

Tishk International University
Faculty of Administrative Sciences and
Economics



MATHEMATICS

FOR ECONOMICS AND BUSINESS

BUS 143
Part 1

I Grade- Fall

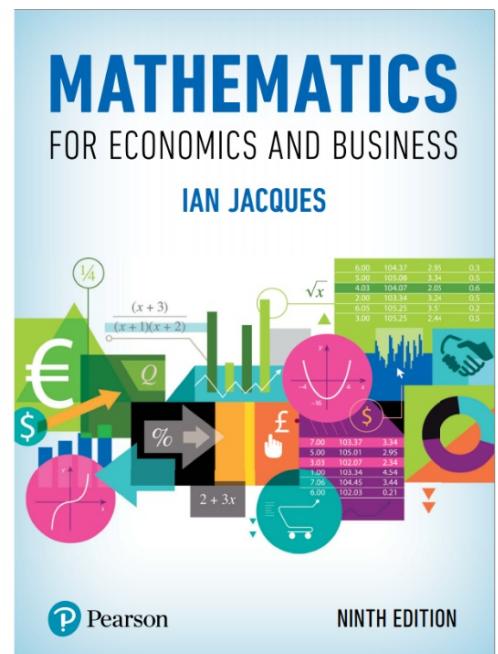
Assist. Prof. Dr. Hamdi Serin

Syllabus

Course 3 credits

Course Language: *English*

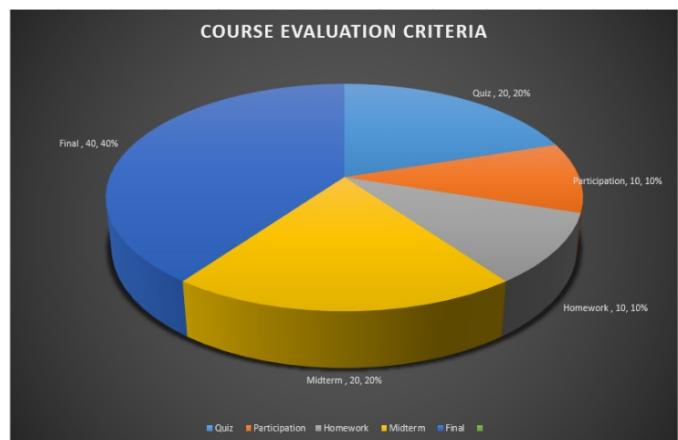
Course Book/Textbook:Mathematics for Economic and Business. Ian Jacques Person 9th Edition



Student's obligation (Special Requirements): *Taking notes, attending minimum 80 percent and follow all assignment.*

COURSE EVALUATION CRITERIA

Quiz	$2 \times 10 = 20$
Participation	1×10
Homework	1×10
Midterm	1×20
Final	<u>1×40</u>
Total:	100



Order of Operations

BODMAS

Operations

"**Operations**" mean things like add, subtract, multiply, divide, squaring, etc. If it isn't a number it is probably an operation.

But, when you see something like...

$$7 + (6 \times 5^2 + 3)$$

... what part should you calculate first?

Start at the left and go to the right?
Or go from right to left?

Warning: Calculate them in the wrong order, and you can get a wrong answer !

So, long ago people agreed to follow rules when doing calculations, and they are:

Order of Operations

Do things in Brackets First

✓ $6 \times (5 + 3) = 6 \times 8 = 48$

✗ $6 \times (5 + 3) = 30 + 3 = 33$ (wrong)

Exponents (Powers, Roots) before Multiply, Divide, Add or Subtract

✓ $5 \times 2^2 = 5 \times 4 = 20$

✗ $5 \times 2^2 = 10^2 = 100$ (wrong)

Multiply or Divide before you Add or Subtract

✓ $2 + 5 \times 3 = 2 + 15 = 17$

✗ $2 + 5 \times 3 = 7 \times 3 = 21$ (wrong)

Otherwise just go left to right

✓ $30 \div 5 \times 3 = 6 \times 3 = 18$

✗ $30 \div 5 \times 3 = 30 \div 15 = 2$ (wrong)

How Do I Remember It All ... ? BODMAS !

- B** Brackets first
- O** Orders (i.e. Powers and Square Roots, etc.)
- DM** Division and Multiplication (left-to-right)
- AS** Addition and Subtraction (left-to-right)

Divide and Multiply rank equally (and go left to right).

Add and Subtract rank equally (and go left to right)

So do it this way:

1. 2. 3. 4.
B **O** **D** **A**
or
M **S**

After you have done "B" and "O", just go from left to right doing any "D" **or** "M" as you find them.

Then go from left to right doing any "A" **or** "S" as you find them.

Note: the only strange name is "Orders". "Exponents" is used in Canada, and so you might prefer "BEDMAS". There is also "Indices" which makes it "BIDMAS". In the US they say "Parentheses" instead of Brackets, so it is "[PEMDAS](#)"



Order of Operations

PEMDAS

Operations

"**Operations**" mean things like add, subtract, multiply, divide, squaring, etc. If it isn't a number it is probably an operation.

But, when you see something like ...

$$7 + (6 \times 5^2 + 3)$$

... what part should you calculate first?

Start at the left and go to the right?
Or go from right to left?

Warning: Calculate them in the wrong order, and you can get a wrong answer !

So, long ago people agreed to follow rules when doing calculations, and they are:

Order of Operations

Do things in Parentheses First

✓ $6 \times (5 + 3) = 6 \times 8 = 48$

✗ $6 \times (5 + 3) = 30 + 3 = 33$ (wrong)

Exponents (Powers, Roots) before Multiply, Divide, Add or Subtract

✓ $5 \times 2^2 = 5 \times 4 = 20$

✗ $5 \times 2^2 = 10^2 = 100$ (wrong)

Multiply or Divide before you Add or Subtract

✓ $2 + 5 \times 3 = 2 + 15 = 17$

✗ $2 + 5 \times 3 = 7 \times 3 = 21$ (wrong)

Otherwise just go left to right

✓ $30 \div 5 \times 3 = 6 \times 3 = 18$

✗ $30 \div 5 \times 3 = 30 \div 15 = 2$ (wrong)

How Do I Remember It All ... ? PEMDAS !

- P** Parentheses first
- E** Exponents (ie Powers and Square Roots, etc.)
- MD** Multiplication and Division (left-to-right)
- AS** Addition and Subtraction (left-to-right)

Divide and Multiply rank equally (and go left to right).

Add and Subtract rank equally (and go left to right)

So do it this way:

1. 2. 3. 4. After you have done "P" and "E", just go from left to right doing any "M" **or** "D" as you find them.
- P E M or D A or S Then go from left to right doing any "A" **or** "S" as you find them.

Exponents of Exponents ...

What about this example?

$$4^{3^2}$$

Exponents are special: **they go top-down** (do the exponent at the top first). So we calculate this way:

$$\begin{aligned} \text{Start with: } & 4^{3^2} \\ 3^2 = 3 \times 3: & 4^9 \\ 4^9 = 4 \times 4: & 262144 \end{aligned}$$

So $4^{3^2} = 4^{(3^2)}$, not $(4^3)^2$

And finally, what about the example from the beginning?

Start with: $7 + (6 \times 5^2 + 3)$

Brackets first and then "Orders": $7 + (6 \times 25 + 3)$

Then Multiply: $7 + (150 + 3)$

Then Add: $7 + (153)$

Brackets completed: $7 + 153$

Last operation is an Add: **160**

What is the value of $3 + 6 \div 3 \times 2$?

A 7

B 6

C 4

D 1.5

- [Basis Operations](#)
- [Financial Results](#)
- [Geographic Profit / Loss by Sector](#)
- [Major Business Initiatives](#)
- [Additional Information](#)

What is the value of $5 \times 3 - 12 \div 4 + 8$?

A 3

B 4

C 14

D 20



What is the value of $30 - (5 \times 2^3 - 15)$?

A -25

B 5

C 15

D -15



What is the value of $(15 \div 3 + 4) - (3^2 - 7 \times 2)$?

A -1.86

B 4

C 5

D 14

What is the value of $(4^2 - 6 + 5) / (3^2 + 8 - 7 \times 2)$?

