IT 347

Department of Cyber Security

Probability and Statistics

Lecture 6: Combination and Permutation

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Learning outcomes of this lecture

By the end of this lecture, the students will be able to:

- understand the concept of basic counting principles in probability.
- master the application of the multiplication principle to calculate probabilities of compound events.
- apply the addition principle to determine probabilities of mutually exclusive events.
- differentiate between combination and permutation and identify scenarios where each is applicable.
- demonstrate proficiency in calculating combinations and permutations in various problem-solving contexts.

Introduction

The study of permutations and combinations is concerned with determining the number of different ways of arranging and selecting objects out of a given number of objects, without actually listing them. There are some basic counting techniques which will be useful in determining the number of different ways of arranging or selecting objects. The two basic counting principles are given below:

Multiplication principle (Fundamental Principle of Counting):

- **Suppose an event E can occur in m different ways and associated with each way of occurring of E, and**
- **another event F can occur in n different ways, then**
- **the total number of occurrence of the two events in the given order is m × n .**

Addition principle:

- **If an event E can occur in m ways and another event F can occur in n ways, and**
- **suppose that both can not occur together,**
- **then E or F can occur in m + n ways.**

Theorem:

The total number of possible arrangements of n different objects is $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$.

Examples

Example 1: In a city, the bus route numbers consist of a natural number less than 100, followed by one of the letters A,B,C,D,E and F. How many different bus routes are possible?

Solution: The number can be any one of the natural numbers from 1 to 99. There are 99 choices for the number.

The letter can be chosen in 6 ways.

So, number of possible bus routes are $99 \times 6 = 594$.

Example 2: In how many ways can you choose a male and a female from a group of 5 males and 6 females.

Solution: The male can be chosen in 5 ways, and after choosing them, the female can be chosen in 6 ways. The total number of ways to choose a male and female is $5 \times 6 \; = \; 30$.

Example 3: How many different car number plates are possible with 2 letters followed by 4 digits?

Solution: for each letter, there are 26 possibilities, and for each digit there are 10 possibilities.
So, $26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6760000$. possible different car plate numbers.

Example 4: In how many ways can 6 people be arranged in a row?

Solution: $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

How many arrangements are possible if only 3 of them are chosen?

Solution: $6 \times 5 \times 4 \; = \; 120$.

Examples

Example 5: In how many ways can 5 children be arranged in a line such that

i. two particular children of them are always together
ii. two particular children of them are never together.

Solution: i. We consider the arrangements by taking 2 particular children together as one and hence the remaining 4 can be arranged in $4! = 24$ ways. Again, two particular children taken together can be arranged in two ways. Therefore, there are $24 \times 2 = 48$ total ways of arrangement.

ii. Among the $5! = 120$ permutations of 5 children, there are 48 in which two children are together. In the remaining $120 - 48 = 72$ permutations, two particular children are never together.

Example 6: In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together.

Solution:

First we take books of a particular subject as one unit. Thus there are 4 units which can be arranged in $4! = 24$ ways.

Now in each of arrangements, mathematics books can be arranged in $3!$ ways, history books in $4!$ ways, chemistry books in $3!$ ways and biology books in $2!$ ways.

Thus the total number of ways = $4! \times 3! \times 4! \times 3! \times 2! = 41472$.

Combinations & Permutations

In English we use the word "combination" loosely, without thinking if the order of things is important. In other words:

The salad is a combination of tomatoes, cucumbers and onion. We don't care what order the ingredients are in, they could also be cucumbers, onion and tomatoes or tomatoes onion and tomatoes, its the same salad.

The combination to the safe was 742. Now we do care about the order. "724" would not work, nor would "247". It has to be exactly 4-7-2.

So, in Mathematics we use more precise language:

- If the order <u>doesn't</u> matter, it is a **Combination**.
- If the order <u>does</u> matter, it is a **Permutation**.

Permutations

Permutations are a grouping of items in which order matters.
With permutations, we have to reduce the number of available choices each time.

The number of permutations of n different objects taken r at a time is: $\mathbf{P_r^n} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!}$**.**

where n = number of objects, and r = number of position.

Example 1: A debating team consists of 4 speakers.

1) In how many ways can all 4 speakers be arranged in a row for a photo?

Solution: $4! = 24$

2) How many ways can the captain and vice-captain be chosen?

Solution: $P_2^4 = \frac{4!}{(4-2)!} = 4 \times 3 = 12.$

Example 2: 6 people entered a room with 10 chairs. How can these chairs be occupied by these six people?

