

Tishk International University  
Architecture Department  
First Grade  
Fall semester 2024-2025

# Calculus

## Lecture -3- Non-Linear Functions

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1



***Non-Linear Function***

2

## Non-Linear Functions



- ❖ A function or equation having a degree greater than 1 with dependent and independent variable(s) will be called a nonlinear function.
- ❖ Such functions, when plotted, do not form a straight line. Alternatively, if any function is not linear, then it will certainly be a nonlinear function. Nonlinear equations are generally written as:

$$f(x) = y = ax^2 + bx + c$$

- ❖ A **nonlinear function** is a function whose graph is NOT a straight line. Its graph can be any curve other than a straight line. For example, if there are 100 fishes in a pond initially and they become double every week, then this situation can be modeled by the function  $f(x) = 100(2)^x$ , where  $x$  is the number of weeks and  $f(x)$  is the number of fishes. Let us make a table and graph this function making use of the table.

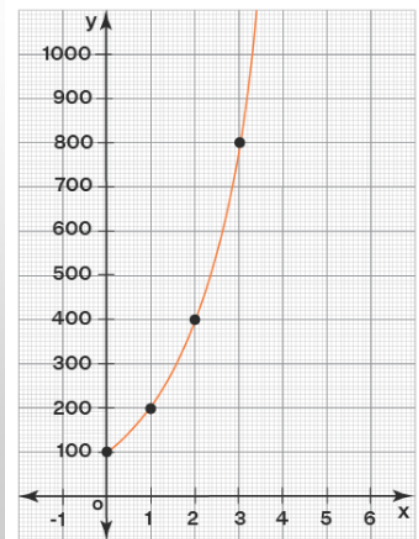
3

$$f(x) = 100(2)^x$$

| x | y   |
|---|-----|
| 0 | 100 |
| 1 | 200 |
| 2 | 400 |
| 3 | 800 |

Let's graph the table now.

- ❖ The shown graph is NOT a line and hence it represents a nonlinear function. From the graph, we can say that the slope is not uniform on a nonlinear function. A nonlinear function can be described using a table of values, an equation, or a graph. Let us see each of them now. Some of the examples of nonlinear functions include quadratic functions, cubic functions, polynomial functions.



4



The steps to determine whether a table of values determine a linear function are:

1. Find the differences between every two consecutive x values.
2. Find the differences between every two consecutive y values.
3. Find the corresponding ratios of differences of y and differences of x.
4. If all the ratios are NOT same, then only the function is linear.

Consider the following table of values.

| x  | y  |
|----|----|
| 3  | 15 |
| 5  | 23 |
| 9  | 33 |
| 11 | 41 |
| 13 | 43 |

- Let us determine whether this table denotes a nonlinear function by using the steps mentioned above.

- Since all the ratios of differences of y to the differences of x are NOT same, the function is a nonlinear function.

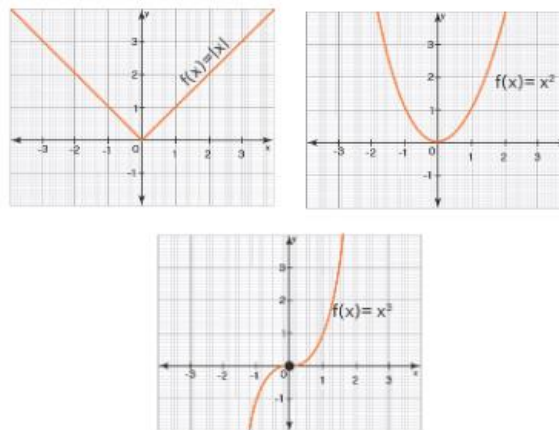
| x  | y   | Difference in y<br>Difference in x |
|----|-----|------------------------------------|
| 3  | 15  |                                    |
| +2 | +8  | $\frac{8}{2} = 4$                  |
| 5  | 23  |                                    |
| +4 | +10 | $\frac{10}{4} = 2.5$               |
| 9  | 33  |                                    |
| +2 | +8  | $\frac{8}{2} = 4$                  |
| 11 | 41  |                                    |
| +2 | +12 | $\frac{12}{2} = 6$                 |
| 13 | 43  |                                    |

All are not same  
Hence, its Nonlinear

5

➤ Some examples of nonlinear functions are:

- $f(x) = x^2$  is nonlinear as it is a quadratic function.
- $f(x) = 2^x$  is nonlinear as it is an exponential function.
- $f(x) = x^3 - 3x$  is nonlinear as it is a cubic function.