A 5

B 11

C $1\frac{4}{11}$

D $\frac{3}{4}$

What is the value of this?

$$\frac{2^4 + (16 - 3 \times 4)}{(6 + 3^2) \div (7 - 4)}$$

A 2.4

B 4

C 5

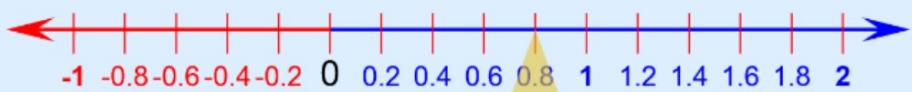
D 13.6

Example Values

Here is a table of commonly used values shown in Percent, Decimal and Fraction form:

Percent	Decimal	Fraction
1%	0.01	$\frac{1}{100}$
5%	0.05	$\frac{1}{20}$
10%	0.1	$\frac{1}{10}$
12½%	0.125	$\frac{1}{8}$
20%	0.2	$\frac{1}{5}$
25%	0.25	$\frac{1}{4}$
33 $\frac{1}{3}$ %	0.333...	$\frac{1}{3}$
50%	0.5	$\frac{1}{2}$
75%	0.75	$\frac{3}{4}$
80%	0.8	$\frac{4}{5}$
90%	0.9	$\frac{9}{10}$
99%	0.99	$\frac{99}{100}$
100%	1	
125%	1.25	$\frac{5}{4}$
150%	1.5	$\frac{3}{2}$
200%	2	

On the Number Line!



0.8

80%

$$\frac{8}{10} \text{ or } \frac{4}{5}$$

Conversions

FROM PERCENT TO DECIMAL

To [convert from percent to decimal](#) : divide by 100, and remove the "%" sign.

The easiest way to divide by 100 is to **move the decimal point 2 places to the left**:

From Percent	To Decimal	
75% 	0.75 	move the decimal point 2 places to the left , and remove the "%" sign.

FROM DECIMAL TO PERCENT

To [convert from decimal to percent](#) : multiply by 100, and add a "%" sign.

The easiest way to multiply by 100 is to **move the decimal point 2 places to the right**:

From Decimal	To Percent	
0.125	 12.5%	move the decimal point 2 places to the right , and add the "%" sign.

FROM FRACTION TO DECIMAL

The easiest way to [convert a fraction to a decimal](#) is to divide the top number by the bottom number (divide the numerator by the denominator in mathematical language)

Example: Convert $\frac{2}{5}$ to a decimal

Divide 2 by 5: $2 \div 5 = 0.4$

Answer: $\frac{2}{5} = 0.4$

FROM DECIMAL TO FRACTION

To [convert a decimal to a fraction](#) needs a little more work.

Example: To convert 0.75 to a fraction

Steps

First, write down the decimal "over" the number 1

Example

$$\begin{array}{r} 0.75 \\ \hline 1 \end{array}$$

Multiply top and bottom by 10 for every number after the decimal point (10 for 1 number, 100 for 2 numbers, etc)

$$\begin{array}{r} 0.75 \times 100 \\ \hline 1 \times 100 \end{array}$$

(This makes a correctly formed fraction)

$$\begin{array}{r} 75 \\ \hline 100 \end{array}$$

Then [Simplify](#) the fraction

$$\begin{array}{r} 3 \\ \hline 4 \end{array}$$

FROM FRACTION TO PERCENTAGE

The easiest way to [convert a fraction to a percentage](#) is to divide the top number by the bottom number, then multiply the result by 100, and add the "%" sign.

Example: Convert $\frac{3}{8}$ to a percentage

First divide 3 by 8: $3 \div 8 = 0.375$,
Then multiply by 100: $0.375 \times 100 = 37.5$
Add the "%" sign: 37.5%

Answer: $\frac{3}{8} = 37.5\%$

FROM PERCENTAGE TO FRACTION

To [convert a percentage to a fraction](#), first convert to a decimal (divide by 100), then use the steps for converting decimal to fractions (like above).

Example: To convert 80% to a fraction

Steps

Convert 80% to a decimal ($=80/100$):

Example

0.8

Write down the decimal "over" the number 1

$\frac{0.8}{1}$

Multiply top and bottom by 10 for every number after the decimal point (10 for 1 number, 100 for 2 numbers, etc)

$\frac{0.8 \times 10}{1 \times 10}$

(This makes a correctly formed fraction)

$\frac{8}{10}$

Then [Simplify](#) the fraction

$\frac{4}{5}$

Scientific Notation

Scientific Notation (also called **Standard Form** in Britain) is a special way of writing numbers:

Like this:

$$\begin{array}{ccc} 700 & \rightarrow & 7 \times 10^2 \\ \text{A Number} & & \text{In Scientific Notation} \end{array}$$

Or this:

$$\begin{array}{ccc} 4,900,000,000 & \rightarrow & 4.9 \times 10^9 \\ \text{A Number} & & \text{In Scientific Notation} \end{array}$$

It makes it easy to use big and small values.

Example: 700

Why is 700 written as 7×10^2 in Scientific Notation ?

- ➡ $700 = 7 \times 100$
- ➡ and $100 = 10^2$ (see [powers of 10](#))
- ➡ so $700 = 7 \times 10^2$

Both **700** and **7×10^2** have the same value, just shown in different ways.

Example: 4,900,000,000

$1,000,000,000 = 10^9$,
so $4,900,000,000 = 4.9 \times 10^9$ in Scientific Notation

So the number is written in **two parts**:

- Just the **digits** (with the decimal point placed after the first digit), followed by
- $\times 10$ to a **power** that puts the decimal point where it should be
(i.e. it shows how many places to move the decimal point).

$$5326.6 = \begin{matrix} \text{Digits} \\ \text{A Number} \end{matrix} \times \begin{matrix} \text{Power of 10} \\ \text{In Scientific Notation} \end{matrix} 10^3$$

In this example, 5326.6 is written as 5.3266×10^3 ,
because $5326.6 = 5.3266 \times 1000 = 5.3266 \times 10^3$

Other Ways of Writing It

We can use the \wedge symbol (above the 6 on a keyboard), as it is easy to type.

3.1×10^8

Example: **3×10^4 is the same as 3×10^4**

- $3 \times 10^4 = 3 \times 10 \times 10 \times 10 \times 10 = 30,000$

Calculators often use "E" or "e" like this:

$1.8004E+94$

Example: **$6E+5$ is the same as 6×10^5**

- $6E+5 = 6 \times 10 \times 10 \times 10 \times 10 \times 10 = 600,000$

Example: **$3.12E4$ is the same as 3.12×10^4**

- $3.12E4 = 3.12 \times 10 \times 10 \times 10 \times 10 = 31,200$

Rule	Example
$a^m \times a^n = a^{m+n}$	$2^5 \times 2^3 = 2^8$
$a^m \div a^n = a^{m-n}$	$5^7 \div 5^3 = 5^4$
$(a^m)^n = a^{m \times n}$	$(10^3)^7 = 10^{21}$
$a^1 = a$	$17^1 = 17$
$a^0 = 1$	$34^0 = 1$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{6}\right)^2 = \frac{25}{36}$
$a^{-m} = \frac{1}{a^m}$	$9^{-2} = \frac{1}{81}$
$a^{\frac{x}{y}} = \sqrt[y]{a^x}$	$49^{\frac{1}{2}} = \sqrt[2]{49} = 7$

Changing the Subject

A very powerful thing that Algebra can do is to "rearrange" a formula so that another variable is the subject.

Example: Rearrange the volume of a box formula ($V = lwh$) so that the width is the subject

Start with: $V = lwh$

divide both sides by h : $V/h = lw$

divide both sides by l : $V/(hl) = w$

swap sides: $w = V/(hl)$

So if we want a box with a volume of 12, a length of 2, and a height of 2, we can calculate its width:

$$\begin{aligned} w &= V/(hl) \\ &= 12 / (2 \times 2) \\ &= 12 / 4 \\ &= 3 \end{aligned}$$

For the formula $x = 3y - z$, what is the value of x when $y = 4$ and $z = 1$?

