Tishk International University
Faculty of Administrative Sciences and Economics

MATHEMATICS
FOR ECONOMICS AND BUSINESS

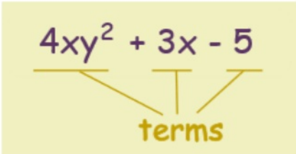
BUS 143
Part 2

I Grade- Fall

Assist. Prof. Dr. Hamdi Serin

Polynomials

A polynomial looks like this:



example of a polynomial
this one has 3 terms

Polynomial comes from *poly-* (meaning "many") and *-nomial* (in this case meaning "term") ... so it says "many terms"

A polynomial can have:

- **constants** (like **3**, **−20**, or **½**)
- **variables** (like ***x*** and ***y***)
- **exponents** (like the 2 in y^2), but only **0, 1, 2, 3, ...** etc are allowed

that can be combined using **addition, subtraction, multiplication and division** ...

... except ...

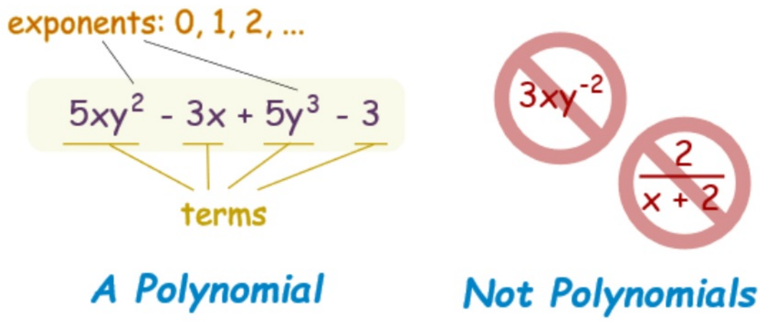
... **not** division by a variable (so something like **2/x** is right out)

So:

> A polynomial can have constants, variables and exponents,
> but never division by a variable.

Also they can have one or more terms, but not an infinite number of terms.

Polynomial or Not?



These **are** polynomials:

- $\mathbf{3x}$
- $\mathbf{x - 2}$
- $\mathbf{-6y^2 - (\frac{7}{9})x}$
- $\mathbf{3xyz + 3xy^2z - 0.1xz - 200y + 0.5}$
- $\mathbf{512v^5 + 99w^5}$
- $\mathbf{5}$

(Yes, "5" is a polynomial, **one term is allowed**, and it can be just a constant!)

These are **not** polynomials

- $3xy^{-2}$ is not, because the exponent is "-2" (exponents can only be 0,1,2,...)
- **2/(x+2)** is not, because dividing by a variable is not allowed
- **1/x** is not either
- **√x** is not, because the exponent is "½" (see fractional exponents)

But these **are** allowed:

- **x/2 is allowed**, because you can divide by a constant
- also **3x/8** for the same reason
- **√2** is allowed, because it is a constant (= 1.4142...etc)

Monomial, Binomial, Trinomial

There are special names for polynomials with 1, 2 or 3 terms:

$3xy^2$	$5x - 1$	$3x + 5y^2 - 3$
Monomial (1 term)	Binomial (2 terms)	Trinomial (3 terms)

How do you remember the names? Think cycles!

There is also quadrinomial (4 terms) and quintinomial (5 terms), but those names are not often used.

Variables

Polynomials can have no variable at all

Example: 21 is a polynomial. It has just one term, which is a constant.

Or one variable

Example: $x^4 - 2x^2 + x$ has three terms, but only one variable (x)

Or two or more variables

Example: $xy^4 - 5x^2z$ has two terms, and three variables (x, y and z)

What is Special About Polynomials?

Because of the strict definition, polynomials are **easy to work with**.

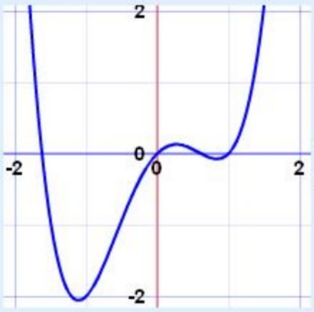
For example we know that:

- If you add polynomials you get a polynomial
- If you multiply polynomials you get a polynomial

So you can do lots of additions and multiplications, and still have a polynomial as the result.

Also, polynomials of one variable are easy to graph, as they have smooth and continuous lines.

Example: x^4-2x^2+x



See how nice and smooth the curve is?

Degree

The **degree** of a polynomial with only one variable is the **largest exponent** of that variable.

Example:

$4x^3 - x + 3$ The Degree is **3** (the largest exponent of $\boldsymbol{x}$)

Standard Form

The Standard Form for writing a polynomial is to put the terms with the highest degree first.

Example: Put this in Standard Form: $3\mathbf{x}^2 - 7 + 4\mathbf{x}^3 + \mathbf{x}^6$

The highest degree is 6, so that goes first, then 3, 2 and then the constant last:

$$\mathbf{x}^6 + 4\mathbf{x}^3 + 3\mathbf{x}^2 - 7$$

Which of the following is **not** a polynomial?

A $5x^3 - x/2$

B $3xy - 4yz + 2xz$

C $52x^3 - 19y^5$

D $3/(x - 5)$

[illegible]

What is the degree of the polynomial $5x^3 - 8x + 3x^5 + 4x^2 - 7x^4 + 1$?

A 3	B 4
C 5	D 6

Put this into the Standard Form for writing a polynomial

[illegible]

The degree of the polynomial is that the largest exponent = 5

Which of the following is a polynomial?

A $3x^2 - 3x^{½} + 2y^{2½}$

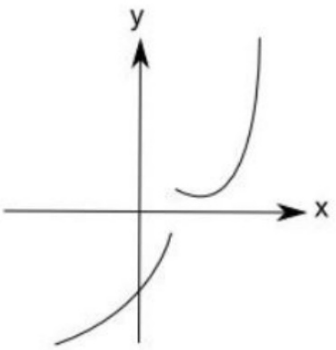
B $-7x^3y^{-1}$

C $5x^3 - 3xy^2 + 8y^4 - 3$

D $\frac{2}{x-4}$

Which one of the following could be the graph of a polynomial?

