Tishk International University
Faculty of Administrative Sciences and Economics

MATHEMATICS
FOR ECONOMICS AND BUSINESS

BUS 143
Part 1

I Grade- Fall

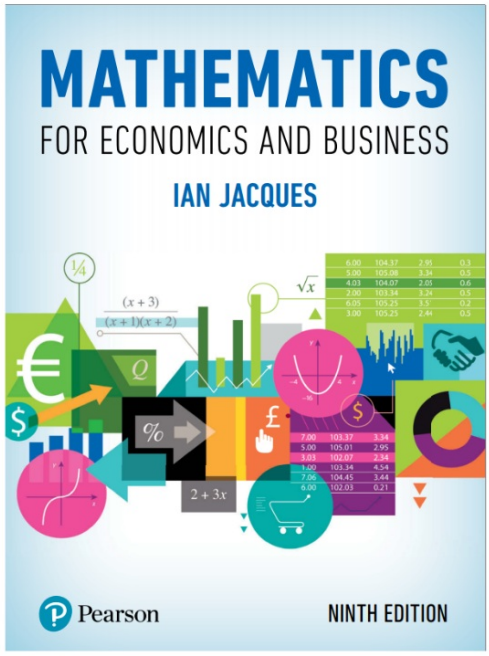
Assist. Prof. Dr. Hamdi Serin

Syllabus

Course 3 credits

Course Language: *English*

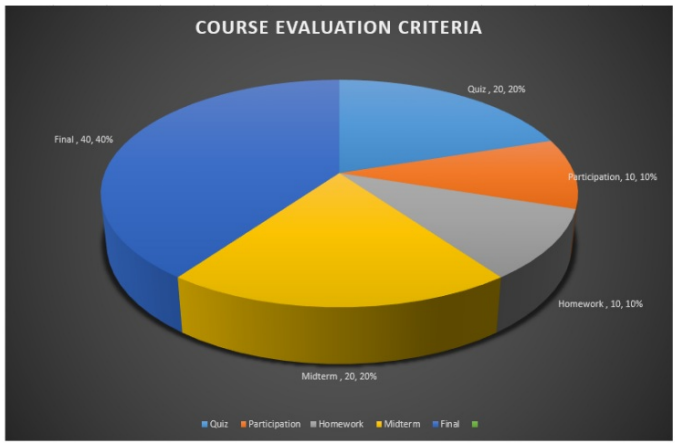
Course Book/Textbook:Mathematics for Economic and Business. Ian Jacques Person 9th Edition



Student's obligation (Special Requirements): *Taking notes, attending minimum 80 percent and follow all assignment.*

COURSE EVALUATION CRITERIA

Quiz	2x10= 20
Participation	1 x 10
Homework	1 x 10
Midterm	1 x 20
Final	1 x 40
Total:	100



Order of Operations
BODMAS

Operations

"Operations" mean things like add, subtract, multiply, divide, squaring, etc. If it isn't a number it is probably an operation.

But, when you see something like...

$$7 + (6 \times 5^2 + 3)$$

... what part should you calculate first?

Start at the left and go to the right?
Or go from right to left?

Warning: Calculate them in the wrong order, and you can get a wrong answer !

So, long ago people agreed to follow rules when doing calculations, and they are:

Order of Operations

Do things in Brackets First

✓ $6 \times (5 + 3) = 6 \times 8 = \mathbf{48}$

✗ $6 \times (5 + 3) = 30 + 3 = 33$ (wrong)

Exponents (Powers, Roots) before Multiply, Divide, Add or Subtract

✓ $5 \times 2^2 = 5 \times 4 = \mathbf{20}$

✗ $5 \times 2^2 = 10^2 = 100$ (wrong)

Multiply or Divide before you Add or Subtract

✓ $2 + 5 \times 3 = 2 + 15 = \mathbf{17}$

✗ $2 + 5 \times 3 = 7 \times 3 = 21$ (wrong)

Otherwise just go left to right

✓ $30 \div 5 \times 3 = 6 \times 3 = \mathbf{18}$

✗ $30 \div 5 \times 3 = 30 \div 15 = 2$ (wrong)

How Do I Remember It All ... ? BODMAS !

B	**B**rackets first
O	**O**rders (i.e. Powers and Square Roots, etc.)
DM	**D**ivision and **M**ultiplication (left-to-right)
AS	**A**ddition and **S**ubtraction (left-to-right)

Divide and Multiply rank equally (and go left to right).

Add and Subtract rank equally (and go left to right)

So do it this way:

After you have done "B" and "O", just go from left to right doing any "D" ***or*** "M" as you find them.

Then go from left to right doing any "A" ***or*** "S" as you find them.

Note: the only strange name is "Orders". "Exponents" is used in Canada, and so you might prefer "BEDMAS". There is also "Indices" which makes it "BIDMAS". In the US they say "Parentheses" instead of Brackets, so it is "PEMDAS"

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PEMDAS

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How Do I Remember It All ... ? PEMDAS !

P	**P**arentheses first
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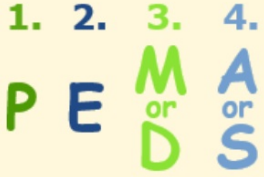
So do it this way:



After you have done "P" and "E", just go from left to right doing any "M" ***or*** "D" as you find them.

Then go from left to right doing any "A" ***or*** "S" as you find them.

Exponents of Exponents ...

What about this example?

$$4^{3^2}$$

Exponents are special: **they go top-down** (do the exponent at the top first). So we calculate this way:

Start with: 4^{3^2}

$3^2 = 3 \times 3$: 4^9

$4^9 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$: 262144

So $4^{3^2} = 4^{(3^2)}$, not $(4^3)^2$

And finally, what about the example from the beginning?