A -1

B 9

C 11

D 12

$x = 3y - z$
Substitute $y = 4$ and $z = 1$ into the formula.
Therefore $x = 3(4) - 1$

For the formula $c^2 = a^2 + b^2$, what is the value of c when $a = 7$ and $b = 24$?

A 31

B 25

C $\sqrt{527}$

D $\sqrt{31}$

$c^2 = a^2 + b^2$
Substituting 7 for a and 24 for b
Therefore c^2
= $7^2 + 24^2$
= $49 + 576$
= 625
Therefore $c^2 = 625$
As c must be a positive number
So we have to find the number that when multiplied by itself equals 625
That number is 25
Therefore $c = \sqrt{625} = 25$

For the formula

$$d = \left(\frac{u + v}{2} \right) t$$

what is the value of d when $u = 3$, $v = 11$ and $t = 5$?

A 12

B 17.5

C 25

D 35



End of page



For the formula

$$v^2 = u^2 + 2ad$$

what is the value of v when $u = 15$, $a = 10$ and $d = 20$?

A 25

B $\sqrt{257}$

C 35

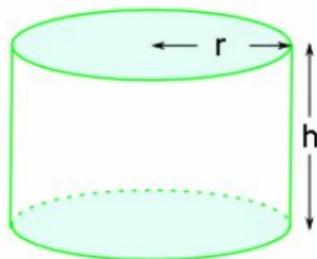
D 415

Level 2

Level 3

Level 4

Level 5



The formula for the total surface area of a cylinder of radius r and height h is:

$$A = 2\pi r^2 + 2\pi r h$$

What is the value of A when $r = 7$ and $h = 10$?

Use $22/7$ as an approximation of π .

A 528

B 748

C 1,056

D 1,408

$A = 2\pi r^2 + 2\pi r h$
Substituting $r = 7$, $h = 10$ and $\pi = 22/7$:
 $= 2 \times \frac{22}{7} \times 7^2 + 2 \times \frac{22}{7} \times 7 \times 10$
 $= 2 \times 22 \times 7 + 2 \times 22 \times 10$
 $= 2 \times 22 \times 17$
 $= 748$

Rearrange the formula

$$P = 2a + 2b$$

to make b the subject

A $b = (P - 2a)/2$

B $b = (P - a)/2$

C $b = (2a - P)/2$

D $b = P/2 - 2a$

$$P = 2a + 2b$$

To make "b" the subject, first let's move "2a" to the other side, by subtracting it from both sides:

$$\Rightarrow P - 2a = 2b - 2a$$

$$\Rightarrow P - 2a = 2b$$

Now, bring "2b" to the other side:

$$\Rightarrow 2b = P - 2a$$

Now divide both sides by 2:

$$\Rightarrow 2b/2 = (P - 2a)/2$$

$$\Rightarrow b = (P - 2a)/2$$

Rearrange the formula:

$$C = \frac{5}{9}(F - 32)$$

to make F the subject

A $F = \frac{5}{9}(C + 32)$

B $F = \frac{C}{32} - \frac{5}{9}$

C $F = \frac{9}{5}C + 32$

D $F = \frac{9}{5}(C + 32)$

$C = \frac{5}{9}(F - 32)$
Multiply both sides of the formula by 9 to get rid of the fraction
 $C \times 9 = \frac{5}{9}(F - 32) \times 9$
 $9C = 5(F - 32)$
Expand the left side of the formula
 $9C = 5F - 160$
Add 160 to both sides
 $9C + 160 = 5F$
Simplify
 $3C + 48 = 5F$
Divide all terms by 5
 $3F + 5 = 5F + 48 \div 5$
 $F = \frac{48}{5} + 32$

Rearrange this formula:

$$A = 2a^2 + 4ab$$

So that b is the Subject of the formula.

A $b = \frac{A}{2a^2} - 4a$

B $b = A - \frac{a}{2}$

C $b = \frac{A + 2a^2}{4a}$

D $b = \frac{A - 2a^2}{4a}$

$$\begin{aligned} A &= 2a^2 + 4ab \\ \text{Subtract } 2a^2 \text{ from both sides: } A - 2a^2 &= 4ab \\ \text{Swap Sides: } 4ab &= A - 2a^2 \\ \text{Now, divide both sides by } 4a: \\ \frac{4ab}{4a} &= \frac{A - 2a^2}{4a} \\ b &= \frac{A - 2a^2}{4a} \end{aligned}$$

$$A = \frac{C^2}{4\pi}$$

is a formula to calculate the area of a circle from its circumference.

Make C the subject of the formula.

A $C = 2\pi A$

B $C = 16\pi^2 A^2$

C $C = \sqrt{4\pi A}$

D $C = \frac{A^2}{4\pi}$



$s = ut + \frac{1}{2}at^2$ is a formula used in Physics to calculate distance.

Make "a" the subject of the formula.

A $a = \frac{s}{ut} - \frac{1}{2}t^2$

B $a = \frac{2(ut - s)}{t^2}$

C $a = \frac{2s - ut}{t^2}$

D $a = \frac{2(s - ut)}{t^2}$

$s = ut + \frac{1}{2}at^2$
Subtract ut from both sides:
 $\Rightarrow s - ut = \frac{1}{2}at^2$
 $\Rightarrow s - ut = \frac{1}{2}at^2$
Multiply both sides by 2:
 $\Rightarrow 2s - 2ut = at^2$
 $\Rightarrow at^2 = 2s - 2ut$
Divide both sides by t^2
 $\Rightarrow a = \frac{2s - 2ut}{t^2}$
Add 2ut to both sides:
 $a = \frac{2s}{t^2}$

Solving Equations

What is a Solution?

A Solution is a value we can put in place of a variable (such as x) that makes the equation **true**.

Example: $x - 2 = 4$

When we put **6** in place of **X** we get:

$$6 - 2 = 4$$

which is **true**

So **x = 6** is a solution.

How about other values for **X** ?

- For $x=5$ we get " $5-2=4$ " which is **not true**, so **x=5 is not a solution**.
- For $x=9$ we get " $9-2=4$ " which is **not true**, so **x=9 is not a solution**.
- etc

In this case **X = 6** is the only solution.

More Than One Solution

There can be **more than one** solution.

Example: $(x-3)(x-2) = 0$

When **x** is **3** we get:

$$(3-3)(3-2) = 0 \times 1 = 0$$

which is **true**

And when **x** is **2** we get:

$$(2-3)(2-2) = (-1) \times 0 = 0$$

which is also **true**

So the solutions are:

$$x = 3, \text{ or } x = 2$$

When we gather all solutions together it is called a **Solution Set**

The above solution set is: **{2, 3}**

Solutions Everywhere!

Some equations are true for all allowed values and are then called **Identities**

Example: $\sin(-\theta) = -\sin(\theta)$ is one of the Trigonometric Identities

Let's try $\theta = 30^\circ$:

$$\begin{aligned} \rightarrow \sin(-30^\circ) &= -0.5 \text{ and} \\ \rightarrow -\sin(30^\circ) &= -0.5 \end{aligned}$$

So it is **true** for $\theta = 30^\circ$

Let's try $\theta = 90^\circ$:

$$\begin{aligned} \rightarrow \sin(-90^\circ) &= -1 \text{ and} \\ \rightarrow -\sin(90^\circ) &= -1 \end{aligned}$$

So it is also **true** for $\theta = 90^\circ$

Is it true for **all values of θ** ? Try some values for yourself!

How to Solve an Equation

There is no "one perfect way" to solve all equations.

A Useful Goal

But we often get success when **our goal** is to end up with:

$$\mathbf{x = something}$$

In other words, we want to move everything except "x" (or whatever name the variable has) over to the right hand side.

Example: Solve $3x - 6 = 9$

Start with: $3x - 6 = 9$

Add 6 to both sides: $3x = 9 + 6$

Divide by 3: $x = (9+6)/3$

Now we have $\mathbf{x = something}$,

and a short calculation reveals that $\mathbf{x = 5}$

How To Check

Take the solution(s) and put them in the **original equation** to see if they really work.

Example: solve for x :

$$\frac{2x}{x-3} + 3 = \frac{6}{x-3} \quad (x \neq 3)$$

We have said $x \neq 3$ to avoid a division by zero.