A



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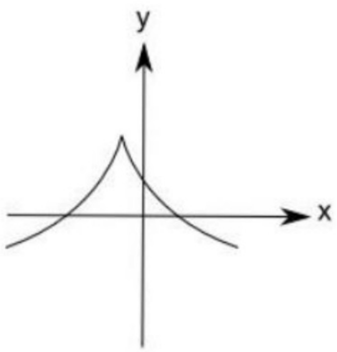

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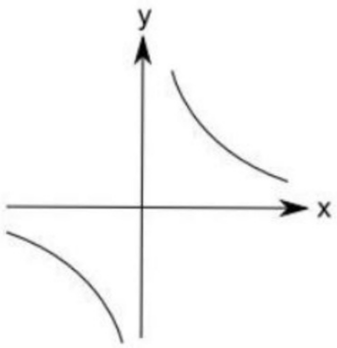

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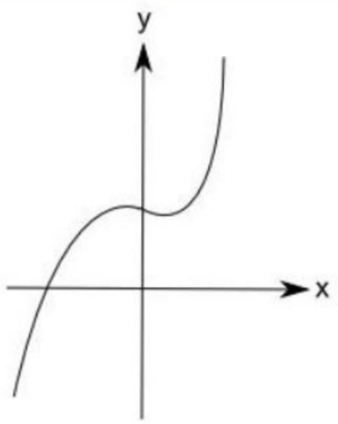


Adding Polynomials

Two Steps:

- Place **like terms** together
- Add the like terms

Example: Add **$2x^2 + 6x + 5$** and **$3x^2 - 2x - 1$**

Start with:	$2x^2 + 6x + 5 \quad + \quad 3x^2 - 2x - 1$
Place like terms together:	$2x^2+3x^2 \quad + \quad 6x-2x \quad + \quad 5-1$
Which is:	$(2+3)x^2 \quad + \quad (6-2)x \quad + \quad (5-1)$
Add the like terms:	$5x^2 \quad + \quad 4x \quad + \quad 4$

Adding Several Polynomials

We can add several polynomials together like that.

Example: Add **$(2x^2 + 6y + 3xy)$** , **$(3x^2 - 5xy - x)$** and **$(6xy + 5)$**

Line them up in columns and add:

$$\begin{array}{lllll} 2x^2 & + 6y & + 3xy & & \\ 3x^2 & & - 5xy & - x & \\ & & 6xy & & + 5 \\ \hline 5x^2 & + 6y & + 4xy & - x & + 5 \end{array}$$

Add the polynomials $(3x^2 - 6x + xy)$, $(2x^3 - 5x^2 -3y)$ and $(7x + 8y)$

A $2x^3 - 8x^2 + x + xy + 5y$

B $2x^3 - 2x^2 + x + xy - 5y$

C $2x^3 - 2x^2 - x + xy + 5y$

D $2x^3 - 2x^2 + x + xy + 5y$

Subtract ($-3x^2 + 5y - 4xy + y^2$) from ($2x^2 - 4y + 7xy - 6y^2$)

A $-5x^2 + 9y - 11xy + 7y^2$

B $5x^2 - 9y + 11xy - 7y^2$

C $-x^2 + y + 3xy - 5y^2$

D $-x^2 - 9y + 11xy - 7y^2$

Subtract $(4x^2 + 2x - 3y + 7xy - 3y^2)$ from $(3x^2 - 5x + 8y + 7xy + 2y^2)$

A $-x^2 - 7x + 11y + 5y^2$

B $-x^2 - 7x + 11y + 14xy + 5y^2$

C $x^2 + 7x + 11y - y^2$

D $x^2 + 7x - 11y - 5y^2$

If P = $5x^4 - 2x^2 + 4x - 3$ and Q = $5x^4 + 3x^3 - 4x + 3$, what is P - Q?

A $-3x^3 - 5x^2 + 8x - 6$

B $-3x^3 - 2x^2 + 8x - 6$

C $-3x^3 + 2x^2 + 8x - 6$

D $-3x^3 - 2x^2$

P - Q

[illegible]

If P = $4x^4 - 3x^3 + x^2 - 5x + 11$
and Q = $-3x^4 + 6x^3 - 8x^2 + 4x - 3$
what is 2P + Q?

A $5x^4 - 6x^2 - 6x + 19$

B $5x^4 - 12x^3 - 6x^2 - 6x + 19$

C $2x^4 + 3x^3 - 7x^2 - x + 8$

D $5x^4 - 6x^2 - 14x + 19$

If P = $4x^4 - 3x^3 + x^2 - 5x + 11$ and Q = $-3x^4 + 6x^3 - 8x^2 + 4x - 3$, what is P - 2Q?

A $-2x^4 + 9x^3 - 15x^2 + 3x + 5$

B $10x^4 + 9x^3 + 17x^2 - 13x + 17$

C $10x^4 - 15x^3 + 17x^2 - 13x + 17$

D $14x^4 - 18x^3 + 18x^2 - 18x + 28$

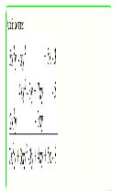
Add the polynomials $(5x^2y - xy^2 + 5x - 3)$, $(4xy^2 - 3y + 7xy - 5)$ and $(2x^2y - 3xy)$

A $7x^2y + 3xy^2 - 3y + 4xy + 5x + 2$

B $3x^2y + 3xy^2 - 3y + 4xy + 5x - 8$

C $7x^2y - 5xy^2 - 3y + 4xy + 5x - 8$

D $7x^2y + 3xy^2 - 3y + 4xy + 5x - 8$



If P = $3x^2 + xy - 5y^2$, Q = $2x^2 - xy + 3y^2$ and R = $-6x^2 + 4xy - 7y^2$,
what is P + Q - R ?

A $11x^2 - 4xy + 5y^2$

B $-x^2 - 4xy + 5y^2$

C $-x^2 - 2xy + 5y^2$

D $11x^2 - 4xy - 9y^2$

[illegible]

$P = 3x^2 + xy - 5y^2$,

$Q = 2x^2 - xy + 3y^2$ and

$R = -6x^2 + 4xy - 7y^2$

What is $P - (Q + R)$?