Solution: Since only 6 chairs can be occupied by the six persons, the number of seating arrangements is equal to the number of substitutions for 10 items, 6 of which are withdrawn at once. i.e.:

$$P_6^{10} = \frac{10!}{(10-6)!} = \frac{10!}{4!} = 10 \times 9 \times \cdots \times 5$$

Combination

Combinations are a grouping of items in which order does NOT matter.

The number of combinations of n different objects taken r at a time is: $\boldsymbol{C_r^n = \frac{n!}{r!(n-r)!}}$.

where n = number of objects, and r = number of position.

Example 1: How many ways can a basketball team of 5 players be chosen from 8 players?

Solution: $C_5^8 = \frac{8!}{5!(8-5)!} = \frac{8!}{5!\cdot 3!} = \frac{8\times 7\times 6}{6} = 56.$

Example 2: A company needs the services of 6 men and 3 boys. In how many ways can the company choose them if 9 men and 5 boys come to the interview?

Solution: $C_6^9 \times C_3^5 = \frac{9!}{6!(9-6)!} \times \frac{5!}{3!(5-3)!} = \frac{9\times 8\times 7}{6} \times \frac{5\times 4}{2} = 84 \times 10 = 840$

Example 3: A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many committees are possible if

a) there are no restrictions?

Solution: C_5^{10}

c) one particular woman must be excluded from the committee?

Solution: C_5^9

b) one particular person must be chosen for the committee?

Solution: $1 \times C_4^9$

Examples

Example 4: In how many ways can three cards be drawn from a back of 52 cards if:

a. Each drawn card is returned back before drawing the next one.
b. The drawn card does not return.

Example 5: If in an examination it is required to answer only 8 questions out of 10.

a. In how many ways one can answer 8 of these 10 questions?
b. In how many ways one can answer 8 of these 10 questions if the first 3 questions must be answered?

Examples

How many arrangements of the letters of the word **REMAND** are possible if:

a) There are no restrictions? $= 6! \quad or\ P_6^6$

b) They begin with RE? $= \mathbf{R\ E}\ \_\ \_\ \_\ \_ \ = 4! \quad or\ P_4^4$

c) They do not begin with RE? $= Total\ - (b) = 6! - 4! = 696$

Examples

In how many ways can we pick any one item from 3 pens and 2 markers?

1. Pick any one item.

P1, P2, P3, M1, M2 – total 5 ways.

2. Pick one pen **and** one marker.

P1 → M1, M2

P2 → M1, M2

P3 → M1, M2

$3\ ways\ \times 2\ ways = 6\ ways$

Examples

(a) How many different car number plates are possible with 3 letters followed by 3 digits?

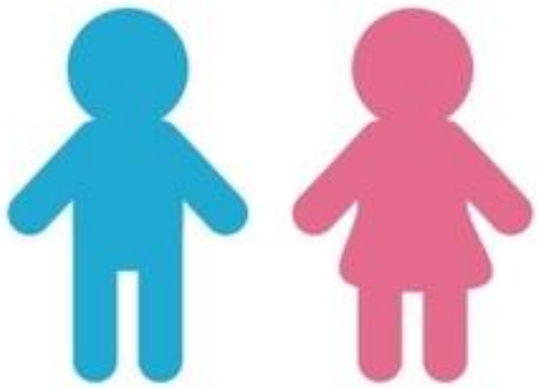
$$= 26 \times 26 \times 26 \times 10 \times 10 \times 10$$

(b) How many of these number plates begin with ABC?

$$= 1 \times 1 \times 1 \times 10 \times 10 \times 10$$

Examples

In how many ways can 5 boys and 4 girls be arranged on a bench if



a) no restrictions?

Solution : 9! or $_9P_9$

c) boys and girls alternate?

Solution : A boy will be on each end

BGBGBGBGB = $5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1$

c) boys and girls are in separate groups?

Solution : Boys & Girls or Girls & Boys

$= 5! \times 4! + 4! \times 5!$

d) Anne and Tom wish to stay together?

Solution : (AT) _ _ _ _ _ _ _

$= 2 \times 8!$

Thank you!

End of slides

References

- Introduction to Probability, Statistics, and Random Processes
  Textbook by Hossein Pishro-Nik

- A Modern Introduction to Probability and Statistics: Understanding Why and How
  Book by Frederik Michel Dekking

- D. C. Montgomery and G.C. Runger, “Applied Statistics and Probability for Engineers”, 5th edition, John Wiley & Sons, (2009).Online lecture notes,