Let's multiply through by $(x - 3)$:

$$2x + 3(x-3) = 6$$

Bring the 6 to the left:

$$2x + 3(x-3) - 6 = 0$$

Expand and solve:

$$2x + 3x - 9 - 6 = 0$$

$$5x - 15 = 0$$

$$5(x - 3) = 0$$

$$x - 3 = 0$$

That can be solved by having $x = 3$

Let us check:

$$\frac{2 \times 3}{3-3} + 3 = \frac{6}{3-3}$$

Hang On!

That means Dividing by Zero!

And anyway, we said at the top that $x \neq 3$, so ...

$x = 3$ does not actually work, and so:

There is **No Solution!**

Solve $5x + 2 = -8$

A $x = -2$

B $x = -1.2$

C $x = 1.2$

D $x = 2$

Simplify $x + 2 = -8$
Subtract 2 from both sides: $x + 2 - 2 = -8 - 2$
Divide by 5: $x = -10 / 5$

Solve $3y - 7 = 26$

A $y = -11$

B $y = -6\frac{1}{3}$

C $y = 6\frac{1}{3}$

D $y = 11$

Start with $3y - 7 = 26$
Add 7 to both sides: $3y - 7 + 7 = 26 + 7$
Calculate: $3y = 33$
Divide by 3: $3y \div 3 = 33 \div 3$
Calculate: $y = 11$

Solve $(x + 2)(2x - 1) = 0$

A $x = 0$

B $x = 2$ or $-1/2$

C $x = -2$ or $1/2$

D $x = -1$ or 1

Solve $(7 - 2y)(5 + y) = 0$

A $y = -3\frac{1}{2}$ or -5

B $y = 3\frac{1}{2}$ or -5

C $y = -3\frac{1}{2}$ or 5

D $y = 3\frac{1}{2}$ or 5

7. $3x^2 + 9 = 0$
B. $x^2 + 3 = 0$
Simplify
 $x^2 = -3$
 $x = \sqrt{-3}$
No real solution

Solve $\frac{3x}{x-2} = \frac{3x+10}{x}$

A $x=4$

B $x=5$

C $x = 2$ or $-3\frac{1}{3}$

D $x = 1\frac{1}{9}$

$$\frac{3x}{x-2} = \frac{3x+10}{x}$$

Multiply both sides by $x(x-2)$

$$\frac{3x}{x-2} \times x(x-2) = \frac{3x+10}{x} \times x(x-2)$$
$$3x^2 = 3x(x-2) + 10(x-2)$$
$$3x^2 = 3x^2 - 6x + 10x - 20$$
$$3x^2 = 3x^2 + 4x - 20$$

Both sides have x^2 , so it can be removed from both sides

$$0 = 4x - 20$$
$$4x = 20$$
$$x = 5$$

Solve $\frac{1}{4}x - 3 = 4$

A $x = 4$

B $x = 19$

C $x = 24$

D $x = 28$

Multiply both sides
 $\frac{1}{4}x - 3 = 4$
 $\times 4$
 $x - 12 = 16$

Multiply both sides by 4
 $x - 12 = 16$
 $\times 4$
 $x = 80$

Solve $\frac{2}{3}x + 5 = -7$

A $x = -18$

B $x = -3$

C $x = 3$

D $x = 18$

$$\begin{aligned} \frac{2}{3}x + 5 &= -7 \\ \text{Subtract 5 from both sides:} \\ \frac{2}{3}x + 5 - 5 &= -7 - 5 \\ \Rightarrow \frac{2}{3}x &= -12 \\ \text{Multiply both sides by } \frac{3}{2}: \\ \frac{2}{3}x \times \frac{3}{2} &= -12 \times \frac{3}{2} \\ \Rightarrow x &= -18 \end{aligned}$$

Solve $\frac{1-7x}{x+2} = \frac{5-7x}{x}$

A $x = 1$

B $x = \frac{5}{7}$

C $x = -1$

D $x = -2$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----

$$\text{Solve } \frac{3x}{x+1} + 6 = \frac{-3}{x+1}$$

$$A \ x = -1$$

B x = 1

$$C \quad x = 3$$

D There is no solution

More difficult to see: $\frac{x-1}{x+1} > \frac{2x-3}{x-1}$

$$\frac{x-1}{x+1} - \frac{2x-3}{x-1} > 0 \quad \text{or} \quad \frac{-x^2 + 4x - 4}{x^2 - 1} > 0$$

$$\frac{-x^2 + 4x - 4}{x^2 - 1} > 0$$

Separate into two cases:

$$\frac{-x^2 + 4x - 4}{x^2 - 1} > 0 \quad \text{and} \quad \frac{-x^2 + 4x - 4}{x^2 - 1} < 0$$

For $\frac{-x^2 + 4x - 4}{x^2 - 1} > 0$, we have two cases:

$$\frac{-x^2 + 4x - 4}{x^2 - 1} > 0 \quad \text{and} \quad \frac{-x^2 + 4x - 4}{x^2 - 1} < 0$$

Break up into two cases:

$$\frac{-x^2 + 4x - 4}{x^2 - 1} > 0 \quad \text{and} \quad \frac{-x^2 + 4x - 4}{x^2 - 1} < 0$$

Maximize $x^2 - 1$ by $x^2 = 1$ in the original equation gives

$$x = 1 \quad \text{or} \quad x = -1$$

Our intervals by now are $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$.

Test intervals by hand (or calculator).

Therefore $x = -1$ does not actually work, and we have two solutions.

Solve $\frac{5x - 4}{2x - 3} = \frac{10(3 - x)}{17 - 4x}$

A $x = -4$

B $x = -3$

C $x = -2$

D $x = -1$



Zero Product Property

The "Zero Product Property" says that:

If $a \times b = 0$ then $a = 0$ or $b = 0$
(or both $a=0$ and $b=0$)

It can help us solve equations:

Example: Solve $(x-5)(x-3) = 0$

The "Zero Product Property" says:

If $(x-5)(x-3) = 0$ then $(x-5) = 0$ or $(x-3) = 0$

Now we just solve each of those:

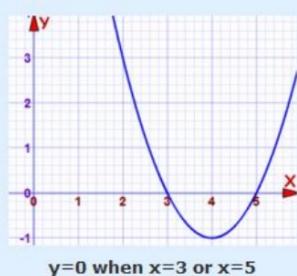
For $(x-5) = 0$ we get $x = 5$

For $(x-3) = 0$ we get $x = 3$

And the solutions are:

$x = 5$, or $x = 3$

Here it is on a graph:



Standard Form of an Equation

Sometimes we can solve an equation by putting it into Standard Form and then using the Zero Product Property:

The "Standard Form" of an equation is:

$$(some\ expression) = 0$$

In other words, " $= 0$ " is on the right, and everything else is on the left.

Example: Put $x^2 = 7$ into Standard Form

Answer:

$$x^2 - 7 = 0$$

Standard Form and the Zero Product Property

So let's try it out:

Example: Solve $5(x+3) = 5x(x+3)$

It is tempting to divide by $(x+3)$, but that is [dividing by zero](#) when $x = -3$

So instead we can use "Standard Form":

$$5(x+3) - 5x(x+3) = 0$$

Which can be simplified to:

$$(5-5x)(x+3) = 0$$

$$5(1-x)(x+3) = 0$$

Then the "Zero Product Property" says:

$$(1-x) = 0, \text{ or } (x+3) = 0$$

And the solutions are:

$$x = 1, \text{ or } x = -3$$

Example: Solve $x^3 = 25x$

It is tempting to divide by x , but that is dividing by zero when $x = 0$

So let's use Standard Form and the Zero Product Property.