A $-5x^2 - 2xy - 15y^2$

B $7x^2 + 2xy - y^2$

C $7x^2 - 2xy - y^2$

D $-5x^2 + 6xy - 15y^2$

1 term × 1 term (monomial times monomial)

To multiply one term by another term, first multiply the **constants**, then multiply **each variable** together and combine the result, like this (press play):

$$(2xy)(4y) = 2 \cdot 4 \cdot xy \cdot y = 8xy^2$$

1 term × 2 terms (monomial times binomial)

Multiply the single term by each of the two terms, like this:

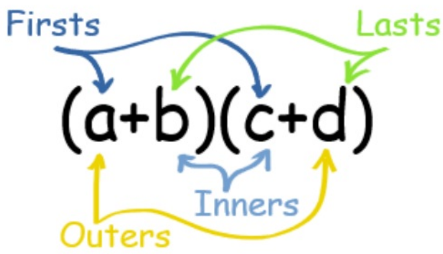
$$2x(x + 3xy) = 2x \cdot x + 2x \cdot 3xy$$
$$= 2x^2 + 6x^2y$$

2 term × 1 terms (binomial times monomial)

Multiply each of the two terms by the single term, like this:

$$(x^2 - x)3y = 3x^2y - 3xy$$

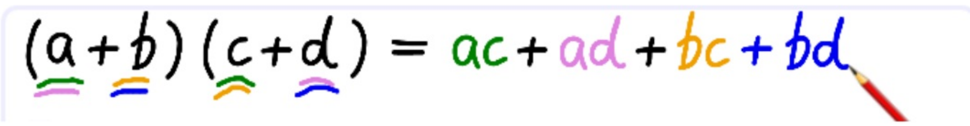
It stands for "**F**irsts, **O**uters, **I**nners, **L**asts":



- Firsts: **ac**
- Outers: **ad**
- Inners: **bc**
- Lasts: **bd**

It is the same when we multiply binomials!

Instead of Alice and Betty, let's just use **a** and **b**, and Charles and David can be **c** and **d**:



2 terms × 3 terms (binomial times trinomial)

"FOIL" won't work here, because there are more terms now. But just remember:

Multiply each term in the first polynomial by each term in the second polynomial

$$(x+a)(2x+3y-5) =$$

$$2x^2+3xy-5x+2ax+3ay-5a$$

Like Terms

And always remember to add Like Terms:

Example: $(x + 2y)(3x - 4y + 5)$

$$(x + 2y)(3x - 4y + 5)$$

$$= 3x^2 - 4xy + 5x + 6xy - 8y^2 + 10y$$

$$= 3x^2 + 2xy + 5x - 8y^2 + 10y$$

Note: **−4xy** and **6xy** are added because they are Like Terms.

Also note: **6yx** means the same thing as **6xy**

Multiply out (3x + 2)(4x - 5)

A $12x^2 - 7x - 10$

B $12x^2 - 23x - 10$

C $12x^2 + 7x - 10$

D $12x^2 - 10$

[illegible]

What is the product of (5x - 2) and (2x + 7)?

A $10x^2 + 31x + 14$

B $10x^2 + 39x - 14$

C $7x^2 + 31x - 14$

D $10x^2 + 31x - 14$

[illegible]

Multiply out $(x - 4)(3x - y + 3)$

A $3x^2 + 4y - 12$

B $3x^2 - 6xy - 12$

C $3x^2 - xy - 9x + 4y - 12$

D $3x^2 - xy - 9x - 4y - 12$

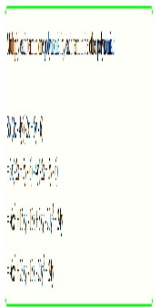
What is the product of (3x - 4y) and (-2x + 5y - 6)?

A $-6x^2 + 7xy - 18x - 20y^2 + 24y$

B $-6x^2 + 23xy - 18x - 20y^2 + 24y$

C $6x^2 + 23xy - 18x - 20y^2 + 24y$

D $-6x^2 + 23xy - 18x + 20y^2 - 24y$



What is the product of (3a - 4b + 1) and (2a + 5b - 3)?

A $6a^2 + 20b^2 + 7ab - 7a + 17b - 3$

B $6a^2 - 20b^2 + 7ab - 7a + 17b - 3$

C $6a^2 - 20b^2 + 23ab - 7a + 17b - 3$

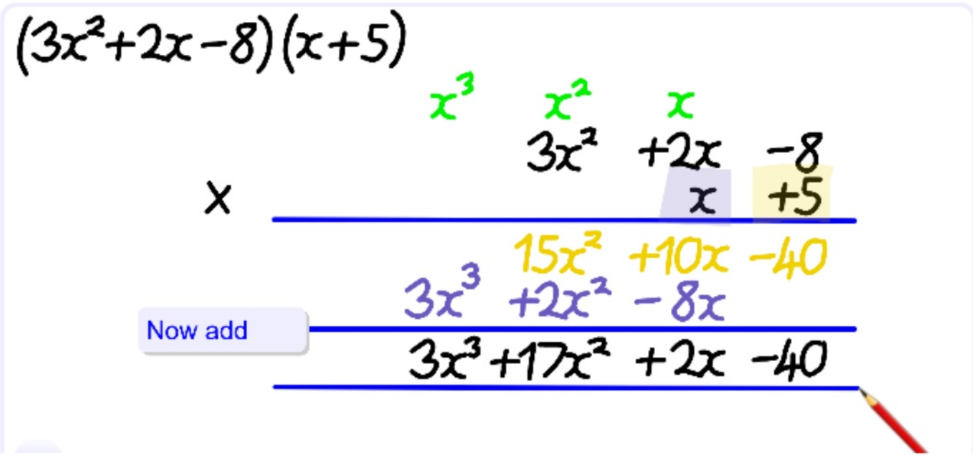
D $6a^2 - 20b^2 + 7ab - 7a - 7b - 3$

The Method

Choose one polynomial (the longest is a good choice) and then:

- multiply it by **the first term** of the other polynomial, writing the result down
- then multiply it by **the second term** of the other polynomial, writing the result under the matching terms from the first multiplication
- then multiply it by **the third term** of the other polynomial (if any) etc ...
- lastly, add up the columns.

