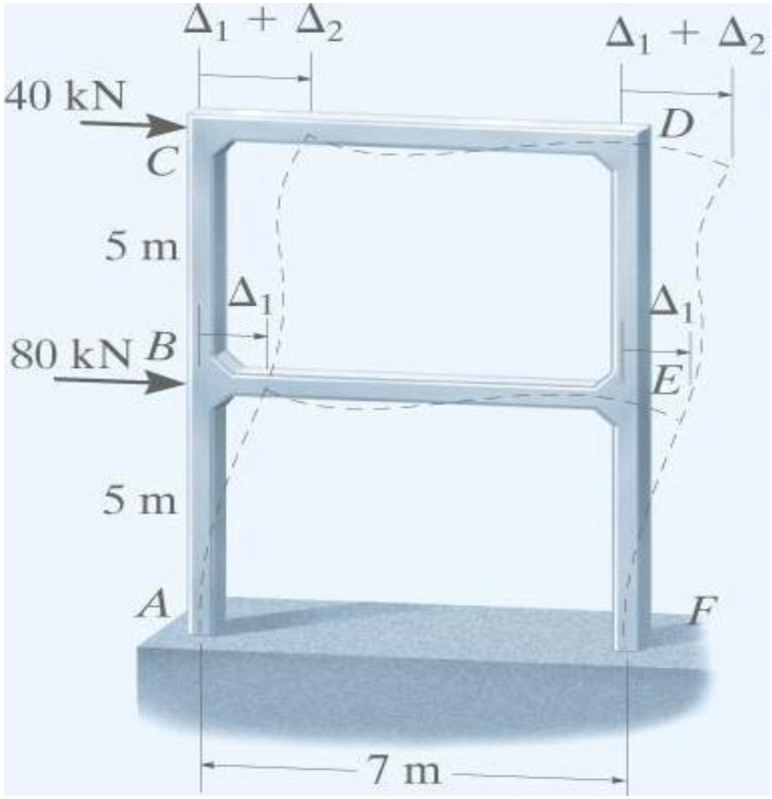
(f)

Example 9

– Draw the bending moment diagram. EI constant

Fixed End Moment

As there is no span loading in any of the member FEM for all the members is zero



Slope Deflection Equations

$$M_{AB} = 2E\left(\frac{I}{5}\right)\left[0. + \theta_B - 3\left(\frac{\Delta_1}{5}\right)\right] + 0$$

$$= \frac{2}{5}EI\theta_B - \frac{6}{25}EI\Delta_1$$

$$M_{BA} = 2E\left(\frac{I}{5}\right)\left[2\theta_B + 0 - 3\left(\frac{\Delta_1}{5}\right)\right] + 0$$

$$= \frac{4}{5}EI\theta_B - \frac{6}{25}EI\Delta_1$$

$$M_{BE} = 2E\left(\frac{I}{7}\right)\left[2\theta_B + \theta_E - 3\left(\frac{0}{7}\right)\right] + 0 \qquad = \frac{4}{7}EI\theta_B + \frac{2}{7}EI\theta_E$$

$$M_{EB} = 2E\left(\frac{I}{7}\right)\left[2\theta_E + \theta_B - 3\left(\frac{0}{7}\right)\right] + 0 \qquad = \frac{2}{7}EI\theta_B + \frac{4}{7}EI\theta_E$$

$$M_{FE} = 2E\left(\frac{I}{5}\right)\left[0. + \theta_E - 3\left(\frac{\Delta_1}{5}\right)\right] + 0 \qquad = \frac{2}{5}EI\theta_E - \frac{6}{25}EI\Delta_1$$

$$M_{EF} = 2E\left(\frac{I}{5}\right)\left[2\theta_E + 0 - 3\left(\frac{\Delta_1}{5}\right)\right] + 0 \qquad = \frac{4}{5}EI\theta_E - \frac{6}{25}EI\Delta_1$$

$$M_{BC} = 2E\left(\frac{I}{5}\right)\left[2\theta_B + \theta_C - 3\left(\frac{\Delta_2}{5}\right)\right] + 0 \qquad = \frac{4}{5}EI\theta_B + \frac{2}{5}EI\theta_C - \frac{6}{25}EI\Delta_2$$

$$M_{CB} = 2E\left(\frac{I}{5}\right)\left[2\theta_C + \theta_B - 3\left(\frac{\Delta_2}{5}\right)\right] + 0 \qquad = \frac{2}{5}EI\theta_B + \frac{4}{5}EI\theta_C - \frac{6}{25}EI\Delta_2$$

$$M_{CD} = 2E\left(\frac{I}{7}\right)\left[2\theta_C + \theta_D - 3\left(\frac{0}{7}\right)\right] + 0 \qquad = \frac{4}{7}EI\theta_C + \frac{2}{7}EI\theta_D$$

$$M_{DC} = 2E\left(\frac{I}{7}\right)\left[2\theta_D + \theta_C - 3\left(\frac{0}{7}\right)\right] + 0 \qquad = \frac{2}{7}EI\theta_C + \frac{4}{7}EI\theta_D$$

$$M_{DE} = 2E\left(\frac{I}{5}\right)\left[2\theta_D + \theta_E - 3\left(\frac{\Delta_2}{5}\right)\right] + 0 \qquad = \frac{4}{5}EI\theta_D + \frac{2}{5}EI\theta_E - \frac{6}{25}EI\Delta_2$$

$$M_{ED} = 2E\left(\frac{I}{5}\right)\left[2\theta_E + \theta_D - 3\left(\frac{\Delta_2}{5}\right)\right] + 0 \qquad = \frac{2}{5}EI\theta_D + \frac{4}{5}EI\theta_E - \frac{6}{25}EI\Delta_2$$

Equilibrium Equations

Joint B $\Sigma M_B = 0$

$$M_{BA} + M_{BC} + M_{BE} = 0$$

$$\frac{4}{5}EI\theta_B - \frac{6}{25}EI\Delta_1 + \frac{4}{5}EI\theta_B + \frac{2}{5}EI\theta_C - \frac{6}{25}EI\Delta_2 + \frac{4}{7}EI\theta_B + \frac{2}{7}EI\theta_E = 0$$

$$380EI\theta_B + 70EI\theta_C + 50EI\theta_E - 42EI\Delta_1 - 42EI\Delta_2 = 0 \quad \text{-----} \quad (1)$$

Joint E $\Sigma M_E = 0$

$$M_{EB} + M_{ED} + M_{EF} = 0$$

$$\frac{2}{7}EI\theta_B + \frac{4}{7}EI\theta_E + \frac{2}{5}EI\theta_D + \frac{4}{5}EI\theta_E - \frac{6}{25}EI\Delta_2 + \frac{4}{5}EI\theta_E - \frac{6}{25}EI\Delta_1 = 0$$

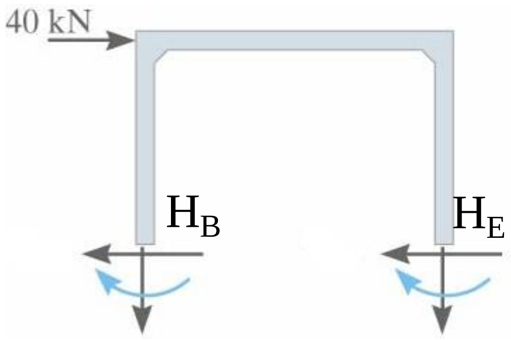
$$50EI\theta_B + 70EI\theta_D + 380EI\theta_E - 42EI\Delta_1 - 42EI\Delta_2 = 0 \quad \text{-----} \quad (2)$$