6

Here are the differences between linear and nonlinear functions.



| Linear Functions                                                                                   | Nonlinear Functions                                                                                       |
|----------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| A linear function is a function whose graph is a line.                                             | A nonlinear function is a function whose graph is NOT a line.                                             |
| Its equation is of the form $f(x) = ax + b$ .                                                      | Its equation can be in any form except of the form $f(x) = ax + b$ .                                      |
| Its slope is constant for any two points on the curve.                                             | The slope of every two points on the graph is NOT the same.                                               |
| In the table of a linear function, the ratio of difference of y and difference of x is a constant. | In the table of a nonlinear function, the ratio of difference of y and difference of x is NOT a constant. |

7

### Example -1-

Plot the graph for the nonlinear function  $f(x) = x^2 - 6x + 12$



#### Solution:

We will solve the nonlinear function at  $x=1,2,3,4$  and  $5$

$$y = x^2 - 6x + 12$$

When  $x = 1$

$$y = 1^2 - 6(1) + 12 = 7$$

When  $x = 2$

$$y = 2^2 - 6(2) + 12 = 4$$

When  $x = 4$

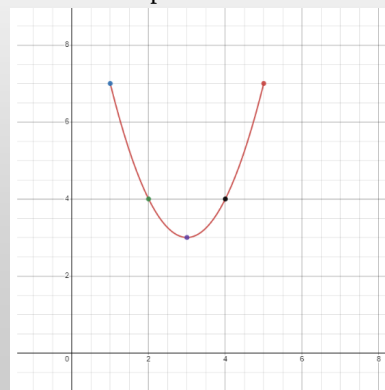
$$y = 4^2 - 6(4) + 12 = 4$$

When  $x = 5$

$$y = 5^2 - 6(5) + 12 = 7$$

Let us form the table so we can easily plot our ordered pairs.

| x | y |
|---|---|
| 1 | 7 |
| 2 | 4 |
| 3 | 3 |
| 4 | 4 |
| 5 | 7 |



8

### Example -2-



Plot the graph for the nonlinear function  $y = |x|$ .

#### Solution:

As “y” is equal to the absolute of “x,” “y” cannot be negative. Hence, we will have a bell-shaped graph. The value of “y” will be the same for every value of x.

When  $x = 1$

$$y = |1| = 1$$

When  $x = -1$

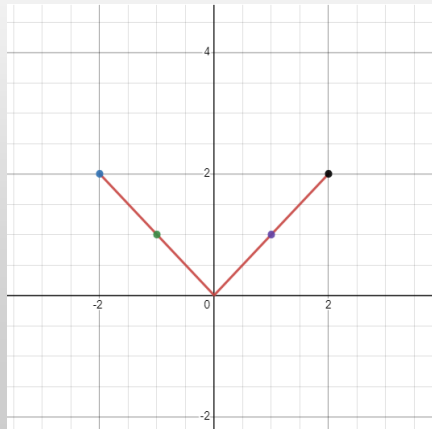
$$y = |-1| = 1$$

When  $x = 2$

$$y = |2| = 2$$

When  $x = -2$

$$y = |-2| = 2$$



We will have a “V” shaped graph, but as it is not a straight line, it is a nonlinear graph.

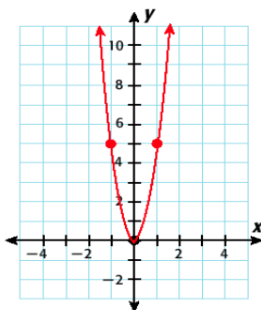
9

### Example -3-



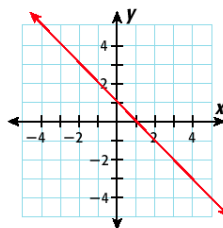
Tell whether the graph is linear or nonlinear.