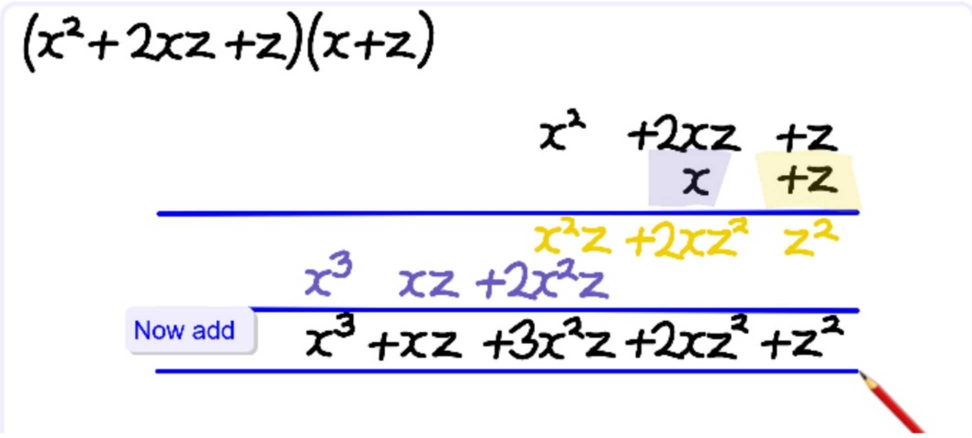
Laying the work out neatly in columns is the key, like this:



More than One Variable

So far we have been multiplying polynomials with only one variable (**x**), but how do we handle polynomials with two or more variables (such as **x** and **y**)? What are the column headings?

Just ignore the columns in the question, write down the answers as they come, **always checking** to see if we could put an answer under a matching answer:



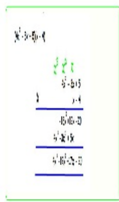
Multiply out $(4x^2 - 3x + 5)(x - 4)$ using polynomial long multiplication.

A $4x^3 + 13x^2 + 17x - 20$

B $4x^3 - 19x^2 - 7x - 20$

C $4x^3 + 13x^2 - 7x - 20$

D $4x^3 - 19x^2 + 17x - 20$



Multiply out $(3x^3 + 4x - 5)(2x^3 - 8x^2 + 2)$ using polynomial long multiplication.

A $5x^3 - 8x^2 + 4x - 3$

B $6x^6 - 24x^5 + 8x^4 - 36x^3 + 40x^2 + 8x - 10$

C $6x^6 + 24x^5 + 8x^4 - 36x^3 + 40x^2 + 8x - 10$

D $6x^6 + 24x^5 + 8x^4 + 28x^3 + 40x^2 + 8x - 10$

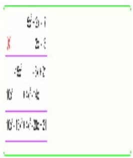
Multiply out $(5x^3 + 2x - 7)(2x - 3)$ using polynomial long multiplication.

A $10x^4 - 15x^3 + 4x^2 + 8x + 21$

B $10x^4 - 15x^3 + 4x^2 - 8x + 21$

C $10x^4 - 15x^3 + 4x^2 - 20x - 21$

D $10x^4 - 15x^3 + 4x^2 - 20x + 21$



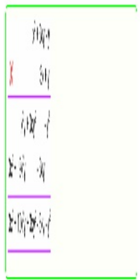
Multiply out $(x^2 + 3xy - y)(3x + y)$ using polynomial long multiplication.

A $3x^3 + 12x^2y + xy^2 - 3xy - y^2$

B $3x^3 + 10x^2y + 3xy^2 - 3xy + y^2$

C $3x^3 + 10x^2y + 3xy^2 - 3xy - y^2$

D $3x^3 + 8x^2y + 3xy^2 - 3xy - y^2$



Multiply out $(3y^2 + 5y - 7)(y + 4)$ using polynomial long multiplication.

A $3y^3 + 7y^2 - 13y - 28$

B $3y^3 + 7y^2 + 27y - 28$

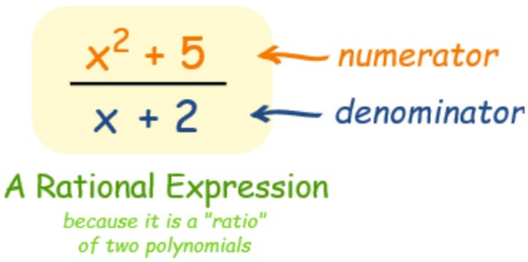
C $3y^3 + 17y^2 + 13y - 28$

D $3y^3 + 17y^2 + 27y - 28$

$$
\begin{array}{r}
3y^2 + 5y - 7 \\
\times \qquad y + 4 \\
\hline
12y^2 + 20y - 28 \\
3y^3 + 5y^2 - 7y \qquad \\
\hline
3y^3 + 17y^2 + 13y - 28 \\
\hline
\end{array}
$$

Rational Expressions

An expression that is the ratio of two polynomials :



It is just like a fraction, but with polynomials.

Other Examples:

$$\frac{x^3 + 2x - 1}{6x^2} \qquad \frac{2x + 9}{x^4 - x^2}$$

Also

$$\frac{1}{2 - x^2}$$

The top polynomial is "1" which is fine.

$$2x^2 + 3$$

Yes it is! As it could also be written:

$$\frac{2x^2 + 3}{1}$$

But Not

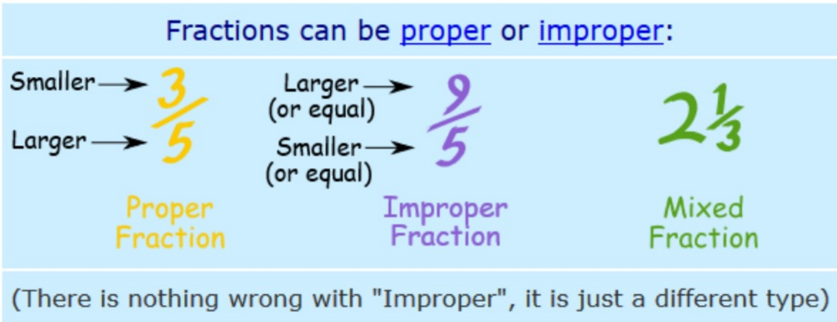
✗ $$\frac{2 - \sqrt{(x)}}{4 - x}$$

the top is not a polynomial (a square root of a variable is not allowed)

✗ $$\frac{1 - x}{1 + \frac{1}{x}}$$

1/x is not allowed in a polynomial

Proper vs Improper



Express $\dfrac{2x^5 + 4x^3}{8x^2}$ in its lowest terms ($x \neq 0$).