Joint C $\Sigma M_C = 0$

$$M_{CB} + M_{CD} = 0$$

$$\frac{2}{5}EI\theta_B + \frac{4}{5}EI\theta_C - \frac{6}{25}EI\Delta_2 + \frac{4}{7}EI\theta_C + \frac{2}{7}EI\theta_D = 0$$

$$70EI\theta_B + 240EI\theta_C + 50EI\theta_D - 42EI\Delta_2 = 0 \quad \text{......} \quad (3)$$



Joint D $\Sigma M_D = 0$

$$M_{DC} + M_{DE} = 0$$

$$\frac{2}{7}EI\theta_C + \frac{4}{7}EI\theta_D + \frac{4}{5}EI\theta_D + \frac{2}{5}EI\theta_E - \frac{6}{25}EI\Delta_2 = 0$$

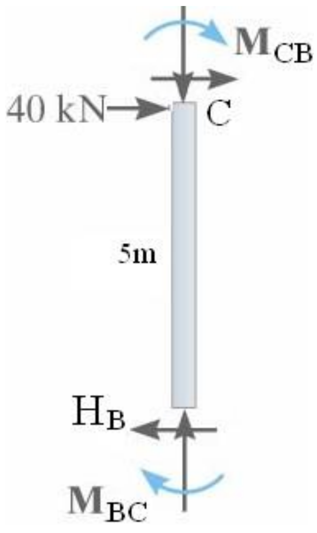
$$50EI\theta_C + 240EI\theta_D + 70EI\theta_E - 42EI\Delta_2 = 0 \quad \text{......} \quad (4)$$

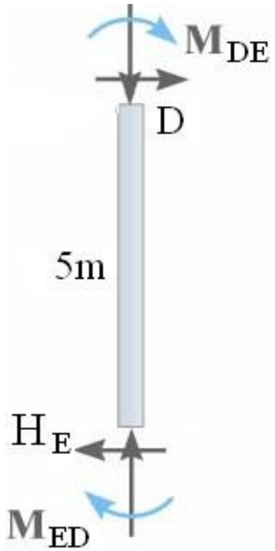
Top story $\Sigma F_X = 0$

$$40 - H_B - H_E = 0$$

$$H_B = -\frac{M_{CB} + M_{BC}}{5}$$

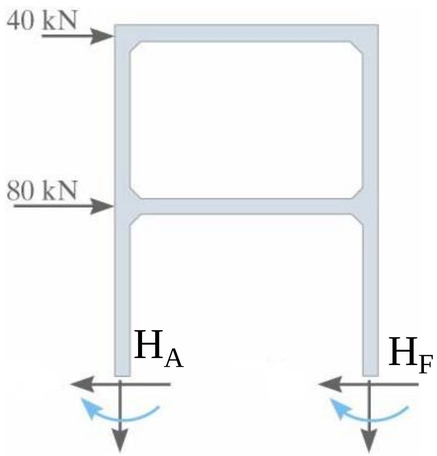
$$H_E = -\frac{M_{DE} + M_{ED}}{5}$$





$$200+\frac{2}{5}EI\theta_B+\frac{4}{5}EI\theta_C-\frac{6}{25}EI\Delta_2+\frac{4}{5}EI\theta_B+\frac{2}{5}EI\theta_C-\frac{6}{25}EI\Delta_2$$

$$+\frac{4}{5}EI\theta_D+\frac{2}{5}EI\theta_E-\frac{6}{25}EI\Delta_2+\frac{2}{5}EI\theta_D+\frac{4}{5}EI\theta_E-\frac{6}{25}EI\Delta_2=0$$



$$6EI\theta_B+6EI\theta_C+6EI\theta_D+6EI\theta_E-4.8EI\Delta_2=-1000 \quad \text{......} \quad (5)$$

Bottom story $\Sigma F_X = 0$

$$40+80-H_A-H_F=0$$

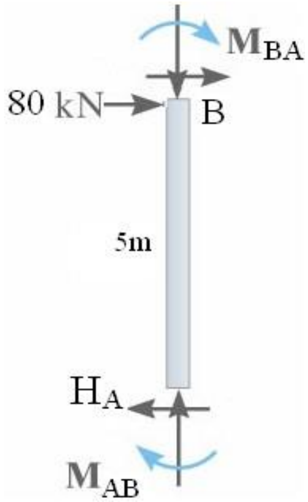
$$H_A=-\frac{M_{BA}+M_{AB}}{5}$$

$$H_F=-\frac{M_{EF}+M_{FE}}{5}$$

$$600+\frac{4}{5}EI\theta_B-\frac{6}{25}EI\Delta_1+\frac{2}{5}EI\theta_B-\frac{6}{25}EI\Delta_1$$

$$+\frac{4}{5}EI\theta_E-\frac{6}{25}EI\Delta_1+\frac{2}{5}EI\theta_E-\frac{6}{25}EI\Delta_1=0$$

$$30EI\theta_B+30EI\theta_E-24EI\Delta_1=-15000 \quad \text{......} \quad (6)$$



6 unknown and 6 equation

$$\begin{bmatrix} 380 & 70 & 0 & 50 & -42 & -42 \\ 50 & 0 & 70 & 380 & -42 & -42 \\ 70 & 240 & 50 & 0 & 0 & -42 \\ 0 & 50 & 240 & 70 & 0 & -42 \\ 6 & 6 & 6 & 6 & 0 & -4.8 \\ 30 & 0 & 0 & 30 & -24 & 0 \end{bmatrix} \begin{Bmatrix} EI\theta_B \\ EI\theta_C \\ EI\theta_D \\ EI\theta_E \\ EI\Delta_1 \\ EI\Delta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1000 \\ -15000 \end{Bmatrix} \longrightarrow \begin{matrix} EI\,\theta_B = 171.79 \\ EI\,\theta_C = 79.80 \\ EI\,\theta_D = 79.80 \\ EI\,\theta_E = 171.79 \\ EI\,\Delta_1 = 1054.46 \\ EI\,\Delta_2 = 837.29 \end{matrix}$$