Bring all to the left hand side:

$$x^3 - 25x = 0$$

Factor out x :

$$x(x^2 - 25) = 0$$

$x^2 - 25$ is a difference of squares, and can be factored into $(x - 5)(x + 5)$:

$$x(x - 5)(x + 5) = 0$$

Now we can see three possible ways it could end up as zero:

$$x = 0, \text{ or } x = 5, \text{ or } x = -5$$

Solve $3(x - 2) = 3x(x - 2)$

A $x = -1$ only

B $x = -1$ or $x = -2$

C $x = 1$ only

D $x = 1$ or $x = 2$

$$3(x - 2) = 3x(x - 2)$$

Use "Standard Form":

$$3(x - 2) - 3x(x - 2) = 0$$

Work can be simplified to:

$$(3 - 3x)(x - 2) = 0$$

$$x(3 - 3x) = 0$$

Then the "Zero Product Property" says:

$$(1 - 0) = 0 \text{ or } x = 0$$

$$3x = 0 \text{ or } x = 2$$

$$x = 0 \text{ or } x = 2$$

Solve $2x^3 = 72x$

A $x = -6$ or 6

B $x = -6, 0$ or 6

C $x = 0$ or $\pm 6\sqrt{2}$

D $x = \pm 6\sqrt{2}$

Pre
Algebra
Geometry
Trigonometry
Calculus
Statistics
Probability
Mathematical
Reasoning

Solve $(x + 5)(2x - 1) = 3(x + 5)$

A $x = -5$ only

B $x = -5$ or $x = 2$

C $x = -5$ or $x = \frac{1}{2}$

D $x = 2$ or $x = 5$

$(x + 5)(2x - 1) = 3(x + 5)$
Use Standard Form:
 $(x + 5)(2x - 1) - 3(x + 5) = 0$
 $(x + 5)$ is a common factor:
Therefore $(x + 5)(2x - 1 - 3) = 0$
 $\alpha (x + 5)(2x - 4) = 0$
Then the 'Zero Product Property' says:
 $(x + 5) = 0, \alpha (2x - 4) = 0$
 $x = -5$ or $x = 2$

Solve: $(3x - 2)(2x - 5) = -8(2x - 5)$

A $x = 2/3$ or $2\frac{1}{2}$

B $x = -2\frac{1}{2}$ or $x = 2$

C $x = 2\frac{1}{2}$ or $x = 2$

D $x = 2\frac{1}{2}$ or $x = -2$

$(x - 2)(x - 5) = -8(x - 5)$
Use "Symbolic Factoring"
 $(x - 2)(x - 5) + 8(x - 5) = 0$
 $(x - 5)$ is a common factor
 $(x - 5)(x - 2) + 8 = 0$
 $x(x - 5) + 8 = 0$
Use the "Zero Product Property" again:
 $(x - 5) + 8 = 0$ or $(x + 6) = 0$
 $x = 5$ or $x = -6$

Solve $x(x + 2)^2 = 49x$

A $x = 0$, or $x = -7$, or $x = 7$

B $x = 0$, or $x = -5$, or $x = 9$

C $x = 0$, or $x = -9$, or $x = 5$

D $x = 0$, or $x = -9$, or $x = 9$

Graph the function $y = x^2 - 6x + 9$
5x + 10x + 4x^2 - 6x + 9
y = 4x^2 - 2x + 9
y = 4(x^2 - 1/2x) + 9
y = 4(x^2 - 1/2x + 1/4) + 9 - 1/4
y = 4(x - 1/4)^2 + 35/4
y = 4(x - 1/4)^2 + 8.75

Solve $(x - 5)(x - 3)^2 = 25(x - 5)$

A $x = 5$, or $x = -2$, or $x = 8$

B $x = -8$, or $x = 5$, or $x = 2$

C $x = 2$, or $x = 5$, or $x = 8$

D $x = 3$, or $x = \pm 5$

Start with: $(x - 5)(x - 3)^2 = 25(x - 5)$
Use "Standard Form": $(x - 5)(x - 3)^2 + 25(x - 5) = 0$
Factor out $(x - 5)$: $(x - 5)[(x - 3)^2 + 25] = 0$
 $(x - 3)^2 + 25$ is a difference of two squares, and can be factored into $[(x - 3) + 5][(x - 3) - 5] = (x + 2)(x - 8)$
And so the whole equation is now: $(x - 5)(x + 2)(x - 8) = 0$
Which is true when $x = 5$, or $x = -2$, or $x = 8$

Solve: $x(x - 1)^2 = 2(x - 1)$

A $x = -1, 1$ or 2

B $x = 1$ or 2

C $x = -1$ or 1

D $x = 0$ or 1

$x(x - 1)^2 = 2(x - 1)$
Use 'Standard Form'.
 $x(x - 1)^2 - 2(x - 1) = 0$
($x - 1$) is a common factor.
 $\text{Therefore, } (x - 1)[x(x - 1) - 2] = 0$
 $\Rightarrow (x - 1)(x^2 - x - 2) = 0$
 $\Rightarrow (x - 1)(x - 1)(x + 2) = 0$
Then the 'Zero Product Property' says:
 $(x - 1) = 0 \text{ or } (x + 1) = 0 \text{ or } (x - 2) = 0$
So $x = 1, x = -1, x = 2$

Solve: $x(x + 5)^2 = -4(x + 5)$

A $x = -5$ or $x = 0$

B $x = -5$ or $x = -4$ or $x = -1$

C $x = -5$ or $x = 1$ or $x = 4$

D $x = -4$ or $x = -1$ or $x = 5$

$x(x + 5)^2 = -4(x + 5)$
Divide both sides by $(x + 5)$.
 $x(x + 5) = -4$
Factor out an x .
 $x(x + 5 + 4) = 0$
 $x(x + 9) = 0$
Set each factor equal to zero.
 $x = 0$ or $x + 9 = 0$
 $x = 0$ or $x = -9$

Solve: $2(2x - 3) = x(2x - 3)^2$

A $x = 0$ or $1\frac{1}{2}$

B $x = 1\frac{1}{2}$ or 2

C $x = -\frac{1}{2}$, $1\frac{1}{2}$ or 2

D $x = -\frac{1}{2}$ or $1\frac{1}{2}$

$2(2x - 3) = x(2x - 3)^2$
Is this true for?
2x = 2x? \checkmark
(2x - 3) = (2x - 3) \checkmark
 $2x(2x - 3) = x(2x - 3)^2$
 $4x^2 - 6x = 4x^3 - 12x^2 + 9x$
Is this true for? \times
2x = 4x \times
4x² - 6x = 4x³ \times
4x² - 6x = 4x²(x - 1) \times
Is this true for? \times

Solve: $(x - 2)(4x - 5)^2 = 7(4x - 5)$

A $x = 1.25$ or $x = 9$

B $x = 1.25$ or $x = 3$

C $x = 0.25$ or $x = 1.25$ or $x = 9$

D $x = 0.25$ or $x = 1.25$ or $x = 3$

100%

60%

30%

10%

5%

100%

50%

0%

Introduction to Inequalities

Inequality tells us about the **relative size** of two values.

Mathematics is not always about "equals", sometimes we only know that something is greater or less than.

Example: Alex and Billy have a race, and Billy wins!

What do we know?

We don't know **how fast** they ran, but we do know that Billy was faster than Alex:

Billy was faster than Alex

We can write that down like this:

$$b > a$$

(Where "b" means how fast Billy was, ">" means "greater than", and "a" means how fast Alex was)

We call things like that **inequalities** (because they are not "equal")

Example: Alex plays in the under 15s soccer. How old is Alex?

We don't know **exactly** how old Alex is, because it doesn't say "equals"

But we **do know** "less than 15", so we can write:

$$\text{Age} < 15$$

The small end points to "Age" because the age is smaller than 15.

... Or Equal To!

We can also have inequalities that include "equals", like:

Symbol	Words	Example Use
\geq	greater than or equal to	$x \geq 1$
\leq	less than or equal to	$y \leq 3$

Example: you must be 13 or older to watch a movie.

The "inequality" is between **your age** and the **age of 13**.

Your age must be "greater than **or** equal to 13", which is written:

$$\text{Age} \geq 13$$

Solving Inequalities

Sometimes we need to solve **Inequalities** like these:

Symbol	Words	Example
$>$	greater than	$x + 3 > 2$
$<$	less than	$7x < 28$
\geq	greater than or equal to	$5 \geq x - 1$
\leq	less than or equal to	$2y + 1 \leq 7$

Solving

Our aim is to have **X** (or whatever the variable is) **on its own** on the left of the inequality sign:

Something like: $x < 5$
or: $y \geq 11$

We call that "solved".