A $\dfrac{x(x^2+2)}{4}$

B $\dfrac{x^2(x^2+2)}{4x}$

C $\dfrac{2x(x^2+2)}{8}$

D $\dfrac{(x^2+2x)}{4}$

Dividing

Sometimes it is easy to divide a polynomial by splitting it at the "+" and "−" signs, like this (press play):

$$\frac{6x-3}{3} = \frac{6x}{3} - \frac{3}{3} = 2x-1$$

$$\frac{2x^2+2x}{x+1} = \frac{2x(x+1)}{x+1} = 2x$$

$$\frac{9x^2+12x+1}{3x}$$

$$= \frac{9x^2}{3x} + \frac{12x}{3x} + \frac{1}{3x}$$

$$= 3x + 4 + \frac{1}{3x}$$

Divide $8x^2 + 12x - 3$ by $4x$

A $2x + 3 + \frac{3}{4x}$

B $2x + 3 - \frac{3}{4x}$

C $2x - 3 - \frac{3}{4x}$

D $8x^2 + 12x - \frac{3}{4x}$

Divide $(5x^2 - 10x)$ by $(x - 2)$, where $x \neq 2$

A $5x$

B $5x + 5$

C $5x - 5$

D $5x + 10$

Divide ($12x^3 + 9x^2 - 3$) by 3x

A $12x^2 + 3x - \frac{1}{x}$

B $4x^2 + 3x - \frac{1}{x}$

C $4x^2 + 6x - \frac{1}{x}$

D $4x^2 + 3x - 1$

Divide ($-12x^2$ - 18x) by (2x + 3)

A 6x

B -12x

C $\frac{-6x(2x-3)}{2x+3}$

D -6x

First factor the numerator.

The terms $-12x^2$ and $-18x$ on the numerator both have a common factor of $-6x$.

Therefore $(-12x^2 - 18x) = -6x(2x + 3)$

Now the numerator is factored, we can see the same factor $(2x + 3)$ on both the numerator and the denominator, so it can be cancelled.

$$\frac{-12x^2 - 18x}{2x + 3} = \frac{-6x(2x + 3)}{2x + 3} = -6x$$

Note that $x \neq -\frac{3}{2}$, otherwise we would be dividing 0 by 0, which is undefined.

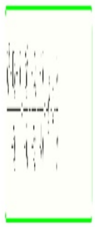
Divide (5y^3 - 10y - 15) by -5y

A $y^2 + 2 + \frac{3}{y}$

B $y^2 + 2 - \frac{3}{y}$

C $-y^2 + 2 + \frac{3}{y}$

D $-y^2 + 2 - \frac{3}{y}$



Divide ($6x^3$ - $2x^2$ - 15x + 5) by (3x - 1)

A $2x^2 + \dfrac{2x^2 + 15x - 5}{3x - 1}$

B $2x^2 - \dfrac{2x^2 + 15x - 5}{3x - 1}$

C $2x^2 + 5$

D $2x^2 - 5$

Divide $(a^3 + b^3)$ by $(a + b)$

A $a^2 + b^2$

B $a^2 - b^2$

C $a^2 + ab + b^2$

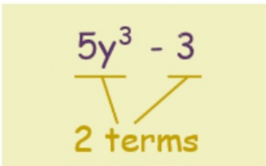
D $a^2 - ab + b^2$

[illegible]

Conjugate

The conjugate is where we **change the sign in the middle** of two terms like this:

We only use it in expressions with **two terms**, called "binomials":



example of a binomial

Here are some more examples:

Expression		Its Conjugate
$x^2 - 3$	⇒	$x^2 + 3$
$a + b$	⇒	$a - b$
$a - b^3$	⇒	$a + b^3$

What is the result of multiplying $(3 - 2x)$ by its conjugate?

A $9 - 12x + 4x^2$

B $9 + 4x^2$

C $9 - 4x^2$

D 6

Special Binomial Products

Special Binomial Products

So when we multiply binomials we get ... Binomial Products!

And we will look at **three special cases** of multiplying binomials ... so they are **Special Binomial Products**.

1. Multiplying a Binomial by Itself

What happens when we square a binomial (in other words, multiply it by itself) .. ?

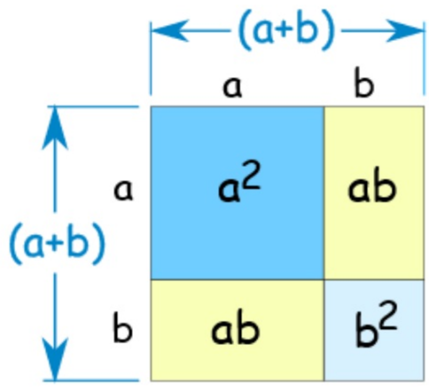
$$(a+b)^2 = (a+b)(a+b) = \dots ?$$

$$(a+b)(a+b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

The result:

$$(a+b)^2 = a^2 + 2ab + b^2$$

This illustration shows why it works:



2. Subtract Times Subtract

And what happens when we square a binomial with a **minus** inside?

$$(a-b)^2 = (a-b)(a-b) = \ldots ?$$

$$(a-b)(a-b) = a^2 - ab - ab + b^2$$

$$= a^2 - 2ab + b^2$$

3. Add Times Subtract

And then there is one more special case ... what about (a+b) times (a−b) ?

$$(a+b)(a-b) = \dots ?$$

$$(a+b)(a-b) = a^2 - ab + ab - b^2$$

$$= a^2 + 0 - b^2 = a^2 - b^2$$

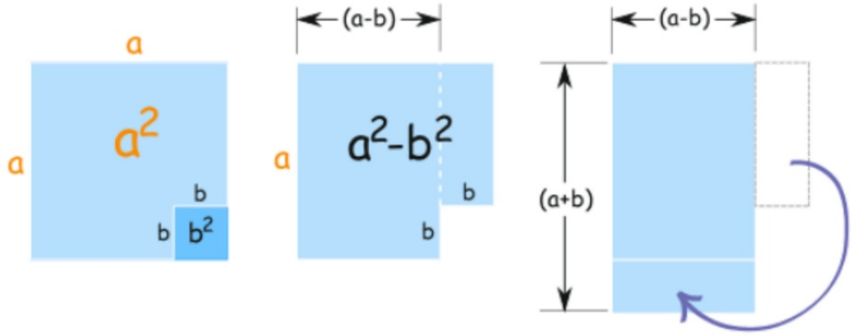
The result:

$$(a+b)(a-b) = a^2 - b^2$$

That was interesting! It ended up very simple.