Start with:	$7 + (6 \times 5^2 + 3)$
Brackets first and then *"Orders"*:	$7 + (6 \times 25 + 3)$
Then *Multiply*:	$7 + (150 + 3)$
Then *Add*:	$7 + (153)$
Brackets completed:	$7 + 153$
Last operation is an *Add*:	**160**

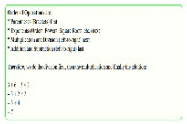
What is the value of $3 + 6 \div 3 \times 2$?

A 7

B 6

C 4

D 1.5



What is the value of $5 \times 3 - 12 \div 4 + 8$?

A 3

B 4

C 14

D 20

What is the value of $30 - (5 \times 2^3 - 15)$?

A -25

B 5

C 15

D -15

What is the value of $(15 \div 3 + 4) - (3^2 - 7 \times 2)$?

A -1.86

B 4

C 5

D 14

What is the value of $(4^2 - 6 + 5) / (3^2 + 8 - 7 \times 2)$?

A 5

B 11

C $1\frac{4}{11}$

D $\frac{3}{4}$

What is the value of this?

$$\frac{2^4 + (16 - 3 \times 4)}{(6 + 3^2) \div (7 - 4)}$$

A 2.4

B 4

C 5

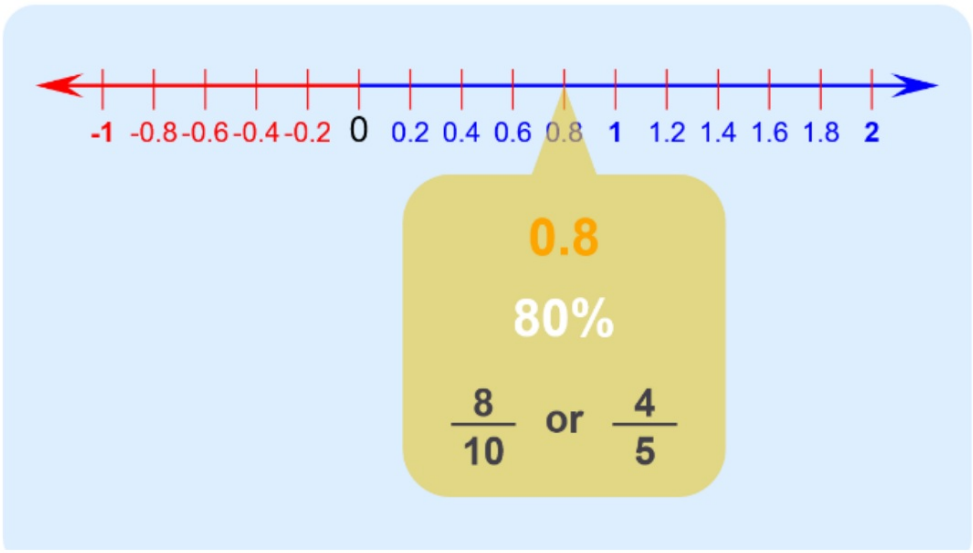
D 13.6

Example Values

Here is a table of commonly used values shown in Percent, Decimal and Fraction form:

Percent	Decimal	Fraction
1%	0.01	$^{1}/_{100}$
5%	0.05	$^{1}/_{20}$
10%	0.1	$^{1}/_{10}$
12½%	0.125	$^{1}/_{8}$
20%	0.2	$^{1}/_{5}$
25%	0.25	$^{1}/_{4}$
$33^{1}/_{3}$%	0.333...	$^{1}/_{3}$
50%	0.5	$^{1}/_{2}$
75%	0.75	$^{3}/_{4}$
80%	0.8	$^{4}/_{5}$
90%	0.9	$^{9}/_{10}$
99%	0.99	$^{99}/_{100}$
100%	**1**	
125%	1.25	$^{5}/_{4}$
150%	1.5	$^{3}/_{2}$
200%	2	

On the Number Line!



Conversions

From Percent to Decimal

To convert from percent to decimal: divide by 100, and remove the "%" sign.

The easiest way to divide by 100 is to **move the decimal point 2 places to the left**:

From Percent	To Decimal	
75% 	0.75	move the decimal point **2 places to the left**, and remove the "%" sign.

From Decimal to Percent

To convert from decimal to percent: multiply by 100, and add a "%" sign.

The easiest way to multiply by 100 is to **move the decimal point 2 places to the right**:

From Decimal		To Percent	
0.125		12.5%	move the decimal point **2 places to the right**, and add the "%" sign.

From Fraction to Decimal

The easiest way to convert a fraction to a decimal is to divide the top number by the bottom number (divide the numerator by the denominator in mathematical language)

Example: Convert $^2/_5$ to a decimal

Divide 2 by 5: 2 ÷ 5 = 0.4

Answer: $^2/_5$ = 0.4

From Decimal to Fraction

To convert a decimal to a fraction needs a little more work.

Example: To convert 0.75 to a fraction

Steps	Example
First, write down the decimal "over" the number 1	$\frac{0.75}{1}$
Multiply top and bottom by 10 for every number after the decimal point (10 for 1 number, 100 for 2 numbers, etc)	$\frac{0.75 \times 100}{1 \times 100}$
(This makes a correctly formed fraction)	$\frac{75}{100}$
Then Simplify the fraction	$\frac{3}{4}$

From Fraction to Percentage

The easiest way to convert a fraction to a percentage is to divide the top number by the bottom number. then multiply the result by 100, and add the "%" sign.

Example: Convert $^{3}/_{8}$ to a percentage

First divide 3 by 8: 3 ÷ 8 = 0.375,
Then multiply by 100: 0.375 x 100 = 37.5
Add the "%" sign: 37.5%

Answer: $^{3}/_{8}$ = 37.5%

From Percentage to Fraction

To convert a percentage to a fraction, first convert to a decimal (divide by 100), then use the steps for converting decimal to fractions (like above).