A.



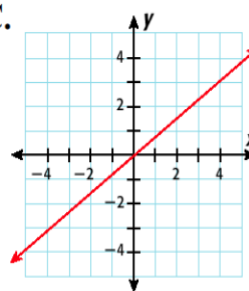
The graph is not a straight line, so it is nonlinear.

B.



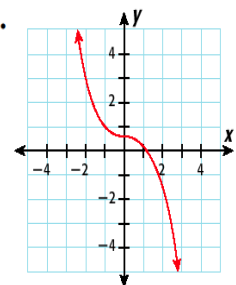
The graph is a straight line, so it is linear.

C.



The graph is a straight line, so it is linear.

D.



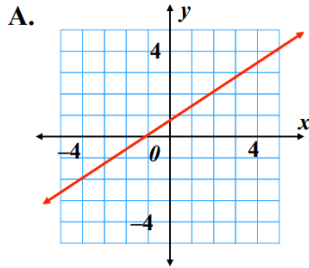
The graph is not a straight line so, it is nonlinear.

10

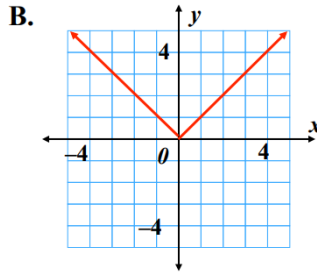
### Example -4-



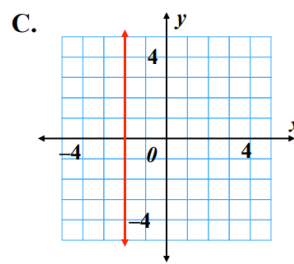
Tell whether the graph is linear or nonlinear.



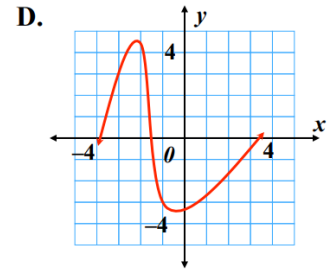
The graph is a straight line, so the graph is linear.



The graph is not a straight line, so it is nonlinear.



The graph is a straight line, so the graph is linear.



The graph is not a straight line, so it is nonlinear.

11

### Example -5-



Tell whether the function in the table has a linear or nonlinear relationship.

A.

| Input | Output |
|-------|--------|
| 1     | 2      |
| 2     | 5      |
| 3     | 11     |

Solution:

A.

| Input | Output |
|-------|--------|
| 1     | 2      |
| 2     | 5      |
| 3     | 11     |

difference = 1  
 difference = 1

difference = 3  
 difference = 6

*The difference between consecutive input values is constant.*  
*The difference between consecutive output values is not constant.*

The function represented in the table is nonlinear.

12



Tell whether the function in the table has a linear or nonlinear relationship.

**B.**

| Input | Output |
|-------|--------|
| 1     | 3      |
| 2     | 6      |
| 3     | 9      |

Solution:

difference = 1  
difference = 1

The difference between consecutive input values is constant.



| Input | Output |
|-------|--------|
| 1     | 3      |
| 2     | 6      |
| 3     | 9      |

difference = 3  
difference = 3

The difference between consecutive output values is constant.

The function represented in the table is linear.

13



Tell whether the function in the table has a linear or nonlinear relationship.

**C.**

| Input | Output |
|-------|--------|
| 1     | 1      |
| 2     | 4      |
| 3     | 9      |

Solution:

difference = 1  
difference = 1

The difference between consecutive input values is constant.



| Input | Output |
|-------|--------|
| 1     | 1      |
| 2     | 4      |
| 3     | 9      |

difference = 3  
difference = 5

The difference between consecutive output values is not constant.