Substituting in slope deflection equations

$M_{AB} = -184.4 kN.m$

$M_{BA} = -115.6 kN.m$

$M_{BE} = 147.2 kN.m$

$M_{EB} = 147.2 kN.m$

$M_{FE} = -184.4 kN.m$

$M_{EF} = -115.6 kN.m$

$M_{BC} = -31.6 kN.m$

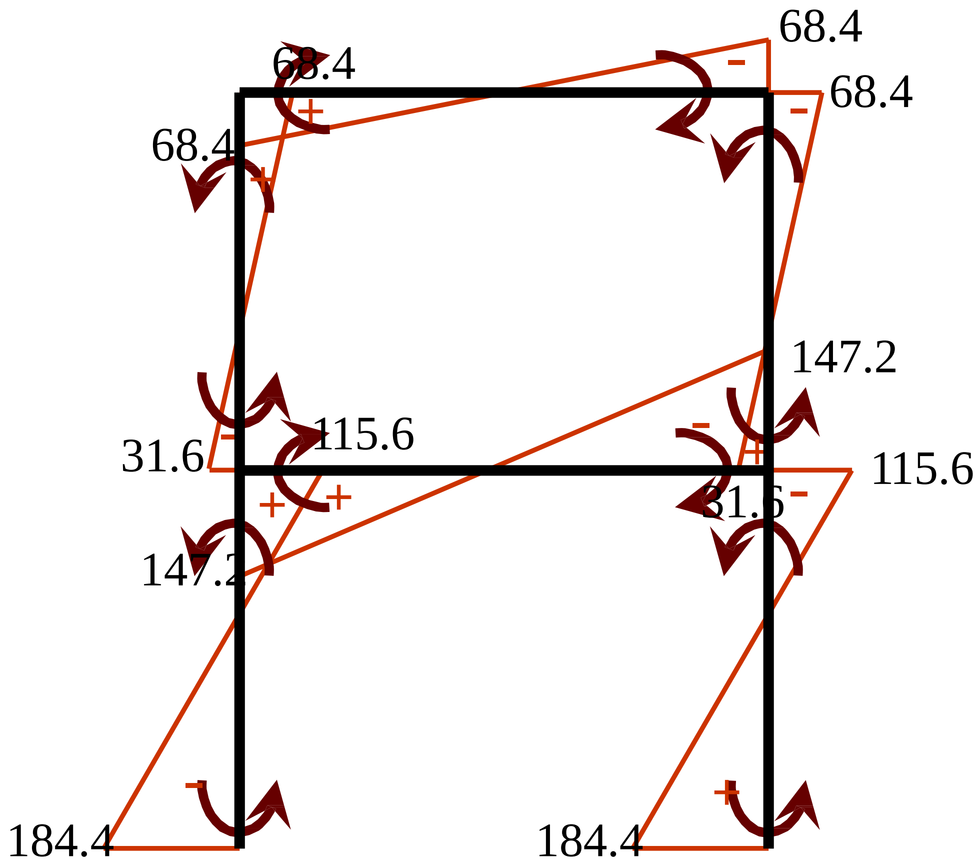
$M_{CB} = -68.4 kN.m$

$M_{CD} = 68.4 kN.m$

$M_{DC} = 68.4 kN.m$

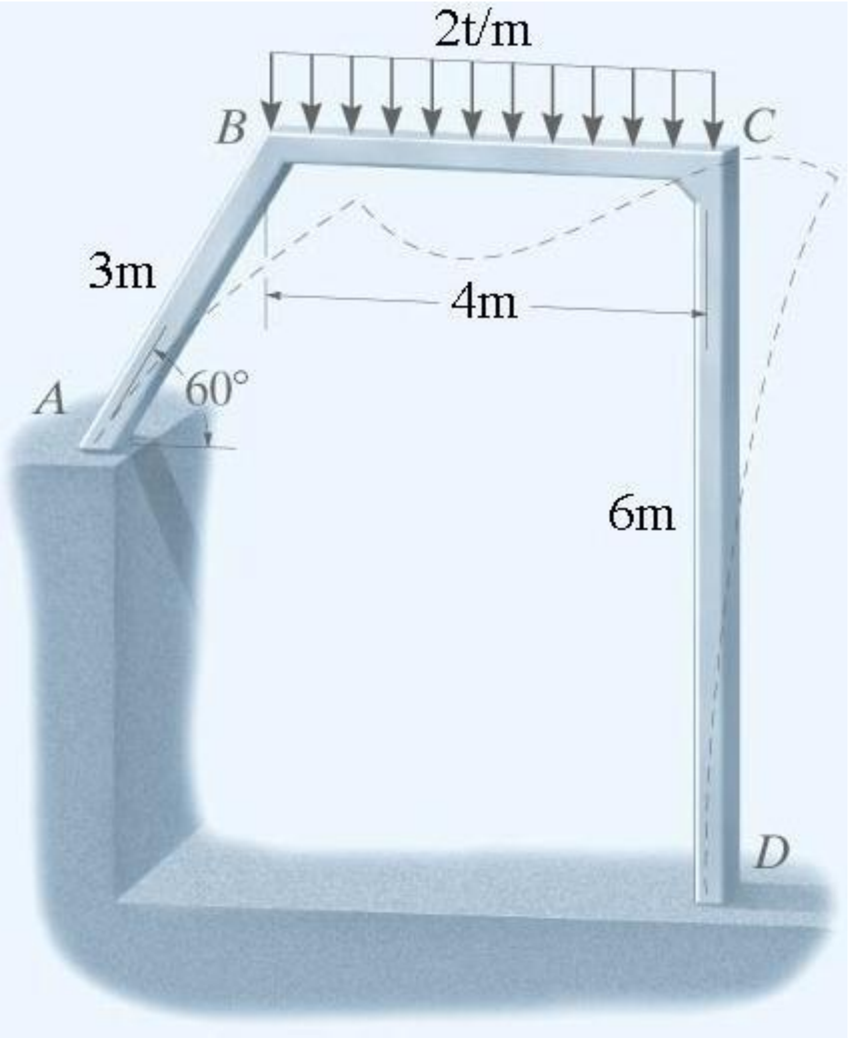
$M_{DE} = -68.4 kN.m$

$M_{ED} = -31.6 kN.m$


68.4
68.4
68.4
68.4
147.2
31.6
115.6
115.6
31.6
147.2
184.4
184.4

Example 10

– Draw the bending moment diagram. EI constant



Degree of freedom

DOF = 3x4-6-3 = 3

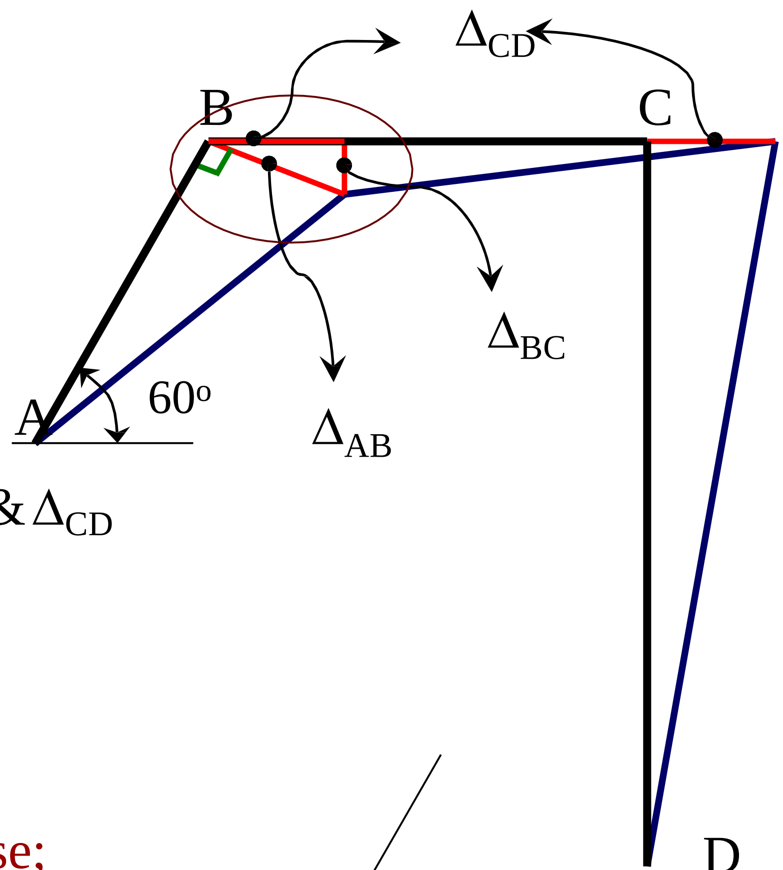
That means we got three unknown & we need three equations