Example: To convert 80% to a fraction

Steps	Example
Convert 80% to a decimal (=80/100):	0.8
Write down the decimal "over" the number 1	$\frac{0.8}{1}$
Multiply top and bottom by 10 for every number after the decimal point (10 for 1 number, 100 for 2 numbers, etc)	$\frac{0.8 \times 10}{1 \times 10}$
(This makes a correctly formed fraction)	$\frac{8}{10}$
Then Simplify the fraction	$\frac{4}{5}$

Scientific Notation

Scientific Notation (also called Standard Form in Britain) is a special way of writing numbers:

Like this: 700 (A Number) → 7×10^2 (In Scientific Notation)

Or this: $4{,}900{,}000{,}000$ (A Number) → 4.9×10^9 (In Scientific Notation)

It makes it easy to use big and small values.

Example: 700

Why is 700 written as $\mathbf{7 \times 10^2}$ in Scientific Notation ?

- $700 = 7 \times 100$
- and $100 = 10^2$ *(see powers of 10)*
- so $700 = \mathbf{7 \times 10^2}$

Both **700** and $\mathbf{7 \times 10^2}$ have the same value, just shown in different ways.

Example: 4,900,000,000

$1{,}000{,}000{,}000 = 10^9$,
so $4{,}900{,}000{,}000 = \mathbf{4.9 \times 10^9}$ in Scientific Notation

So the number is written in **two parts**:

- Just the **digits** (with the decimal point placed after the first digit), followed by
- **× 10 to a power** that puts the decimal point where it should be

(i.e. it shows how many places to move the decimal point).

In this example, 5326.6 is written as **5.3266×10^3**,

because $5326.6 = 5.3266 \times 1000 = 5.3266 \times 10^3$

Other Ways of Writing It

We can use the **^** symbol (above the 6 on a keyboard), as it is easy to type.

3.1 × 10^8

Example: **3 × 10^4 is the same as 3 × 10^4**

- **3 × 10^4 = 3 × 10 × 10 × 10 × 10 = 30,000**

Calculators often use "E" or "e" like this:

Example: **6E+5 is the same as 6 × 10^5**

- **6E+5 = 6 × 10 × 10 × 10 × 10 × 10 = 600,000**

Example: **3.12E4 is the same as 3.12 × 10^4**

- **3.12E4 = 3.12 × 10 × 10 × 10 × 10 = 31,200**

Rule	Example
$a^m \times a^n = a^{m+n}$	$2^5 \times 2^3 = 2^8$
$a^m \div a^n = a^{m-n}$	$5^7 \div 5^3 = 5^4$
$(a^m)^n = a^{m \times n}$	$(10^3)^7 = 10^{21}$
$a^1 = a$	$17^1 = 17$
$a^0 = 1$	$34^0 = 1$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{6}\right)^2 = \frac{25}{36}$
$a^{-m} = \frac{1}{a^m}$	$9^{-2} = \frac{1}{81}$
$a^{\frac{x}{y}} = \sqrt[y]{a^x}$	$49^{\frac{1}{2}} = \sqrt[2]{49} = 7$

Changing the Subject

A very powerful thing that Algebra can do is to "rearrange" a formula so that another variable is the subject.

Example: Rearrange the volume of a box formula (**V** = **lwh**) so that the width is the subject

Start with:	**V** = **lwh**
divide both sides by h:	**V/h** = **lw**
divide both sides by l:	**V/(hl)** = **w**
swap sides:	**w** = **V/(hl)**

So if we want a box with a volume of 12, a length of 2, and a height of 2, we can calculate its width:

$$\begin{aligned} w &= V/(hl) \\ &= 12 / (2 \times 2) \\ &= 12 / 4 \\ &= 3 \end{aligned}$$

For the formula x = 3y − z, what is the value of x when y = 4 and z = 1?

A -1	B 9
C 11	D 12

x = 3y − z

Substitute 4 for y and 1 for z

In other words: where you see y, replace it with 4, and where you see z, replace it with 1

Therefore x = 3 × 4 − 1 = 12 − 1 = 11

For the formula $c^2 = a^2 + b^2$, what is the value of c when a = 7 and b = 24?

A 31

B 25

C $\sqrt{527}$

D $\sqrt{31}$

$c^2 = a^2 + b^2$

Substitute 7 for a and 24 for b

Therefore c^2
$= 7^2 + 24^2$
$= 7 \times 7 + 24 \times 24$
$= 49 + 576$
$= 625$

Therefore $c^2 = 625$

c^2 means $c \times c$
So we have to find the number that, when multiplied by itself, equals 625

That means we have to take the square root:
Therefore $c = \sqrt{625} = 25$

For the formula

$$d = \left(\frac{u+v}{2}\right) t$$

what is the value of d when u = 3, v = 11 and t = 5?

A 12

B 17.5

C 25

D 35

For the formula

$v^2 = u^2 + 2ad$

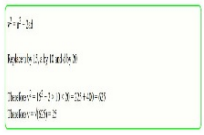
what is the value of v when u = 15, a = 10 and d = 20?

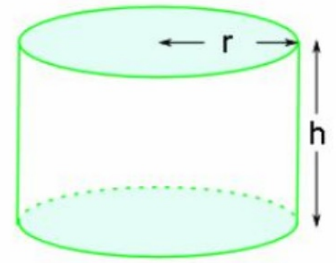
A 25

B $\sqrt{(257)}$

C 35

D 415





The formula for the total surface area of a cylinder of radius r and height h is:

$A = 2\pi r^2 + 2\pi rh$

What is the value of A when r = 7 and h = 10?

Use 22/7 as an approximation of π.