Example: $x + 2 > 12$

Subtract 2 from both sides:

$$x + 2 - 2 > 12 - 2$$

Simplify:

$$x > 10$$

Solved!

Safe Things To Do

These things **do not affect** the direction of the inequality:

- Add (or subtract) a number from both sides
- Multiply (or divide) both sides by a **positive** number
- Simplify a side

Example: $3x < 7+3$

We can simplify $7+3$ without affecting the inequality:

$$3x < 10$$

But these things **do change the direction** of the inequality (" $<$ " becomes " $>$ " for example):

- Multiply (or divide) both sides by a **negative** number
- Swapping left and right hand sides

Example: $2y+7 < 12$

When we swap the left and right hand sides, we must also **change the direction of the inequality**:

$$12 > 2y+7$$

Adding or Subtracting a Value

We can often solve inequalities by adding (or subtracting) a number from both sides (just as in [Introduction to Algebra](#)), like this:

Solve: $x + 3 < 7$

If we subtract 3 from both sides, we get:

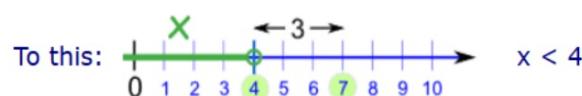
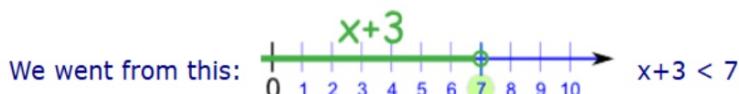
$$x + 3 - 3 < 7 - 3$$

$$x < 4$$

And that is our solution: $x < 4$

In other words, x can be any value less than 4.

What did we do?



What If I Solve It, But "x" Is On The Right?

No matter, just swap sides, but **reverse the sign** so it still "points at" the correct value!

Example: $12 < x + 5$

If we subtract 5 from both sides, we get:

$$12 - 5 < x + 5 - 5$$

$$7 < x$$

That is a solution!

But it is normal to put "x" on the left hand side ...

... so let us flip sides (and the inequality sign!):

$$x > 7$$

Do you see how the inequality sign still "points at" the smaller value (7) ?

And that is our solution: $x > 7$

Note: "x" **can** be on the right, but people usually like to see it on the left hand side.

Multiplying or Dividing by a Value

Another thing we do is multiply or divide both sides by a value (just as in [Algebra - Multiplying](#)).

But we need to be a bit more careful (as you will see).

Positive Values

Everything is fine if we want to multiply or divide by a **positive number**:

Solve: $3y < 15$

If we divide both sides by 3 we get:

$$3y/3 < 15/3$$

$$y < 5$$

And that is our solution: $y < 5$

Negative Values

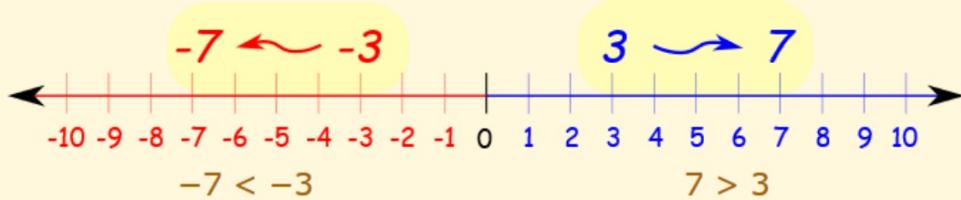


When we multiply or divide by a **negative number** we must **reverse** the inequality.

Why?

Well, just look at the number line!

For example, from 3 to 7 is **an increase**,
but from -3 to -7 is **a decrease**.



See how the inequality sign reverses (from $<$ to $>$) ?

Let us try an example:

Solve: $-2y < -8$

Let us divide both sides by -2 ... and **reverse the inequality!**

$$-2y < -8$$

$$-2y / -2 > -8 / -2$$

$$y > 4$$

And that is the correct solution: **$y > 4$**

(Note that I reversed the inequality **on the same line** I divided by the negative number.)

So, just remember:

When multiplying or dividing by a negative number, **reverse** the inequality

Multiplying or Dividing by Variables

Here is another (tricky!) example:

Solve: $bx < 3b$

It seems easy just to divide both sides by **b**, which gives us:

$$x < 3$$

... but wait ... if **b** is **negative** we need to reverse the inequality like this:

$$x > 3$$

But we don't know if **b** is positive or negative, so **we can't answer this one!**

To help you understand, imagine replacing **b** with **1 or -1** in the example of $bx < 3b$:

- if **b is 1**, then the answer is $x < 3$
- but if **b is -1**, then we are solving $-x < -3$, and the answer is $x > 3$

The answer could be $x < 3$ or $x > 3$ and we can't choose because we don't know **b**.

So:

Do not try dividing by a variable to solve an inequality (unless you know the variable is always positive, or always negative).

A Bigger Example

Solve: $\frac{x-3}{2} < -5$

First, let us clear out the "/2" by multiplying both sides by 2.

Because we are multiplying by a positive number, the inequalities will not change.

$$\frac{x-3}{2} \times 2 < -5 \times 2$$

$$x-3 < -10$$

Now add 3 to both sides:

$$x-3 + 3 < -10 + 3$$

$$x < -7$$

And that is our solution: $x < -7$

Two Inequalities At Once!

How do we solve something with two inequalities at once?

Solve:

$$-2 < \frac{6-2x}{3} < 4$$

First, let us clear out the "/3" by multiplying each part by 3.

Because we are multiplying by a positive number, the inequalities will not change:

$$-6 < 6-2x < 12$$

Now subtract 6 from each part:

$$-12 < -2x < 6$$

Now multiply each part by $-(1/2)$.

Because we are multiplying by a **negative** number, the inequalities **change direction**.

$$6 > x > -3$$

And that is the solution!

But to be neat it is better to have the smaller number on the left, larger on the right. So let us swap them over (and make sure the inequalities point correctly):

$$-3 < x < 6$$

Solve $3x + 2 > 8$

A $x > 10/3$

B $x < 2$

C $x < -2$

D $x > 2$

Score: 1
0.5 (80% correct)
0.5 (50%)
0.5 (C)
Question 1 of 10
Time remaining: 00:30:00
0.5 (D)
0.5 (B)