Before we start let us discus the relative displacement (Δ) of each span

The relative displacement (Δ) for span AB is equal $\Delta_{AB} - 0. = \Delta_{AB}$ (clockwise)

The relative displacement (Δ) for span BC is equal $0. - \Delta_{BC} = -\Delta_{BC}$ (counterclockwise)

The relative displacement (Δ) for span CD is equal $0. - (-\Delta_{CD}) = \Delta_{CD}$ (clockwise)



Let us build a relationship between Δ_{AB}, Δ_{BC} & Δ_{CD}

take $\Delta_{AB} = \Delta$

$\Delta_{BC} = \Delta_{AB} \times \sin 30 = 0.5\Delta$

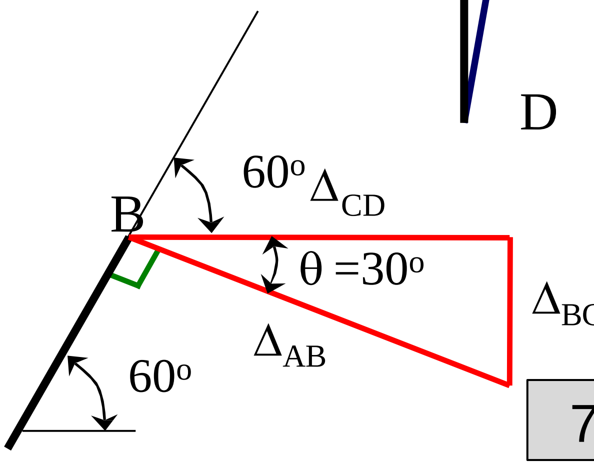
$\Delta_{CD} = \Delta_{AB} \times \cos 30 = 0.866\Delta$

So in the slope deflection equations we will use;

Δ as the relative displacement of span AB.

-0.5Δ as the relative displacement of span BC.

0.866Δ as the relative displacement of span CD.



Fixed End Moment

$$FEM_{AB} = FEM_{BA} = 0.$$

$$FEM_{BC} = -\frac{2\times 4^2}{12} = -2.667\,t.m$$

$$FEM_{CB} = 2.667.m$$

$$FEM_{CD} = FEM_{DC} = 0.$$

Slope Deflection Equations

$$M_{AB} = 2E\left(\frac{I}{3}\right)\left[0. + \theta_B - 3\left(\frac{\Delta}{3}\right)\right] + 0 \qquad = \frac{2}{3}EI\theta_B - \frac{2}{3}EI\Delta$$

$$M_{BA} = 2E\left(\frac{I}{3}\right)\left[2\theta_B + 0 - 3\left(\frac{\Delta}{3}\right)\right] + 0 \qquad = \frac{4}{3}EI\theta_B - \frac{2}{3}EI\Delta$$

$$M_{BC} = 2E\left(\frac{I}{4}\right)\left[2\theta_B + \theta_C - 3\left(\frac{-0.5\Delta}{4}\right)\right] - 2.667 \qquad = EI\theta_B + \frac{1}{2}EI\theta_C + \frac{3}{16}EI\Delta - 2.667$$

$$M_{CB} = 2E\left(\frac{I}{4}\right)\left[\theta_B + 2\theta_C - 3\left(\frac{-0.5\Delta}{4}\right)\right] + 2.667 \qquad = \frac{1}{2}EI\theta_B + EI\theta_C + \frac{3}{16}EI\Delta + 2.667$$

$$M_{CD} = 2E\left(\frac{I}{6}\right)\left[2\theta_C + 0 - 3\left(\frac{0.866\Delta}{6}\right)\right] + 0 \qquad = \frac{2}{3}EI\theta_C - \frac{0.866}{6}EI\Delta$$

$$M_{DC} = 2E\left(\frac{I}{6}\right)\left[0 + \theta_C - 3\left(\frac{0.866\Delta}{6}\right)\right] + 0 \qquad = \frac{1}{3}EI\theta_C - \frac{0.866}{6}EI\Delta$$

Equilibrium Equations

Joint B $\Sigma M_B = 0$

$$M_{BA} + M_{BC} = 0$$

$$\frac{4}{3}EI\theta_B - \frac{2}{3}EI\Delta + EI\theta_B + \frac{1}{2}EI\theta_C + \frac{3}{16}EI\Delta - 2.667 = 0$$

$$112EI\theta_B + 24EI\theta_C - 23EI\Delta = 128 \quad \text{..................} \quad (1)$$

Joint C $\Sigma M_C = 0$

$$M_{CB} + M_{CD} = 0$$

$$\frac{1}{2}EI\theta_B + EI\theta_C + \frac{3}{16}EI\Delta + 2.667 + \frac{2}{3}EI\theta_C - \frac{0.866}{6}EI\Delta = 0$$

$$24EI\theta_B + 80EI\theta_C + 2.072EI\Delta = -128 \quad \text{..................} \quad (2)$$

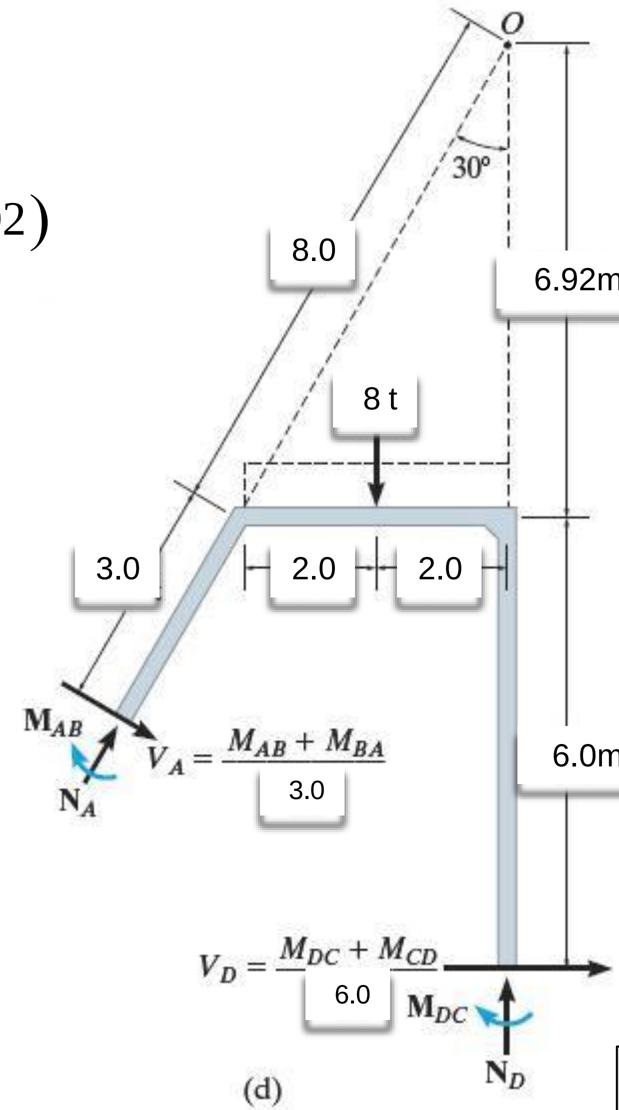
Third Equilibrium Equations (Method 1)