A 528

B 748

C 1,056

D 1,408

Rearrange the formula
$P = 2a + 2b$
to make b the subject

A $b = (P - 2a)/2$

B $b = (P - a)/2$

C $b = (2a - P)/2$

D $b = P/2 - 2a$

$P = 2a + 2b$

To make "b" the subject, first lets move "2a" to the other side, by subtracting it from both sides:
$\Rightarrow P - 2a = 2a + 2b - 2a$
$\Rightarrow P - 2a = 2b$

Now Swap sides:
$\Rightarrow 2b = P - 2a$

Now divide both sides by 2:
$\Rightarrow 2b/2 = (P - 2a)/2$
$\Rightarrow b = (P - 2a)/2$

Rearrange the formula:

$C = \frac{5}{9}(F - 32)$

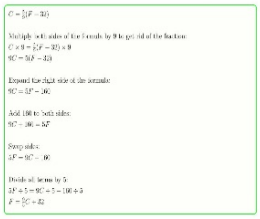
to make F the subject

A $F = \frac{5}{9}(C + 32)$

B $F = \frac{C}{32} - \frac{5}{9}$

C $F = \frac{9}{5}C + 32$

D $F = \frac{9}{5}(C + 32)$



Rearrange this formula:

$A = 2a^2 + 4ab$

So that b is the Subject of the formula.

A $b = \frac{A}{2a^2} - 4a$

B $b = A - \frac{a}{2}$

C $b = \frac{A + 2a^2}{4a}$

D $b = \frac{A - 2a^2}{4a}$

$A = 2a^2 + 4ab$

Subtract $2a^2$ from both sides: $A - 2a^2 = 4ab$

Swap Sides: $4ab = A - 2a^2$

Now, divide both sides by $4a$:

$\frac{4ab}{4a} = \frac{A - 2a^2}{4a}$

$b = \frac{A - 2a^2}{4a}$

$A = \dfrac{C^2}{4\pi}$

is a formula to calculate the area of a circle from its circumference.

Make C the subject of the formula.

A $C = 2\pi A$

B $C = 16\pi^2 A^2$

C $C = \sqrt{4\pi A}$

D $C = \dfrac{A^2}{4\pi}$

$s = ut + \frac{1}{2}at^2$ is a formula used in Physics to calculate distance.

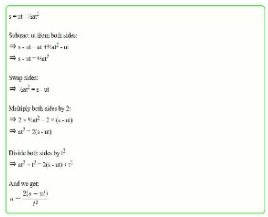
Make **"a"** the subject of the formula.

A $a = \frac{s}{ut} - \frac{1}{2}t^2$

B $a = \frac{2(ut - s)}{t^2}$

C $a = \frac{2s - ut}{t^2}$

D $a = \frac{2(s - ut)}{t^2}$



Solving Equations

What is a Solution?

A Solution is a value we can put in place of a variable (such as x) that makes the equation **true**.

Example: $x - 2 = 4$

When we put 6 in place of x we get:

$$6 - 2 = 4$$

which is **true**

So $x = 6$ is a solution.

How about other values for x ?

- For x=5 we get "5−2=4" which is **not true**, so **x=5 is not a solution**.
- For x=9 we get "9−2=4" which is **not true**, so **x=9 is not a solution**.
- etc

In this case $x = 6$ is the only solution.

More Than One Solution

There can be **more than one** solution.

Example: $(x-3)(x-2) = 0$

When x is 3 we get:

$$(3-3)(3-2) = 0 \times 1 = 0$$

which is **true**

And when x is 2 we get:

$$(2-3)(2-2) = (-1) \times 0 = 0$$

which is also **true**

So the solutions are:

$$x = \mathbf{3}, \text{ or } x = \mathbf{2}$$

When we gather all solutions together it is called a **Solution Set**

The above solution set is: $\{2, 3\}$

Solutions Everywhere!

Some equations are true for all allowed values and are then called **Identities**

Example: $\sin(-\theta) = -\sin(\theta)$ is one of the Trigonometric Identities

Let's try θ = 30°:

- $\sin(-30°) = -0.5$ and
- $-\sin(30°) = -0.5$

So it is **true** for θ = 30°

Let's try θ = 90°:

- $\sin(-90°) = -1$ and
- $-\sin(90°) = -1$

So it is also **true** for θ = 90°

Is it true for **all values of θ**? Try some values for yourself!

How to Solve an Equation

There is no "one perfect way" to solve all equations.

A Useful Goal

But we often get success when **our goal** is to end up with:

$$\mathbf{x} = \textit{something}$$

In other words, we want to move everything except "x" (or whatever name the variable has) over to the right hand side.

Example: Solve $3x-6 = 9$

Start with:	$3x-6 = 9$
Add 6 to both sides:	$3x = 9+6$
Divide by 3:	$x = (9+6)/3$

Now we have $\mathbf{x} = \textbf{\textit{something}}$,

and a short calculation reveals that $\mathbf{x} = \mathbf{5}$

How To Check

Take the solution(s) and put them in the **original equation** to see if they really work.

Example: solve for x:

$$\frac{2x}{x-3} + 3 = \frac{6}{x-3} \qquad (x \neq 3)$$

We have said $x \neq 3$ to avoid a division by zero.

Let's multiply through by $(x - 3)$:

$$2x + 3(x-3) = 6$$

Bring the 6 to the left:

$$2x + 3(x-3) - 6 = 0$$

Expand and solve:

$$2x + 3x - 9 - 6 = 0$$

$$5x - 15 = 0$$

$$5(x - 3) = 0$$

$$x - 3 = 0$$

That can be solved by having $x = 3$

Let us check:

$$\frac{2 \times 3}{3-3} + 3 = \frac{6}{3-3}$$

Hang On!
That means Dividing by Zero!

And anyway, we said at the top that $x \neq 3$, so ...

$x = 3$ does not actually work, and so:

There is **No** Solution!