Solve the inequality $3x - 7 < 5$

A $x < -2/3$

B $x > 4$

C $x < 4$

D $x < -4$

3x - 7 < 5
Add 7 to both sides
3x < 12
Divide by 3
x < 4
Correct answer
B
x < 4
Score
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
3.25
3.5
3.75
4
4.25
4.5
4.75
5
5.25
5.5
5.75
6
6.25
6.5
6.75
7
7.25
7.5
7.75
8
8.25
8.5
8.75
9
9.25
9.5
9.75
10
10.25
10.5
10.75
11
11.25
11.5
11.75
12
12.25
12.5
12.75
13
13.25
13.5
13.75
14
14.25
14.5
14.75
15
15.25
15.5
15.75
16
16.25
16.5
16.75
17
17.25
17.5
17.75
18
18.25
18.5
18.75
19
19.25
19.5
19.75
20
20.25
20.5
20.75
21
21.25
21.5
21.75
22
22.25
22.5
22.75
23
23.25
23.5
23.75
24
24.25
24.5
24.75
25
25.25
25.5
25.75
26
26.25
26.5
26.75
27
27.25
27.5
27.75
28
28.25
28.5
28.75
29
29.25
29.5
29.75
30
30.25
30.5
30.75
31
31.25
31.5
31.75
32
32.25
32.5
32.75
33
33.25
33.5
33.75
34
34.25
34.5
34.75
35
35.25
35.5
35.75
36
36.25
36.5
36.75
37
37.25
37.5
37.75
38
38.25
38.5
38.75
39
39.25
39.5
39.75
40
40.25
40.5
40.75
41
41.25
41.5
41.75
42
42.25
42.5
42.75
43
43.25
43.5
43.75
44
44.25
44.5
44.75
45
45.25
45.5
45.75
46
46.25
46.5
46.75
47
47.25
47.5
47.75
48
48.25
48.5
48.75
49
49.25
49.5
49.75
50
50.25
50.5
50.75
51
51.25
51.5
51.75
52
52.25
52.5
52.75
53
53.25
53.5
53.75
54
54.25
54.5
54.75
55
55.25
55.5
55.75
56
56.25
56.5
56.75
57
57.25
57.5
57.75
58
58.25
58.5
58.75
59
59.25
59.5
59.75
60
60.25
60.5
60.75
61
61.25
61.5
61.75
62
62.25
62.5
62.75
63
63.25
63.5
63.75
64
64.25
64.5
64.75
65
65.25
65.5
65.75
66
66.25
66.5
66.75
67
67.25
67.5
67.75
68
68.25
68.5
68.75
69
69.25
69.5
69.75
70
70.25
70.5
70.75
71
71.25
71.5
71.75
72
72.25
72.5
72.75
73
73.25
73.5
73.75
74
74.25
74.5
74.75
75
75.25
75.5
75.75
76
76.25
76.5
76.75
77
77.25
77.5
77.75
78
78.25
78.5
78.75
79
79.25
79.5
79.75
80
80.25
80.5
80.75
81
81.25
81.5
81.75
82
82.25
82.5
82.75
83
83.25
83.5
83.75
84
84.25
84.5
84.75
85
85.25
85.5
85.75
86
86.25
86.5
86.75
87
87.25
87.5
87.75
88
88.25
88.5
88.75
89
89.25
89.5
89.75
90
90.25
90.5
90.75
91
91.25
91.5
91.75
92
92.25
92.5
92.75
93
93.25
93.5
93.75
94
94.25
94.5
94.75
95
95.25
95.5
95.75
96
96.25
96.5
96.75
97
97.25
97.5
97.75
98
98.25
98.5
98.75
99
99.25
99.5
99.75
100
100.25
100.5
100.75
101
101.25
101.5
101.75
102
102.25
102.5
102.75
103
103.25
103.5
103.75
104
104.25
104.5
104.75
105
105.25
105.5
105.75
106
106.25
106.5
106.75
107
107.25
107.5
107.75
108
108.25
108.5
108.75
109
109.25
109.5
109.75
110
110.25
110.5
110.75
111
111.25
111.5
111.75
112
112.25
112.5
112.75
113
113.25
113.5
113.75
114
114.25
114.5
114.75
115
115.25
115.5
115.75
116
116.25
116.5
116.75
117
117.25
117.5
117.75
118
118.25
118.5
118.75
119
119.25
119.5
119.75
120
120.25
120.5
120.75
121
121.25
121.5
121.75
122
122.25
122.5
122.75
123
123.25
123.5
123.75
124
124.25
124.5
124.75
125
125.25
125.5
125.75
126
126.25
126.5
126.75
127
127.25
127.5
127.75
128
128.25
128.5
128.75
129
129.25
129.5
129.75
130
130.25
130.5
130.75
131
131.25
131.5
131.75
132
132.25
132.5
132.75
133
133.25
133.5
133.75
134
134.25
134.5
134.75
135
135.25
135.5
135.75
136
136.25
136.5
136.75
137
137.25
137.5
137.75
138
138.25
138.5
138.75
139
139.25
139.5
139.75
140
140.25
140.5
140.75
141
141.25
141.5
141.75
142
142.25
142.5
142.75
143
143.25
143.5
143.75
144
144.25
144.5
144.75
145
145.25
145.5
145.75
146
146.25
146.5
146.75
147
147.25
147.5
147.75
148
148.25
148.5
148.75
149
149.25
149.5
149.75
150
150.25
150.5
150.75
151
151.25
151.5
151.75
152
152.25
152.5
152.75
153
153.25
153.5
153.75
154
154.25
154.5
154.75
155
155.25
155.5
155.75
156
156.25
156.5
156.75
157
157.25
157.5
157.75
158
158.25
158.5
158.75
159
159.25
159.5
159.75
160
160.25
160.5
160.75
161
161.25
161.5
161.75
162
162.25
162.5
162.75
163
163.25
163.5
163.75
164
164.25
164.5
164.75
165
165.25
165.5
165.75
166
166.25
166.5
166.75
167
167.25
167.5
167.75
168
168.25
168.5
168.75
169
169.25
169.5
169.75
170
170.25
170.5
170.75
171
171.25
171.5
171.75
172
172.25
172.5
172.75
173
173.25
173.5
173.75
174
174.25
174.5
174.75
175
175.25
175.5
175.75
176
176.25
176.5
176.75
177
177.25
177.5
177.75
178
178.25
178.5
178.75
179
179.25
179.5
179.75
180
180.25
180.5
180.75
181
181.25
181.5
181.75
182
182.25
182.5
182.75
183
183.25
183.5
183.75
184
184.25
184.5
184.75
185
185.25
185.5
185.75
186
186.25
186.5
186.75
187
187.25
187.5
187.75
188
188.25
188.5
188.75
189
189.25
189.5
189.75
190
190.25
190.5
190.75
191
191.25
191.5
191.75
192
192.25
192.5
192.75
193
193.25
193.5
193.75
194
194.25
194.5
194.75
195
195.25
195.5
195.75
196
196.25
196.5
196.75
197
197.25
197.5
197.75
198
198.25
198.5
198.75
199
199.25
199.5
199.75
200
200.25
200.5
200.75
201
201.25
201.5
201.75
202
202.25
202.5
202.75
203
203.25
203.5
203.75
204
204.25
204.5
204.75
205
205.25
205.5
205.75
206
206.25
206.5
206.75
207
207.25
207.5
207.75
208
208.25
208.5
208.75
209
209.25
209.5
209.75
210
210.25
210.5
210.75
211
211.25
211.5
211.75
212
212.25
212.5
212.75
213
213.25
213.5
213.75
214
214.25
214.5
214.75
215
215.25
215.5
215.75
216
216.25
216.5
216.75
217
217.25
217.5
217.75
218
218.25
218.5
218.75
219
219.25
219.5
219.75
220
220.25
220.5
220.75
221
221.25
221.5
221.75
222
222.25
222.5
222.75
223
223.25
223.5
223.75
224
224.25
224.5
224.75
225
225.25
225.5
225.75
226
226.25
226.5
226.75
227
227.25
227.5
227.75
228
228.25
228.5
228.75
229
229.25
229.5
229.75
230
230.25
230.5
230.75
231
231.25
231.5
231.75
232
232.25
232.5
232.75
233
233.25
233.5
233.75
234
234.25
234.5
234.75
235
235.25
235.5
235.75
236
236.25
236.5
236.75
237
237.25
237.5
237.75
238
238.25
238.5
238.75
239
239.25
239.5
239.75
240
240.25
240.5
240.75
241
241.25
241.5
241.75
242
242.25
242.5
242.75
243
243.25
243.5
243.75
244
244.25
244.5
244.75
245
245.25
245.5
245.75
246
246.25
246.5
246.75
247
247.25
247.5
247.75
248
248.25
248.5
248.75
249
249.25
249.5
249.75
250
250.25
250.5
250.75
251
251.25
251.5
251.75
252
252.25
252.5
252.75
253
253.25
253.5
253.75
254
254.25
254.5
254.75
255
255.25
255.5
255.75
256
256.25
256.5
256.75
257
257.25
257.5
257.75
258
258.25
258.5
258.75
259
259.25
259.5
259.75
260
260.25
260.5
260.75
261
261.25
261.5
261.75
262
262.25
262.5
262.75
263
263.25
263.5
263.75
264
264.25
264.5
264.75
265
265.25
265.5
265.75
266
266.25
266.5
266.75
267
267.25
267.5
267.75
268
268.25
268.5
268.75
269
269.25
269.5
269.75
270
270.25
270.5
270.75
271
271.25
271.5
271.75
272
272.25
272.5
272.75
273
273.25
273.5
273.75
274
274.25
274.5
274.75
275
275.25
275.5
275.75
276
276.25
276.5
276.75
277
277.25
277.5
277.75
278
278.25
278.5
278.75
279
279.25
279.5
279.75
280
280.25
280.5
280.75
281
281.25
281.5
281.75
282
282.25
282.5
282.75
283
283.25
283.5
283.75
284
284.25
284.5
284.75
285
285.25
285.5
285.75
286
286.25
286.5
286.75
287
287.25
287.5
287.75
288
288.25
288.5
288.75
289
289.25
289.5
289.75
290
290.25
290.5
290.75
291
291.25
291.5
291.75
292
292.25
292.5
292.75
293
293.25
293.5
293.75
294
294.25
294.5
294.75
295
295.25
295.5
295.75
296
296.25
296.5
296.75
297
297.25
297.5
297.75
298
298.25
298.5
298.75
299
299.25
299.5
299.75
300
300.25
300.5
300.75
301
301.25
301.5
301.75
302
302.25
302.5
302.75
303
303.25
303.5
303.75
304
304.25
304.5
304.75
305
305.25
305.5
305.75
306
306.25
306.5
306.75
307
307.25
307.5
307.75
308
308.25
308.5
308.75
309
309.25
309.5
309.75
310
310.25
310.5
310.75
311
311.25
311.5
311.75
312
312.25
312.5
312.75
313
313.25
313.5
313.75
314
314.25
314.5
314.75
315
315.25
315.5
315.75
316
316.25
316.5
316.75
317
317.25
317.5
317.75
318
318.25
318.5
318.75
319
319.25
319.5
319.75
320
320.25
320.5
320.75
321
321.25
321.5
321.75
322
322.25
322.5
322.75
323
323.25
323.5
323.75
324
324.25
324.5
324.75
325
325.25
325.5
325.75
326
326.25
326.5
326.75
327
327.25
327.5
327.75
328
328.25
328.5
328.75
329
329.25
329.5
329.75
330
330.25
330.5
330.75
331
331.25
331.5
331.75
332
332.25
332.5
332.75
333
333.25
333.5
333.75
334
334.25
334.5
334.75
335
335.25
335.5
335.75
336
336.25
336.5
336.75
337
337.25
337.5
337.75
338
338.25
338.5
338.75
339
339.25
339.5
339.75
340
340.25
340.5
340.75
341
341.25
341.5
341.75
342
342.25
342.5
342.75
343
343.25
343.5
343.75
344
344.25
344.5
344.75
345
345.25
345.5
345.75
346
346.25
346.5
346.75
347
347.25
347.5
347.75
348
348.25
348.5
348.75
349
349.25
349.5
349.75
350
350.25
350.5
350.75
351
351.25
351.5
351.75
352
352.25
352.5
352.75
353
353.25
353.5
353.75
354
354.25
354.5
354.75
355
355.25
355.5
355.75
356
356.25
356.5
356.75
357
357.25
357.5
357.75
358
358.25
358.5
358.75
359
359.25
359.5
359.75
360
360.25
360.5
360.75
361
361.25
361.5
361.75
362
362.25
362.5
362.75
363
363.25
363.5
363.75
364
364.25
364.5
364.75
365
365.25
365.5
365.75
366
366.25
366.5
366.75
367
367.25
367.5
367.75
368
368.25
368.5
368.75
369
369.25
369.5
369.75
370
370.25
370.5
370.75
371
371.25
371.5
371.75
372
372.25
372.5
372.75
373
373.25
373.5
373.75
374
374.25
374.5
374.75
375
375.25
375.5
375.75
376
376.25
376.5
376.75
377
377.25
377.5
377.75
378
378.25
378.5
378.75
379
379.25
379.5
379.75
380
380.25
380.5
380.75
381
381.25
381.5
381.75
382
382.25
382.5
382.75
383
383.25
383.5
383.75
384
384.25
384.5
384.75
385
385.25
385.5
385.75
386
386.25
386.5
386.75
387
387.25
387.5
387.75
388
388.25
388.5
388.75
389
389.25
389.5
389.75
390
390.25
390.5
390.75
391
391.25
391.5
391.75
392
392.25
392.5
392.75
393
393.25
393.5
393.75
394
394.25
394.5
394.75
395
395.25
395.5
395.75
396
396.25
396.5
396.75
397
397.25
397.5
397.75
398
398.25
398.5
398.75
399
399.25
399.5
399.75
400
400.25
400.5
400.75
401
401.25
401.5
401.75
402
402.25
402.5
402.75
403
403.25
403