And it is called the "**difference of two squares**" (the two squares are $\mathbf{a^2}$ and $\mathbf{b^2}$).

This illustration shows why it works:



$a^2 - b^2$ is equal to $(a+b)(a-b)$

Note: (a−b) could be first and (a+b) second:

$$(a-b)(a+b) = a^2 - b^2$$

The Three Cases

Here are the three results we just got:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

} the "perfect square trinomials"

$$(a+b)(a-b) = a^2 - b^2$$

the "difference of squares"

Remember those patterns, they will save you time and help you solve many algebra puzzles.

Using Them

So far we have just used "a" and "b", but they could be anything.

Example: $(y+1)^2$

We can use the $(a+b)^2$ case where "a" is y, and "b" is 1:

$$(y+1)^2 = (y)^2 + 2(y)(1) + (1)^2 = y^2 + 2y + 1$$

Example: $(3x-4)^2$

We can use the $(a-b)^2$ case where "a" is 3x, and "b" is 4:

$$(3x-4)^2 = (3x)^2 - 2(3x)(4) + (4)^2 = 9x^2 - 24x + 16$$

Example: $(4y+2)(4y-2)$

We know the result is the difference of two squares, because:

$$(a+b)(a-b) = a^2 - b^2$$

so:

$$(4y+2)(4y-2) = (4y)^2 - (2)^2 = 16y^2 - 4$$

Solving Word Questions

With LOTS of examples!

In Algebra we often have word questions like:

Example: Sam and Alex play tennis.

On the weekend Sam played 4 more games than Alex did, and together they played 12 games.

How many games did Alex play?

How do we solve them?

The trick is to break the solution into two parts:

Turn the English into Algebra.
Then use Algebra to solve.

Turning English into Algebra

To turn the English into Algebra it helps to:

- Read the whole thing first
- Do a **sketch** if possible
- Assign **letters** for the values
- Find or work out **formulas**

You should also write down **what is actually being asked for**, so you know where you are going and when you have arrived!

Also look for key words:

When you see		Think
add, total, sum, increase, more, combined, together, plus, more than		+
minus, less, difference, fewer, decreased, reduced		−
multiplied, times, of, product, factor		×
divided, quotient, per, out of, ratio, ratio, percent, rate		÷
maximize or minimize		geometry formulas
Rate, speed		distance formulas
How long, days, hours, minutes, seconds		time

Thinking Clearly

Some wording can be tricky, making it hard to think "the right way around", such as:

Example: Sam has 2 dollars less than Alex. How do we write this as an equation?

- Let S = dollars Sam has
- Let A = dollars Alex has

Now ... is that: $S - 2 = A$

or should it be: $S = A - 2$

or should it be: $S = 2 - A$

The correct answer is

($\mathbf{S} - \mathbf{2} = \mathbf{A}$ is a common mistake, as the question is written "Sam ... 2 less ... Alex")

Example: on our street there are twice as many dogs as cats. How do we write this as an equation?

- Let D = number of dogs
- Let C = number of cats

Now ... is that: $2D = C$

or should it be: $D = 2C$

Think carefully now!

The correct answer is

(**2D** = **C** is a common mistake, as the question is written "twice ... dogs ... cats")

Examples

Let's start with a **really simple** example so we see how it's done:

Example: A rectangular garden is 12m by 5m, what is its area?

Turn the English into Algebra:

Sketch:

Letters:

- Use **w** for width of rectangle: **w = 12m**
- Use **h** for height of rectangle: **h = 5m**

Formula for Area of a Rectangle : **A = w × h**

We are being asked for the Area.

Solve:

$$A = w \times h = 12 \times 5 = 60\ m^2$$

The area is **60 square meters**.

Now let's try the example from the top of the page:

Example: Sam and Alex play Tennis. On the weekend Sam played 4 more games than Alex did, and together they played 12 games. How many games did Alex play?

Turn the English into Algebra:

Letters:

- Use **S** for how many games Sam played
- Use **A** for how many games Alex played

We know that Sam played 4 more games than Alex, so: S = A + 4

And we know that together they played 12 games: S + A = 12

We are being asked for how many games Alex played: **A**

Solve:

Start with:	S + A = 12
S = A + 4, so we can substitute "A + 4" for S:	(A + 4) + A = 12
Simplify:	2A + 4 = 12
Subtract 4 from both sides:	2A = 12 − 4
Simplify:	2A = 8
Divide both sides by 2:	A = 4

Which means that Alex played 4 games of tennis.

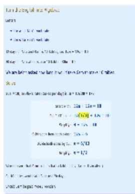
Check: Sam played 4 more games than Alex, so Sam played 8 games. Together they played 8 + 4 = 12 games. Yes!

A slightly harder example:

Example: Alex and Sam also build tables.
Together they make 10 tables in 12 days.

Alex working alone can make 10 in 30 days.

How long would it take Sam working alone to make 10 tables?

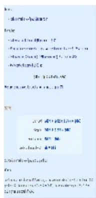


Example: A cube has a volume of 125 mm^3, what is its surface area?

An example about Money:

Example: Joel works at the local pizza parlor. When he works overtime he earns 1¼ times the normal rate.

One week Joel worked for 40 hours at the normal rate of pay and also worked 12 hours overtime. If Joel earned $660 altogether in that week, what is his normal rate of pay?

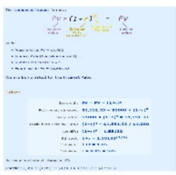


More about Money, with these two examples involving Compound Interest

Example: Alex puts $2000 in the bank at an annual compound interest of 11%. How much will it be worth in 3 years?

Example: Roger deposited $1,000 into a savings account. The money earned interest compounded annually at the same rate. After nine years Roger's deposit has grown to $1,551.33

What was the annual rate of interest for the savings account?



And an example of a Ratio question:

Example: At the start of the year the ratio of boys to girls in a class is 2 : 1

But now, half a year later, four boys have left the class and there are two new girls. The ratio of boys to girls is now 4 : 3

How many students are there altogether now?