Solve $5x + 2 = -8$

A $x = -2$

B $x = -1.2$

C $x = 1.2$

D $x = 2$

Start with $5x - 1 = -9$

Subtract 1 from both sides: $5x = -8 - 1 = -10$

Divide by 5: $x = -10 \div 5 = -2$

Solve 3y - 7 = 26

A y = -11

B $y = -6\frac{1}{3}$

C $y = 6\frac{1}{3}$

D y = 11

Start with 3y - 7 = 26

Add 7 to both sides: 3y - 7 - 7 = 26 + 7

Calculate: 3y = 33

Divide by 3: 3y ÷ 3 = 33 ÷ 3

Calculate: y = 11

Solve $(x + 2)(2x - 1) = 0$

A $x = 0$

B $x = 2$ or $-1/2$

C $x = -2$ or $1/2$

D $x = -1$ or 1

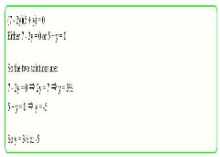
Solve (7 - 2y)(5 + y) = 0

A y = -3½ or -5

B y = 3½ or -5

C y = -3½ or 5

D y = 3½ or 5



Solve $\dfrac{3x}{x-2} = \dfrac{3x+10}{x}$

A x = 4

B x = 5

C x = 2 or $-3\frac{1}{3}$

D $x = 1\frac{1}{9}$

$\frac{3x}{x-2} = \frac{3x+10}{x}$

Multiply both sides by $x(x-2)$:

$\frac{3x}{x-2} \times x(x-2) = \frac{3x+10}{x} \times x(x-2)$

$\Rightarrow 3x^2 = (3x+10)(x-2)$
$\Rightarrow 3x^2 = 3x(x-2) + 10(x-2)$
$\Rightarrow 3x^2 = 3x^2 - 6x + 10x - 20$
$\Rightarrow 3x^2 = 3x^2 + 4x - 20$

Both sides have $3x^2$, so it can be subtracted from both sides:
$\Rightarrow 0 = 4x - 20$
$\Rightarrow 4x = 20$
$\Rightarrow x = 20 \div 4 = 5$

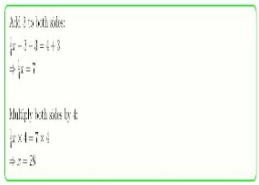
Solve $\frac{1}{4}x - 3 = 4$

A x = 4

B x = 19

C x = 24

D x = 28



Solve $\frac{2}{3}x + 5 = -7$

A x = -18

B x = -3

C x = 3

D x = 18

$\frac{2}{3}x + 5 = -7$

Subtract 5 from both sides:

$\frac{2}{3}x + 5 - 5 = -7 - 5$

$\Rightarrow \frac{2}{3}x = -12$

Multiply both sides by $\frac{3}{2}$:

$\frac{2}{3}x \times \frac{3}{2} = -12 \times \frac{3}{2}$

$\Rightarrow x = -18$

Solve $\frac{1-7x}{x+2} = \frac{5-7x}{x}$

A x = 1

B $x = \frac{5}{7}$

C x = -1

D x = -2

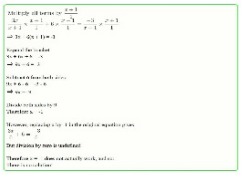
Solve $\dfrac{3x}{x+1} + 6 = \dfrac{-3}{x+1}$

A x = -1

B x = 1

C x = 3

D There is no solution



Solve $\frac{5x-4}{2x-3} = \frac{10(3-x)}{17-4x}$

A x = -4

B x = -3

C x = -2

D x = -1

Zero Product Property

The "Zero Product Property" says that:

If $a \times b = 0$ then $a = 0$ or $b = 0$
(or both a=0 and b=0)

It can help us solve equations:

Example: Solve $(x-5)(x-3) = 0$

The "Zero Product Property" says:

If $(x-5)(x-3) = 0$ then $(x-5) = 0$ **or** $(x-3) = 0$

Now we just solve each of those:

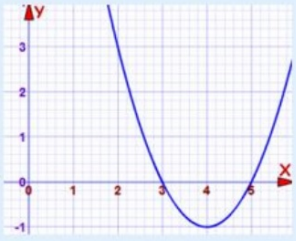
For $(x-5) = 0$ we get $x = 5$

For $(x-3) = 0$ we get $x = 3$

And the solutions are:

$x = \mathbf{5}$, or $x = \mathbf{3}$

Here it is on a graph:



y=0 when x=3 or x=5

Standard Form of an Equation

Sometimes we can solve an equation by putting it into Standard Form and then using the Zero Product Property:

The "Standard Form" of an equation is:

$$\textit{(some expression)} = 0$$

In other words, "= 0" is on the right, and everything else is on the left.

Example: Put $x^2 = 7$ into Standard Form

Answer:

$$x^2 - 7 = 0$$

Standard Form and the Zero Product Property

So let's try it out:

Example: Solve $5(x+3) = 5x(x+3)$

It is tempting to divide by (x+3), but that is dividing by zero when $x = -3$

So instead we can use "Standard Form":

$$5(x+3) - 5x(x+3) = 0$$

Which can be simplified to:

$$(5-5x)(x+3) = 0$$

$$5(1-x)(x+3) = 0$$

Then the "Zero Product Property" says:

$$(1-x) = 0, \text{ or } (x+3) = 0$$

And the solutions are:

$$x = \mathbf{1}, \text{ or } x = \mathbf{-3}$$

Example: Solve $\mathbf{x^3 = 25x}$

It is tempting to divide by x, but that is dividing by zero when $x = 0$

So let's use Standard Form and the Zero Product Property.