$$M_{AB} + M_{DC} - \left(\frac{M_{AB} + M_{BA}}{3}\right)(11) - \left(\frac{M_{DC} + M_{CD}}{6}\right)(12.92)$$

$$-8(2) = 0.0$$

$$-21M_{AB} - 22M_{BA} - 12.92M_{CD} - 11.92M_{DC} - 96 = 0$$

$$34.5EI\theta_B - 3.1EI\theta_C - 38.375EI\Delta = -144 \qquad \text{---------} \quad (3)$$



Third Equilibrium Equations (Method 2)

Third equation $\Sigma F_X = 0$

$$-H_A - H_D = 0$$

From the free body diagram for column CD

$$H_D = -\frac{M_{CD} + M_{DC}}{6}$$

Free body diagram for column AB

$$H_A \times 2.6 + (M_{BA} + M_{AB}) + V_A \times 1.5 = 0$$

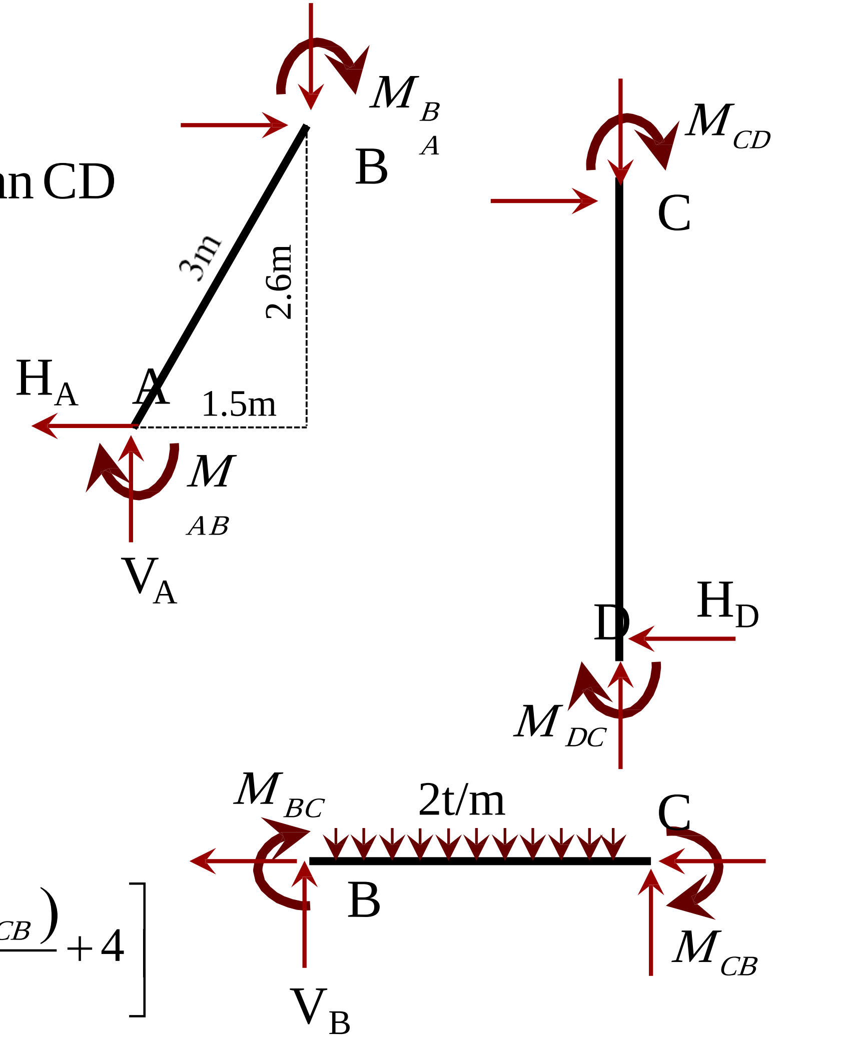
Free body diagram for Beam BC

$$V_B \times 4 + (M_{BC} + M_{CB}) - 2 \times 4 \times 2 = 0$$

$$V_A = V_B = -\frac{(M_{BC} + M_{CB})}{4} + 4$$

$$\therefore H_A = \frac{-(M_{BA} + M_{AB})}{2.6} - \frac{1.5}{2.6}\left[-\frac{(M_{BC} + M_{CB})}{4} + 4\right]$$

$$\frac{(M_{BA} + M_{AB})}{2.6} + \frac{1.5}{2.6}\left[-\frac{(M_{BC} + M_{CB})}{4} + 4\right] + \frac{M_{CD} + M_{DC}}{6} = 0.$$



$$24\left(M_{BA}+M_{AB}\right)-9\left(M_{BC}+M_{CB}\right)+10.4\left(M_{CD}+M_{DC}\right)=-144$$

$$24\left(\frac{4}{3}EI\theta_B-\frac{2}{3}EI\Delta+\frac{2}{3}EI\theta_B-\frac{2}{3}EI\Delta\right)-9\left(EI\theta_B+\frac{1}{2}EI\theta_C\right.$$

$$\left.+\frac{3}{16}EI\Delta-2.667+\frac{1}{2}EI\theta_B+EI\theta_C+\frac{3}{16}EI\Delta+2.667\right)$$

$$+10.4\left(\frac{2}{3}EI\theta_C-\frac{0.866}{6}EI\Delta+\frac{1}{3}EI\theta_C-\frac{0.866}{6}EI\Delta\right)=-144$$

$$34.5EI\theta_B-3.1EI\theta_C-38.375EI\Delta=-144 \quad \text{---------------} \quad (3)$$

Solving the three equation

$$\begin{bmatrix}112 & 24 & -23\\ 24 & 80 & 2.072\\ 34.5 & -3.1 & -38.375\end{bmatrix}\begin{Bmatrix}EI\theta_B\\ EI\theta_C\\ EI\Delta\end{Bmatrix}=\begin{Bmatrix}128\\ -128\\ -144\end{Bmatrix} \longrightarrow$$