A quiz has thirty questions whose answers can only be correct or incorrect.

A correct answer scores 8 points, but 3 points are deducted for every incorrect answer.

Gina did the quiz and scored a total of 152 points. How many questions did she get correct?

A 11	B 18
C 19	D 22

[illegible]

The denominator of a fraction is 3 more than the numerator. If both the numerator and the denominator are increased by 4, the fraction becomes

$$\frac{4}{5}$$

What was the original fraction?

A $\frac{8}{11}$

B $\frac{5}{8}$

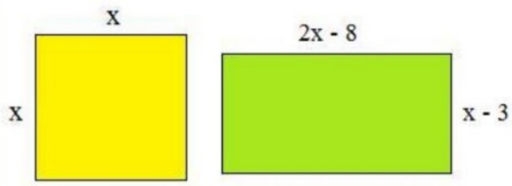
C $\frac{10}{13}$

D $\frac{7}{10}$

The fraction is

According to the second statement of the problem,

Hence, the fraction is



The areas of the square and the rectangle are equal.
What is the value of x?

A $x = 2$

B $x = 11$

C $x = 2$ or 12

D $x = 12$

Area of the square = x^2 units^2

Area of the rectangle = $(2x - 8)(x - 3)$ units^2

But these are equal

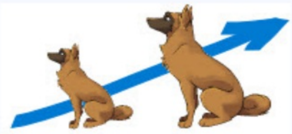
$\Rightarrow (2x - 8)(x - 3) = x^2 \Rightarrow 2x^2 - 6x - 8x + 24 = x^2$

$\Rightarrow 2x^2 - 14x + 24 = x^2 \Rightarrow x^2 - 14x + 24 = 0$

$\Rightarrow (x - 2)(x - 12) = 0 \Rightarrow x = 2$ or 12

But if $x = 2$, the width and height of the rectangle would both be negative.
Then $x = 12$ is the only solution.

Directly Proportional and Inversely Proportional



Directly proportional: as one amount increases, another amount increases at the same rate.

∝ The symbol for "directly proportional" is ∝
(Don't confuse it with the symbol for infinity ∞)

Example: you are paid $20 an hour

How much you earn is **directly proportional** to how many hours you work

Work more hours, get more pay; in direct proportion.

This could be written:

Earnings ∝ Hours worked

- If you work 2 hours you get paid $40
- If you work 3 hours you get paid $60
- etc ...

Constant of Proportionality

The "constant of proportionality" is the value that relates the two amounts

Example: you are paid $20 an hour (continued)

The constant of proportionality is **20** because:

Earnings = 20 × Hours worked

This can be written:

$$y = kx$$

Where **k** is the constant of proportionality

Example: y is directly proportional to x, and when x=3 then y=15.
What is the constant of proportionality?

They are directly proportional, so:

$y = kx$

Put in what we know (y=15 and x=3):

$15 = k \times 3$

Solve (by dividing both sides by 3):

$15/3 = k \times 3/3$

$5 = k \times 1$

$k = 5$

The constant of proportionality is 5:

$$y = 5x$$

When we know the constant of proportionality we can then answer other questions

Example: (continued)

What is the value of y when x = 9?

➜ $y = 5 \times 9 = 45$

What is the value of x when y = 2?

➜ $2 = 5x$

➜ $x = 2/5 = 0.4$

Example: 4 people can paint a fence in 3 hours.

How long will it take 6 people to paint it?

(Assume everyone works at the same rate)

It is an Inverse Proportion:

- As the number of people goes up, the painting time goes down.
- As the number of people goes down, the painting time goes up.

We can use:

$$t = k/n$$

Where:

- t = number of hours
- k = constant of proportionality
- n = number of people

"4 people can paint a fence in 3 hours" means that t = 3 when n = 4

$3 = k/4$

$3 \times 4 = k \times 4 / 4$

$12 = k$

$k = 12$

So now we know:

$t = 12/n$

And when n = 6:

$t = 12/6 = 2$ hours

So 6 people will take 2 hours to paint the fence.

How many people are needed to complete the job in half an hour?

$½ = 12/n$

$n = 12 / ½ = 24$

So it needs 24 people to complete the job in half an hour.
(Assuming they don't all get in each other's way!)

Proportional to ...

It is also possible to be proportional to a square, a cube, an exponential, or other function!

Example: Proportional to x^2

A stone is dropped from the top of a high tower.

The distance it falls is **proportional to the square** of the time of fall.

The stone falls 19.6 m after 2 seconds, how far does it fall after 3 seconds?

We can use:

$$d = kt^2$$

Where:

- d is the distance fallen and
- t is the time of fall

When d = 19.6 then t = 2

→ $19.6 = k \times 2^2$

→ $19.6 = 4k$

→ $k = 4.9$

So now we know:

$$d = 4.9t^2$$

And when t = 3:

→ $d = 4.9 \times 3^2$

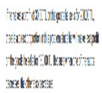
→ $d = 44.1$

So it has fallen 44.1 m after 3 seconds.

Example: If a student reads a 300-page book in 6 days, in how many days will he read a 600-page book?

[illegible]

Example 2: If a seller makes a profit of 30 TL on the goods he sells for 200.00 TL, how much profit will he make when he sells the same goods at a discount for 180 TL?

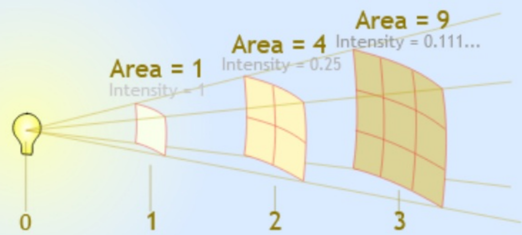


Inverse Square

Inverse Square: when one value **decreases** as the **square** of the other value.

Example: light and distance

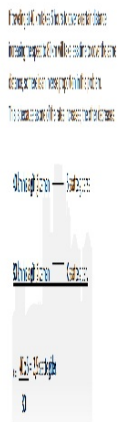
The further away we are from a light, the less bright it is.



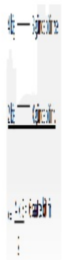
In fact the brightness decreases as the **square** of the distance. Because the light is spreading out in all directions.

So a brightness of "1" at 1 meter is only "0.25" at 2 meters (double the distance leads to a quarter of the brightness), and so on.