Bring all to the left hand side:

$$x^3 - 25x = 0$$

Factor out x:

$$x(x^2 - 25) = 0$$

$x^2 - 25$ is a difference of squares, and can be factored into $(x - 5)(x + 5)$:

$$x(x - 5)(x + 5) = 0$$

Now we can see three possible ways it could end up as zero:

$$x = \mathbf{0}, \text{ or } x = \mathbf{5}, \text{ or } x = \mathbf{-5}$$

Solve $3(x - 2) = 3x(x - 2)$

A $x = -1$ only

B $x = -1$ or $x = -2$

C $x = 1$ only

D $x = 1$ or $x = 2$

$3(x - 2) = 3x(x - 2)$

Use "Standard Form":
$3(x - 2) - 3x(x - 2) = 0$

Which can be simplified to:
$(3 - 3x)(x - 2) = 0$
or $3(1 - x)(x - 2) = 0$

Then the "Zero Product Property" says:
$(1 - x) = 0$, or $(x - 2) = 0$
so $x = 1$ or $x = 2$

Solve $2x^3 = 72x$

A $x = -6$ or 6

B $x = -6, 0$ or 6

C $x = 0$ or $\pm 6\sqrt{2}$

D $x = \pm 6\sqrt{2}$

Solve $(x + 5)(2x - 1) = 3(x + 5)$

A $x = -5$ only

B $x = -5$ or $x = 2$

C $x = -5$ or $x = ½$

D $x = 2$ or $x = 5$

$(x + 5)(2x - 1) = 3(x + 5)$

Use "Standard Form":
$(x + 5)(2x - 1) - 3(x + 5) = 0$

$(x + 5)$ is a common factor:
Therefore $(x + 5)[2x - 1 - 3] = 0$
or $(x + 5)(2x - 4) = 0$

Then the "Zero Product Property" says:
$(x + 5) = 0$, or $(2x - 4) = 0$

so $x = -5$ or $x = 2$

Solve: $(3x - 2)(2x - 5) = -8(2x - 5)$

A $x = 2/3$ or $2½$

B $x = -2½$ or $x = 2$

C $x = 2½$ or $x = 2$

D $x = 2½$ or $x = -2$

$(3x - 2)(2x - 5) = -8(2x - 5)$

Use "Product Form"
$(3x - 2)(2x - 5) + 8(2x - 5) = 0$

$(2x - 5)$ is a common factor
Then $(3x - 2)[2x - 5] + 8[2x - 5] = 0$
or $(3x - 2)(2x - 5) + 8 = 0$

Then the "Zero Product Property" says:
$(2x - 5) = 0$ or $(3x + 6) = 0 \Rightarrow 2x = 5$ or $3x = -6$

So $x = 2½$ or $x = -2$

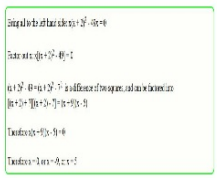
Solve $x(x + 2)^2 = 49x$

A $x = 0$, or $x = -7$, or $x = 7$

B $x = 0$, or $x = -5$, or $x = 9$

C $x = 0$, or $x = -9$, or $x = 5$

D $x = 0$, or $x = -9$, or $x = 9$



Solve $(x - 5)(x - 3)^2 = 25(x - 5)$

A $x = 5$, or $x = -2$, or $x = 8$

B $x = -8$, or $x = 5$, or $x = 2$

C $x = 2$, or $x = 5$, or $x = 8$

D $x = 3$, or $x = \pm 5$

Start with: $(x - 5)(x - 3)^2 = 25(x - 5)$

Use "Standard Form": $(x - 5)(x - 3)^2 - 25(x - 5) = 0$

Factor out $(x - 5)$: $(x - 5)[(x - 3)^2 - 25] = 0$

$(x - 3)^2 - 25 = (x - 3)^2 - 5^2$ is a difference of two squares, and can be factored into:
$[(x - 3) + 5][(x - 3) - 5] = (x + 2)(x - 8)$

And so the whole equation is now: $(x - 5)(x + 2)(x - 8) = 0$

Which is true when $x = 5$, or $x = -2$, or $x = 8$

Solve: $x(x - 1)^2 = 2(x - 1)$

A $x = -1, 1$ or 2

B $x = 1$ or 2

C $x = -1$ or 1

D $x = 0$ or 1

$x(x - 1)^2 = 2(x - 1)$

Use "Standard Form":
$x(x - 1)^2 - 2(x - 1) = 0$

$(x - 1)$ is a common factor.
Then $(x - 1)[(x - 1)x - 2] = 0$
$\Rightarrow (x - 1)(x^2 - x - 2) = 0$
$\Rightarrow (x - 1)(x + 1)(x - 2) = 0$
Then the "Zero Product Property" says:
$(x - 1) = 0$, or $(x + 1) = 0$ or $(x - 2) = 0$

So $x = 1$, $x = -1$, or $x = 2$.

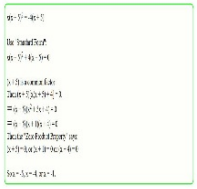
Solve: $x(x + 5)^2 = -4(x + 5)$

A $x = -5$ or $x = 0$

B $x = -5$ or $x = -4$ or $x = -1$

C $x = -5$ or $x = 1$ or $x = 4$

D $x = -4$ or $x = -1$ or $x = 5$



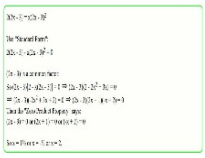
Solve: $2(2x - 3) = x(2x - 3)^2$

A $x = 0$ or $1\frac{1}{2}$

B $x = 1\frac{1}{2}$ or 2

C $x = -\frac{1}{2}$, $1\frac{1}{2}$ or 2

D $x = -\frac{1}{2}$ or $1\frac{1}{2}$



Solve: $(x - 2)(4x - 5)^2 = 7(4x - 5)$

A $x = 1.25$ or $x = 9$

B $x = 1.25$ or $x = 3$

C $x = 0.25$ or $x = 1.25$ or $x = 9$

D $x = 0.25$ or $x = 1.25$ or $x = 3$

Introduction to Inequalities

Inequality tells us about the **relative size** of two values.