Solve the inequality $3(4 - y) \geq 9$

A $y \leq 1$

B $y \geq 1$

C $y \leq -1$

D $y \geq -1$

$3(4 - y) \geq 9$
Divide both sides by 3:
 $\geq 3(4 - y) / 3 > 9 / 3$
 $\geq 4 - y \geq 3$
 $\geq 4 \geq y + 3$
Subtract 4 from both sides:
 $\geq 4 - 4 \geq 4 - y + 3 - 4$
 $\geq -y \geq -1$
Divide both sides by -1 to change y to y and remember to reverse the inequality sign:
 $\geq -y / -1 \leq -1 / -1$
 $\geq y \leq 1$

Solve $-4x > -12$

A $x > 3$

B $x > -3$

C $x < 3$

D $x < -3$



Solve

$$-5y - 7 \leq 3$$

A $y \geq 2$

B $y \leq 2$

C $y \leq -2$

D $y \geq -2$

8-71)
Simplifying
 A $y \geq 1$
 B $y \leq 1$

 C $y \geq -2$
 D $y \leq -2$

Solve the inequality:

$$\frac{x-3}{5} \geq -2$$

A $x \geq 5$

B $x \geq 7$

C $x \geq -7$

D $x \leq -7$

0123456789
9 $\frac{1}{5}$ 102345

• 0

• 1

• 2

• 3

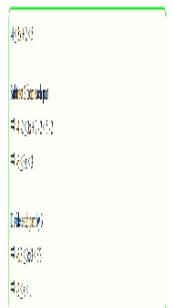
Solve the inequality $-4 \leq 3x + 2 < 5$

A $-1 < x \leq 2$

B $-1 \leq x < 2$

C $-2 < x \leq 1$

D $-2 \leq x < 1$



Solve the inequality $-9 < 5 - 7y \leq 12$

A $-1 \leq y < 2$

B $-1 < y \leq 2$

C $-2 \leq y < 1$

D $2 < y \leq -1$

$9 < 5 - 7y \leq 12$
Subtract 5 from each part
 $\Rightarrow 4 < -7y \leq 7$
 $\Rightarrow -4 > y \geq -1$
Divide each part by -7 and change the direction of the inequality:
 $\Rightarrow 4 > 7y \geq 7$
 $\Rightarrow 3 > y \geq -1$
That is an acceptable answer, but it is easier to pose the values from smaller to larger, so complete the right and change the direction of the inequality:
 $\Rightarrow -1 \leq y < 3$

Solve the inequality $3 \leq -6 - 5x < 12$

A $-3.6 \leq x < -1.8$

B $-3.6 < x \leq -1.8$

C $1.8 < x \leq 3.6$

D $1.8 \leq x < 3.6$

$3 \leq -6 - 5x < 12$

Add 6 to each part:

$$\begin{aligned} \Rightarrow 3 + 6 &\leq -6 - 5x + 6 < 12 + 6 \\ \Rightarrow 9 &\leq -5x < 18 \end{aligned}$$

Divide each part by -5 and change the direction of the inequality:

$$\begin{aligned} \Rightarrow 9/(-5) &\geq -5x/(-5) > 18/(-5) \\ \Rightarrow -1.8 &\geq x > -3.6 \end{aligned}$$

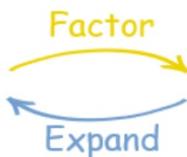
Note: It is an acceptable answer, but it is normal to place the values from smaller to larger, on comp left and right and change the direction of the inequality:

$\Rightarrow -3.6 < x \leq -1.8$

Remember these Identities

Here is a list of common "Identities" (including the "**difference of squares**" used above).

It is worth remembering these, as they can make factoring easier.



$$a^2 - b^2 = (a+b)(a-b)$$

$$a^2 + 2ab + b^2 = (a+b)(a+b)$$

$$a^2 - 2ab + b^2 = (a-b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2-ab+b^2)$$

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$$

$$a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$$

There are many more like those, but those are the most useful ones.



**NEXT
TOPIC**