Example: If a car travels a certain distance in 5 hours when traveling at a speed of 40 km, how many hours will it take to travel the same distance when the speed is increased to 80 km?



Example 2: If four people can whitewash a house in 3 days, how many days can two people whitewash the same house?



y is directly proportional to x, and y = 24 when x = 4.

What is the value of y when x = 3?

A 18

B 20

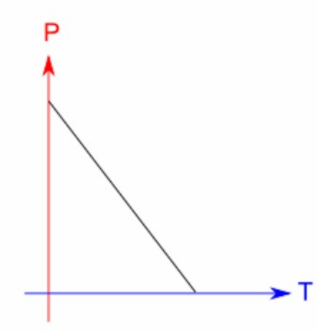
C 23

D 43

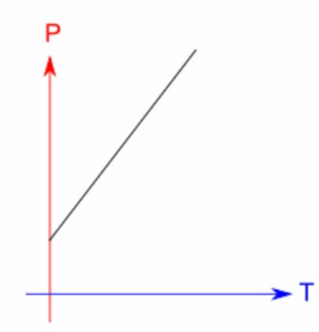
P is directly proportional to T.

Which of the following could be the graph connecting T and P?

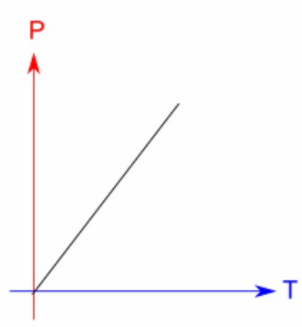
A



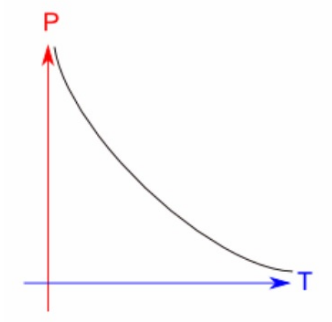
B



C



D



y is inversely proportional to x, and y = 8 when x = 3

What is the value of y when x = 4?

A 6

B 7

C 9

D 10.67

The time (t days) required to build a house is inversely proportional to the number of builders (n), all working at the same rate.

If there are 6 builders, it takes 80 days to complete the house.
How many builders must be employed to build the house in just 16 days?

A 1.2 builders

B 25 builders

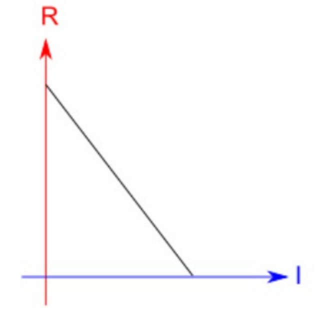
C 30 builders

D 36 builders

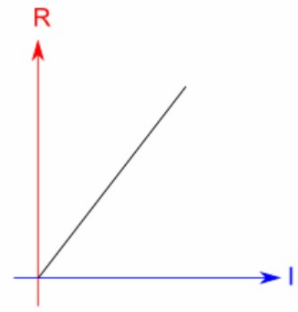
R is inversely proportional to I.

Which of the following could be the graph connecting I and R?

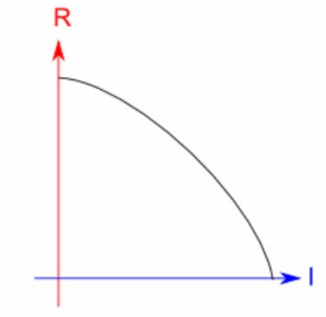
A



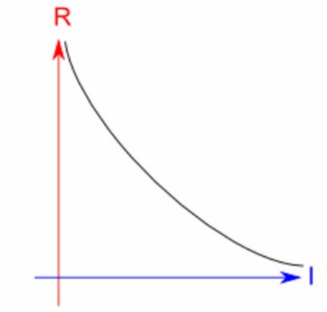
B



C



D



For inverse proportion the graph must show that one variable increases at the same rate as the other variable decreases (and their product is a constant but never zero).

So answer D is the only one that could be correct.

John and Karen both insure their properties with the Budget Insurance Company.
The cost of house insurance premiums with the Budget Insurance Company is directly proportional to the value of the property being insured.

John pays premiums of $76 per month on his property valued at $152,000
How much does Karen pay on her property valued at $259,000?

A $44.60

B $129.50

C $183.00

D $205.50

The amount of sales tax on a new car is directly proportional to the purchase price of the car.
Victor bought a new car for $30,000 and paid $1,500 in sales tax.

Wesley bought a new car from the same dealer and paid $2,375 sales tax.

How much did Wesley pay for his car?

A $18,947	B $30,875
C $45,000	D $47,500

The cost of insurance on a house is directly proportional to the area of the house.
Alan has a 2,500 square-foot house and pays a monthly insurance premium of $67.50
Bella has a 3,800 square-foot house insured with the same company. What is her monthly premium?

A $44.41

B $80.50

C $102.60

D $120.60

The stopping distance (d meters) of a car is directly proportional to the square of its speed (s km/h) when the brakes are applied.

A car travelling at 50 km/h requires a stopping distance of 20 meters.
If the stopping distance is 51.2 meters, what is the speed of the car when the brakes are applied?

A 19.5 km/h

B 80 km/h

C 81.2 km/h

D 128 km/h

The stopping distance (d meters) of a car is directly proportional to the square of its speed (s km/h) when the brakes are applied
$\Rightarrow d \propto s^2$ or $d = ks^2$ where k is the constant of proportionality

$d = 20$ when $s = 50$,

$20 = k \times 50^2 = 2,500k$

$\Rightarrow k = 20 \div 2,500 = 0.008$

$\Rightarrow d = 0.008s^2$

When $d = 51.2$,

$51.2 = 0.008 \times s^2$

$\Rightarrow s^2 = 51.2 \div 0.008 = 6,400$

$\Rightarrow s = \sqrt{6,400} = 80$
So, the car's speed was 80km/h.

The acceleration of a particle is inversely proportional to the square of the time since it was fired.

If the acceleration of the particle 20 seconds after it was fired was 5 m/sec^2, what was its acceleration 5 seconds later?

A 3.2 m/sec^2

B 4 m/sec^2

C 6.4 m/sec^2

D 20 m/sec^2

NEXT TOPIC