Mathematics is not always about "equals", sometimes we only know that something is greater or less than.

Example: Alex and Billy have a race, and Billy wins!

What do we know?

We don't know **how fast** they ran, but we do know that Billy was faster than Alex:

Billy was faster than Alex

We can write that down like this:

$$b > a$$

(Where "b" means how fast Billy was, ">" means "greater than", and "a" means how fast Alex was)

We call things like that **inequalities** (because they are not "equal")

Example: Alex plays in the under 15s soccer. How old is Alex?

We don't know **exactly** how old Alex is, because it doesn't say "equals"

But we **do know** "less than 15", so we can write:

$$Age < 15$$

The small end points to "Age" because the age is smaller than 15.

... Or Equal To!

We can also have inequalities that include "equals", like:

Symbol	Words	Example Use
≥	greater than **or equal to**	x ≥ 1
≤	less than **or equal to**	y ≤ 3

Example: you must be 13 or older to watch a movie.

The "inequality" is between **your age** and the **age of 13**.

Your age must be "greater than **or** equal to 13", which is written:

$$Age \geq 13$$

Solving Inequalities

Sometimes we need to solve Inequalities like these:

Symbol	Words	Example
$>$	greater than	$x + 3 > 2$
$<$	less than	$7x < 28$
$\geq$	greater than or equal to	$5 \geq x - 1$
$\leq$	less than or equal to	$2y + 1 \leq 7$

Solving

Our aim is to have x (or whatever the variable is) **on its own** on the left of the inequality sign:

Something like: $x < 5$

or: $y \geq 11$

We call that "solved".

Example: $x + 2 > 12$

Subtract 2 from both sides:

$$x + 2 - 2 > 12 - 2$$

Simplify:

$$x > 10$$

Solved!

Safe Things To Do

These things **do not affect** the direction of the inequality:

- Add (or subtract) a number from both sides
- Multiply (or divide) both sides by a **positive** number
- Simplify a side

> Example: $3x < 7+3$
>
> We can simplify 7+3 without affecting the inequality:
>
> $$3x < 10$$

But these things **do change the direction** of the inequality ("<" becomes ">" for example):

- Multiply (or divide) both sides by a **negative** number
- Swapping left and right hand sides

> Example: $2y+7 < 12$
>
> When we swap the left and right hand sides, we must also **change the direction of the inequality**:
>
> $$12 > 2y+7$$

Adding or Subtracting a Value

We can often solve inequalities by adding (or subtracting) a number from both sides (just as in Introduction to Algebra), like this:

Solve: x + 3 < 7

If we subtract 3 from both sides, we get:

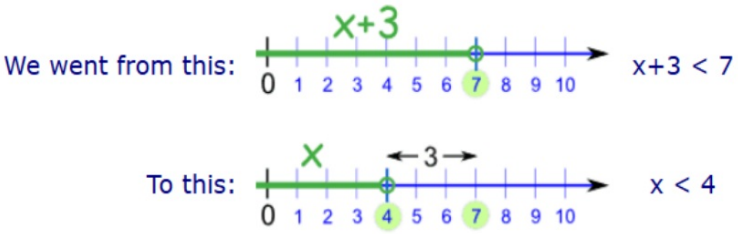
$$x + 3 \mathbf{- 3} < 7 \mathbf{- 3}$$

$$x < 4$$

And that is our solution: **x < 4**

In other words, **x** can be any value less than 4.

What did we do?



What If I Solve It, But "x" Is On The Right?

No matter, just swap sides, but **reverse the sign** so it still "points at" the correct value!

Example: **12 < x + 5**

If we subtract 5 from both sides, we get:

$$12 - 5 < x + 5 - 5$$

$$7 < x$$

That is a solution!

But it is normal to put "x" on the left hand side ...

... so let us flip sides (and the inequality sign!):

$$x > 7$$

Do you see how the inequality sign still "points at" the smaller value (7) ?

And that is our solution: **x > 7**

Note: "x" **can** be on the right, but people usually like to see it on the left hand side.

Multiplying or Dividing by a Value

Another thing we do is multiply or divide both sides by a value (just as in Algebra - Multiplying).

But we need to be a bit more careful (as you will see).

Positive Values

Everything is fine if we want to multiply or divide by a **positive number**:

Solve: 3y < 15

If we divide both sides by 3 we get:

$$3y/3 < 15/3$$

$$y < 5$$

And that is our solution: **y < 5**

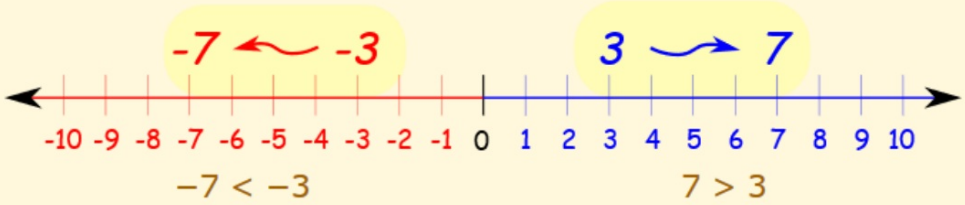
Negative Values

When we multiply or divide by a **negative number** we must **reverse** the inequality.

Why?

Well, just look at the number line!

For example, from 3 to 7 is **an increase**,
but from −3 to −7 is **a decrease.**



See how the inequality sign reverses (from < to >) ?

Let us try an example:

Solve: −2y < −8

Let us divide both sides by −2 ... and **reverse the inequality**!

$$-2y < -8$$

$$-2y/\mathbf{-2} > -8/\mathbf{-2}$$

$$y > 4$$

And that is the correct solution: **y > 4**

(Note that I reversed the inequality **on the same line** I divided by the negative number.)