$EI\,\theta_B = 3.11$

$EI\,\theta_C = -2.71$

$EI\,\Delta = 6.77$

Solving the three equation

$$\begin{bmatrix} 112 & 24 & -23 \\ 24 & 80 & 2.072 \\ 34.5 & -3.1 & -38.375 \end{bmatrix} \begin{Bmatrix} EI\theta_B \\ EI\theta_C \\ EI\Delta \end{Bmatrix} = \begin{Bmatrix} 128 \\ -128 \\ -144 \end{Bmatrix}$$

$$EI\,\theta_B = 3.11$$

$$EI\,\theta_C = -2.71$$

$$EI\,\Delta = 6.77$$

Substituting in slope deflection equations

$$M_{AB} = -2.44 t.m$$

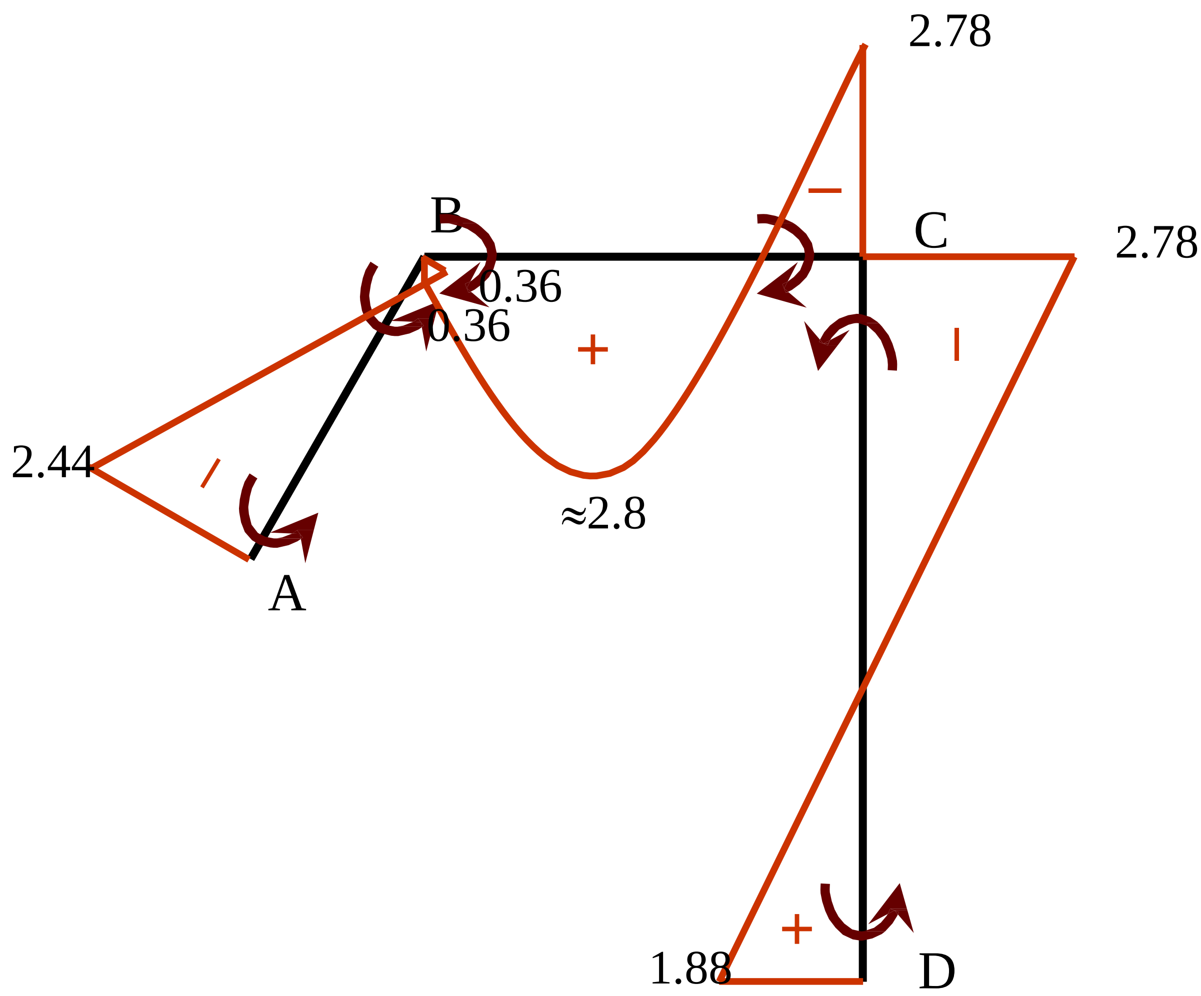
$$M_{BA} = -0.36 t.m$$

$$M_{BC} = 0.36 t.m$$

$$M_{CB} = 2.78 t.m$$

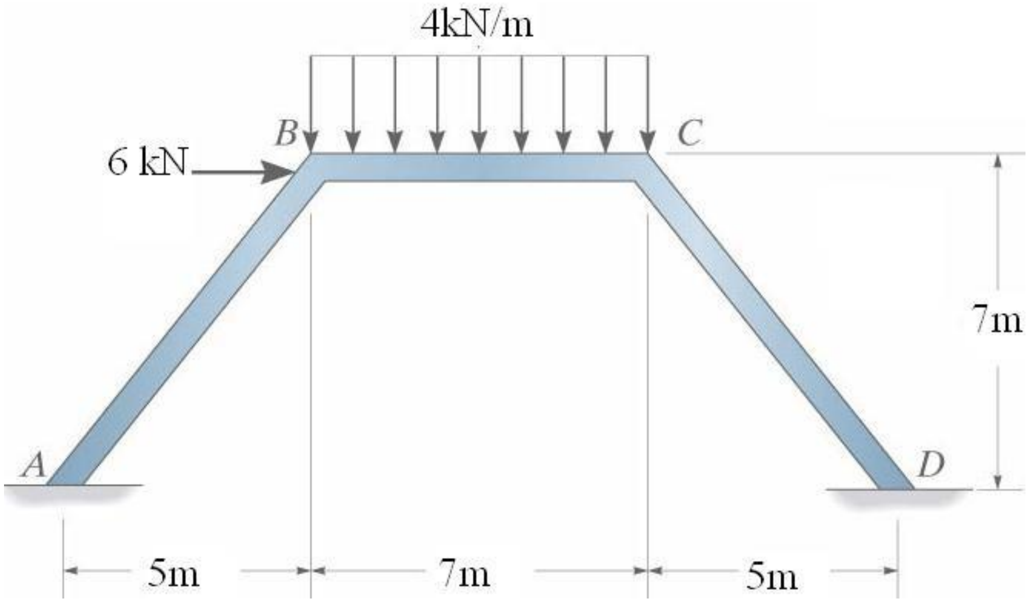
$$M_{CD} = -2.78 t.m$$

$$M_{DC} = -1.88 t.m$$



Example

– Draw the bending moment diagram. EI constant

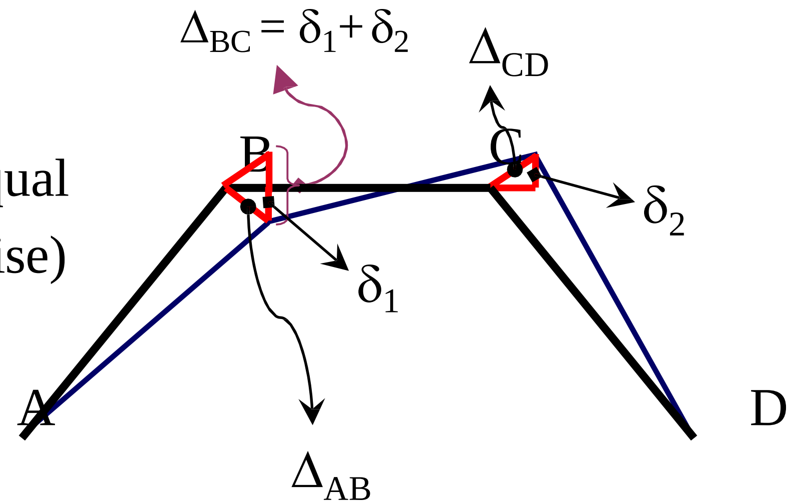


Before we start let us discus the relative displacement (Δ) of each span

The relative displacement (Δ) for span AB is equal $\Delta_{AB} - 0. = \Delta_{AB}$ (clockwise)