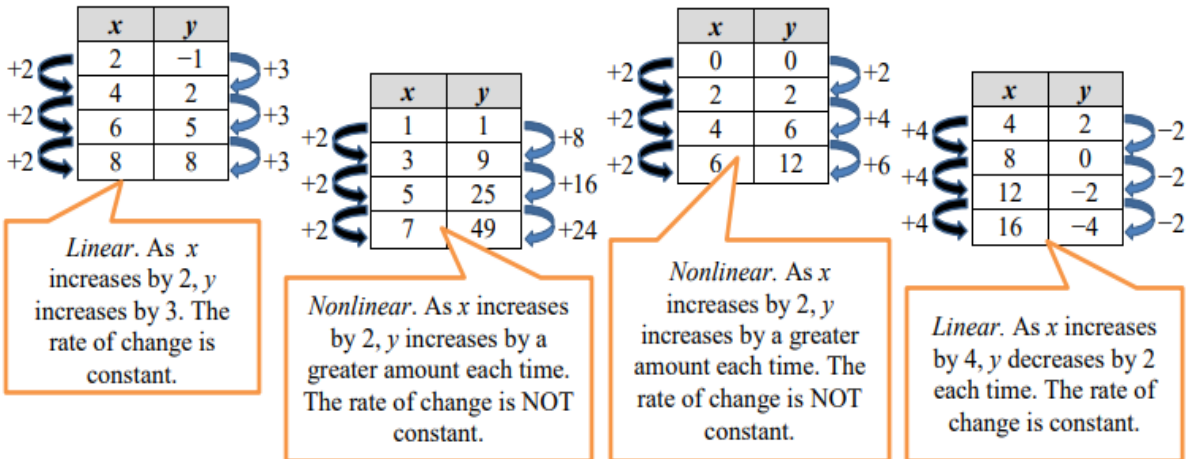
The function represented in the table is nonlinear.

14

### Example -6-



Determine whether each table represents a linear or nonlinear function.

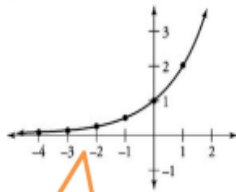


15

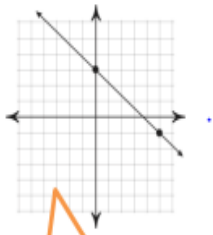
### Example -7-



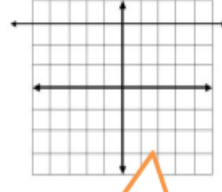
Determine whether each equation represents a linear or nonlinear function.



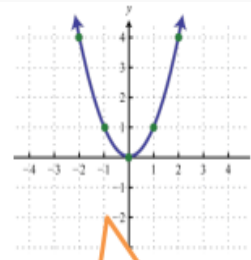
*Nonlinear.*  
Graph is not a straight line



*Linear.*  
Graph is a straight line



*Linear.*  
Graph is a straight line



*Nonlinear.*  
Graph is not a straight line

16

### Example -8-



Determine whether each equation represents a linear or nonlinear function. Remember that all linear functions can be written in the slope-intercept form.

$$y = 3x - 5$$

Linear. In slope-intercept form.

$$y = 2(x - 3)$$

Linear. Can be rewritten in slope-intercept form.

$$y = 2x - 6$$

$$y = \frac{x}{3} - 2$$

Linear. Can be rewritten in slope-intercept form.

$$y = \frac{1}{3}x - 2$$

$$y = \frac{3}{x} + 4$$

Nonlinear. Cannot be rewritten in slope-intercept form.

$$y = 3x$$

Linear. In slope-intercept form.

17

### Example -9-



The following table shows the bank balances of Joe and Mitchell for the last 55 years. Graph the data and check if there has been any constant growth for both.

| Year (x) | Joe   | Mitchell |
|----------|-------|----------|
| 1        | \$110 | \$110    |
| 2        | \$210 | \$250    |
| 3        | \$310 | \$160    |
| 4        | \$410 | \$280    |
| 5        | \$510 | \$400    |

18

**Solution:**

We will plot the points for both Joe and Mitchell.



- Joe's points are displayed on a straight line, while Mitchell's points are on a curved line, both with positive slopes.
- It is evident that Joe's graph maintains a constant growth rate with a consistent rate of change of \$100, while Mitchell's graph portrays an inconsistent growth pattern with a curve.
- These observations indicate that Joe's growth rate has been constant over the past five years.

19

**References**

- Thomas-Calculus-14th-Edition
- Internet sources

20