So, just remember:

When multiplying or dividing by a negative number, **reverse** the inequality

Multiplying or Dividing by Variables

Here is another (tricky!) example:

Solve: $bx < 3b$

It seems easy just to divide both sides by **b**, which gives us:

$$x < 3$$

... but wait ... if **b** is **negative** we need to reverse the inequality like this:

$$x > 3$$

But we don't know if b is positive or negative, so **we can't answer this one**!

To help you understand, imagine replacing **b** with **1 or −1** in the example of **$bx < 3b$**:

- if **b is 1**, then the answer is **$x < 3$**
- but if **b is −1**, then we are solving **$-x < -3$**, and the answer is **$x > 3$**

The answer could be **$x < 3$** or **$x > 3$** and we can't choose because we don't know **b**.

So:

Do not try dividing by a variable to solve an inequality (unless you know the variable is always positive, or always negative).

A Bigger Example

Solve: $\frac{x-3}{2} < -5$

First, let us clear out the "/2" by multiplying both sides by 2.

Because we are multiplying by a positive number, the inequalities will not change.

$$\frac{x-3}{2} \times 2 < -5 \times 2$$

$$x-3 < -10$$

Now add 3 to both sides:

$$x-3 + 3 < -10 + 3$$

$$x < -7$$

And that is our solution: $x < -7$

Two Inequalities At Once!

How do we solve something with two inequalities at once?

Solve:

$$-2 < \frac{6-2x}{3} < 4$$

First, let us clear out the "/3" by multiplying each part by 3.

Because we are multiplying by a positive number, the inequalities will not change:

$$-6 < 6-2x < 12$$

Now subtract 6 from each part:

$$-12 < -2x < 6$$

Now multiply each part by −(1/2).

Because we are multiplying by a **negative** number, the inequalities **change direction**.

$$6 > x > -3$$

And that is the solution!

But to be neat it is better to have the smaller number on the left, larger on the right. So let us swap them over (and make sure the inequalities point correctly):

$$-3 < x < 6$$

Solve $3x + 2 > 8$

A $x > 10/3$

B $x < 2$

C $x < -2$

D $x > 2$

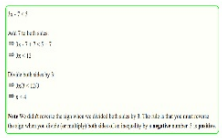
Solve the inequality $3x - 7 < 5$

A $x < -2/3$

B $x > 4$

C $x < 4$

D $x < -4$



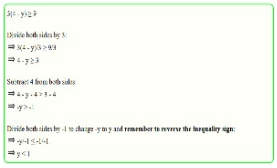
Solve the inequality $3(4 - y) \geq 9$

A $y \leq 1$

B $y \geq 1$

C $y \leq -1$

D $y \geq -1$



Solve $-4x > -12$

A $x > 3$

B $x > -3$

C $x < 3$

D $x < -3$

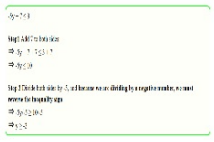
Solve
$-5y - 7 \leq 3$

A $y \geq 2$

B $y \leq 2$

C $y \leq -2$

D $y \geq -2$



Solve the inequality:

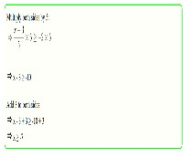
$$\frac{x-3}{5} \geq -2$$

A $x \geq 5$

B $x \geq 7$

C $x \geq -7$

D $x \leq -7$



Solve the inequality $-4 \leq 3x + 2 < 5$

A $-1 < x \leq 2$

B $-1 \leq x < 2$

C $-2 < x \leq 1$

D $-2 \leq x < 1$

Solve the inequality $-9 < 5 - 7y \leq 12$

A $-1 \leq y < 2$

B $-1 < y \leq 2$

C $-2 \leq y < 1$

D $2 < y \leq -1$

$-9 < 5 - 7y \leq 12$

Subtract 5 from each part

$\Rightarrow -9 - 5 < 5 - 7y - 5 \leq 12 - 5$

$\Rightarrow -14 < -7y \leq 7$

Divide each part by -7 and change the direction of the inequalities

$\Rightarrow -14/-7 > -7y/-7 \geq 7/-7$

$\Rightarrow 2 > y \geq -1$

That is an acceptable answer, but it is normal to pass the values from smaller to larger, so swap left and right (and change the direction of the inequalities)

$\Rightarrow -1 \leq y < 2$

Solve the inequality $3 \leq -6 - 5x < 12$

A $-3.6 \leq x < -1.8$

B $-3.6 < x \leq -1.8$

C $1.8 < x \leq 3.6$

D $1.8 \leq x < 3.6$

$3 \leq -6 - 5x < 12$

Add 6 to each part:
$\Rightarrow 3 + 6 \leq -6 - 5x + 6 < 12 + 6$
$\Rightarrow 9 \leq -5x < 18$

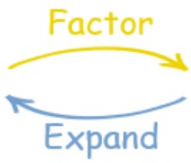
Divide each part by -5 and change the direction of the inequalities:
$\Rightarrow 9/-5 > -5x/-5 > 18/-5$
$\Rightarrow -1.8 \geq x > -3.6$

That is an acceptable answer, but it is normal to place the values from smaller to larger, so swap left and right (and change the direction of the inequalities):
$\Rightarrow -3.6 < x < -1.8$

Remember these Identities

Here is a list of common "Identities" (including the **"difference of squares"** used above).

It is worth remembering these, as they can make factoring easier.



$$a^2 - b^2 = (a+b)(a-b)$$

$$a^2 + 2ab + b^2 = (a+b)(a+b)$$

$$a^2 - 2ab + b^2 = (a-b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2-ab+b^2)$$

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$a^3+3a^2b+3ab^2+b^3 = (a+b)^3$$

$$a^3-3a^2b+3ab^2-b^3 = (a-b)^3$$

There are many more like those, but those are the most useful ones.

NEXT TOPIC