The relative displacement (Δ) for span BC is equal $(-\delta_2) - \delta_1 = -(\delta_1 + \delta_2) = -\Delta_{BC}$ (counterclockwise)

The relative displacement (Δ) for span CD is equal $0. - (-\Delta_{CD}) = \Delta_{CD}$ (clockwise)

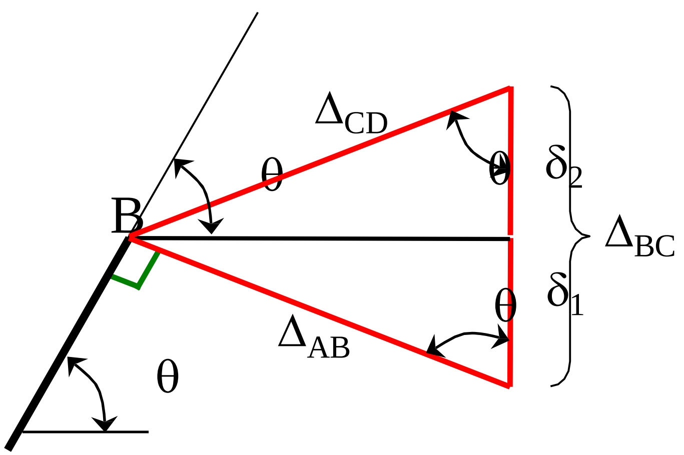


Let us build a relationship between Δ_{AB}, Δ_{BC} & Δ_{CD}

take $\Delta_{AB} = \Delta$

$\Delta_{BC} = 2(\Delta_{AB} \times \cos\theta) = 2\Delta \times 5/8.6 = 1.163\Delta$

$\Delta_{CD} = \Delta_{AB} = \Delta$



$\delta_1 = \delta_2$ & $\Delta_{CD} = \Delta_{AB}$ because of the symmetry in the geometry

<u>So in the slope deflection equations we will use;</u>

<u>Δ</u> as the relative displacement of span AB.

<u>-1.163Δ</u> as the relative displacement of span BC.

<u>Δ</u> as the relative displacement of span CD.

Fixed End Moment

$$FEM_{AB} = FEM_{BA} = 0.$$

$$FEM_{BC} = -\frac{4\times 7^2}{12} = -16.33\,kN.m$$

$$FEM_{CB} = 16.33\,kN.m$$

$$FEM_{CD} = FEM_{DC} = 0.$$

Slope Deflection Equations

$$M_{AB} = 2E\left(\frac{I}{8.6}\right)\left[0.+\theta_B - 3\left(\frac{\Delta}{8.6}\right)\right] + 0 \qquad = \frac{2}{8.6}EI\theta_B - \frac{6}{73.96}EI\Delta$$

$$M_{BA} = 2E\left(\frac{I}{8.6}\right)\left[2\theta_B + 0 - 3\left(\frac{\Delta}{8.6}\right)\right] + 0 \qquad = \frac{4}{8.6}EI\theta_B - \frac{6}{73.96}EI\Delta$$

$$M_{BC} = 2E\left(\frac{I}{7}\right)\left[2\theta_B + \theta_C - 3\left(\frac{-1.163\Delta}{7}\right)\right] - 16.33 = \frac{4}{7}EI\theta_B + \frac{2}{7}EI\theta_C + \frac{6.978}{49}EI\Delta - 16.33$$

$$M_{CB} = 2E\left(\frac{I}{7}\right)\left[2\theta_C + \theta_B - 3\left(\frac{-1.163\Delta}{7}\right)\right] + 16.33 = \frac{2}{7}EI\theta_B + \frac{4}{7}EI\theta_C + \frac{6.978}{49}EI\Delta + 16.33$$

$$M_{CD} = 2E\left(\frac{I}{8.6}\right)\left[2\theta_C + 0 - 3\left(\frac{\Delta}{8.6}\right)\right] + 0 \qquad = \frac{4}{8.6}EI\theta_C - \frac{6}{73.96}EI\Delta$$

$$M_{DC} = 2E\left(\frac{I}{8.6}\right)\left[0 + \theta_C - 3\left(\frac{\Delta}{8.6}\right)\right] + 0 \qquad = \frac{2}{8.6}EI\theta_C - \frac{6}{73.96}EI\Delta$$

Equilibrium Equations

Joint B $\Sigma M_B = 0$

$$M_{BA} + M_{BC} = 0$$

$$\frac{4}{8.6}EI\theta_B - \frac{6}{73.96}EI\Delta + \frac{4}{7}EI\theta_B + \frac{2}{7}EI\theta_C + \frac{6.978}{49}EI\Delta - 16.33 = 0$$

$$103.65EI\theta_B + 28.57EI\theta_C + 6.13EI\Delta = 1633 \quad \text{------------} \; (1)$$

Joint C $\Sigma M_C = 0$

$$M_{CB} + M_{CD} = 0$$

$$\frac{2}{7}EI\theta_B + \frac{4}{7}EI\theta_C + \frac{6.978}{49}EI\Delta + 16.33 + \frac{4}{8.6}EI\theta_C - \frac{6}{73.96}EI\Delta = 0$$

$$28.57EI\theta_B + 103.65EI\theta_C + 6.13EI\Delta = -1633 \quad \text{------------} \; (2)$$

Third equation $\Sigma F_X = 0$

$$6 - H_A - H_D = 0$$

Free body diagram for column AB

$$H_A \times 7 + (M_{BA} + M_{AB}) + V_A \times 5 = 0$$

Free body diagram for Beam BC

$$V_B \times 7 + (M_{BC} + M_{CB}) - 4 \times 7 \times 3.5 = 0$$

$$V_A = V_B = -\frac{(M_{BC} + M_{CB})}{7} + 14$$

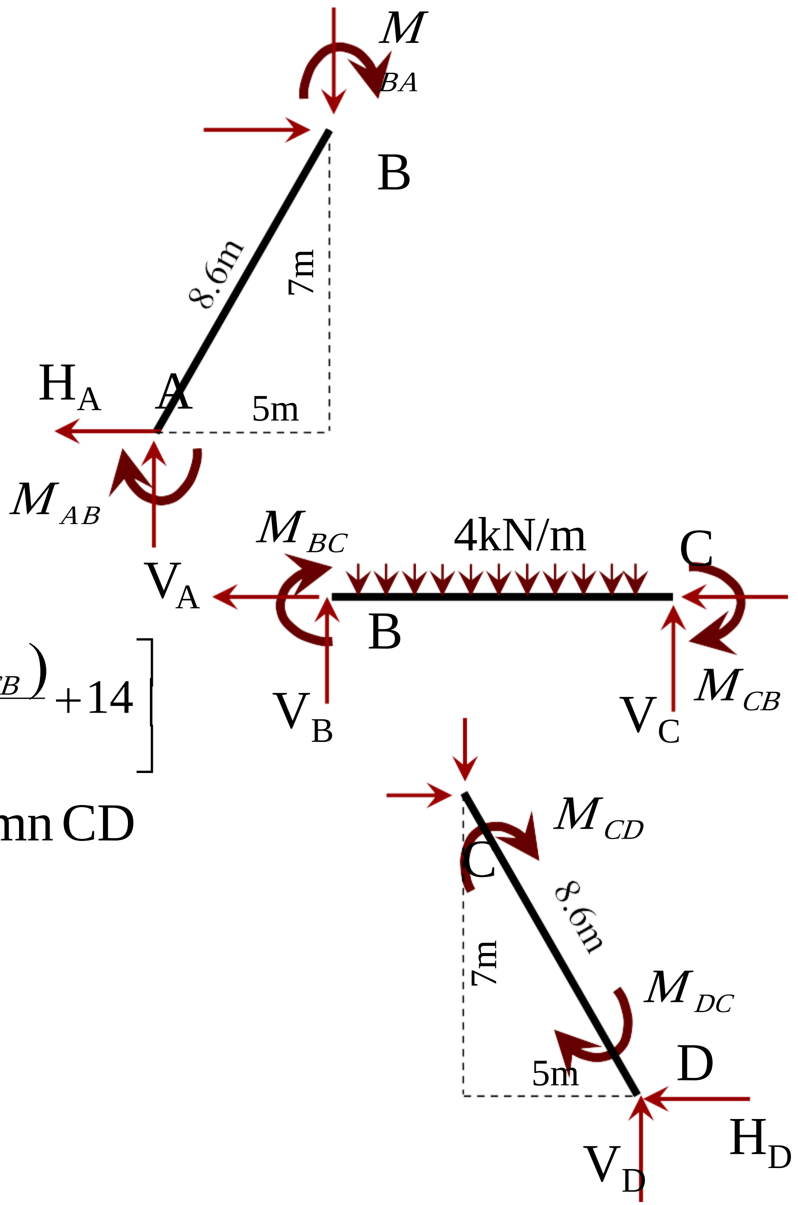
$$\therefore H_A = -\frac{(M_{BA} + M_{AB})}{7} - \frac{5}{7}\left[-\frac{(M_{BC} + M_{CB})}{7} + 14\right]$$

From the free body diagram for column CD

$$H_D \times 7 + (M_{CD} + M_{DC}) - V_D \times 5 = 0$$

$$V_C \times 7 - (M_{BC} + M_{CB}) - 4 \times 7 \times 3.5 = 0$$

$$V_D = V_C = \frac{(M_{BC} + M_{CB})}{7} + 14$$



$$\therefore H_D = -\frac{(M_{CD}+M_{DC})}{7} + \frac{5}{7}\left[\frac{(M_{BC}+M_{CB})}{7} + 14\right]$$

$$\because 6 - H_A - H_D = 0$$

$$6\times 49 + 7(M_{BA}+M_{AB}) - 5(M_{BC}+M_{CB}) + 7(M_{CD}+M_{DC}) - 5(M_{BC}+M_{CB}) = 0$$

$$7(M_{BA}+M_{AB}) + 7(M_{CD}+M_{DC}) - 10(M_{BC}+M_{CB}) = -294$$

$$7\left(\frac{2}{8.6}EI\theta_B - \frac{6}{73.96}EI\Delta + \frac{4}{8.6}EI\theta_B - \frac{6}{73.96}EI\Delta\right) + 7\left(\frac{2}{8.6}EI\theta_C - \frac{6}{73.96}EI\Delta\right.$$
$$\left. + \frac{4}{8.6}EI\theta_C - \frac{6}{73.96}EI\Delta\right) - 10\left(\frac{4}{7}EI\theta_B + \frac{2}{7}EI\theta_C + \frac{6.978}{49}EI\Delta - 16.33\right.$$
$$\left. + \frac{2}{7}EI\theta_B + \frac{4}{7}EI\theta_C + \frac{6.978}{49}EI\Delta + 16.33\right) = -294$$

$$-3.688EI\theta_B - 3.688EI\theta_C - 5.12EI\Delta = -294 \quad \text{......} (3)$$

Solving the three equation

$$\begin{bmatrix} 103.65 & 28.57 & 6.13 \\ 28.57 & 103.65 & 6.13 \\ -3.688 & -3.688 & -5.12 \end{bmatrix} \begin{Bmatrix} EI\theta_B \\ EI\theta_C \\ EI\Delta \end{Bmatrix} = \begin{Bmatrix} 1633 \\ -1633 \\ -294 \end{Bmatrix} \longrightarrow$$

$EI\,\theta_B = 18.897$

$EI\,\theta_C = -24.603$

$EI\Delta = 61.532$

Substituting in slope deflection equations

$$M_{AB} = -0.6 kN.m$$

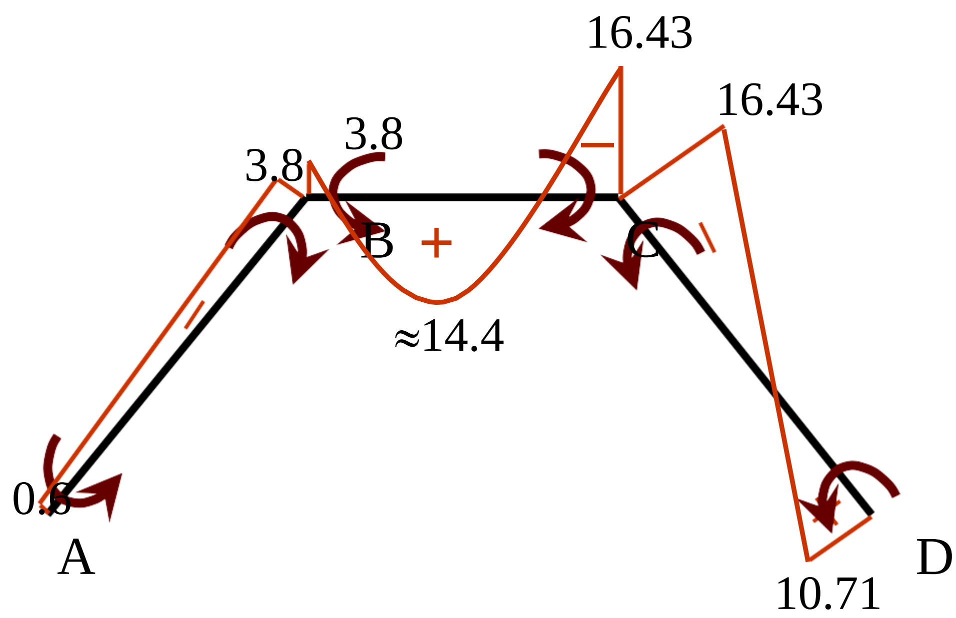
$$M_{BA} = 3.8 kN.m$$

$$M_{BC} = -3.8 kN.m$$

$$M_{CB} = 16.43 kN.m$$

$$M_{CD} = -16.43 kN.m$$

$$M_{DC} = -10.71 kN.m$$



Exercises

- **Problems (Page 466): 21,22,23,24**