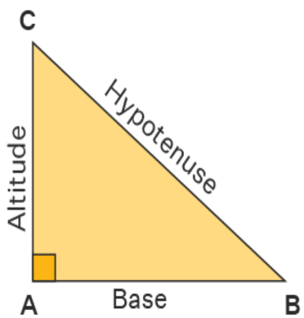
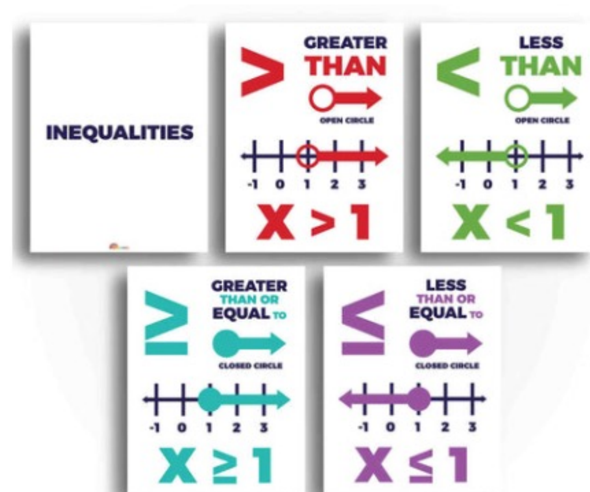
Lecture 2



$BC^2 = AB^2 + AC^2$

- Surd Expressions
- Perfect Square Trinomials
- Theorem of Pythagoras
- Rationalizing the denominator
- Factorization of zero
- Inequalities



$$(a+bi)(a-bi) = a^2 - \cancel{abi} + \cancel{bia} - b^2 i^2$$

Surd Expressions

- A surd is an irrational root that cannot be simplified to a rational (exact decimal or fraction) number.
- The surds have a decimal which goes on forever without repeating, and are Irrational Numbers.
- $\sqrt{2}, \sqrt{3}+1, \frac{\sqrt{13-a}}{\sqrt{b+c}}$, … - surd expressions
- $\sqrt{a} \longrightarrow a \geq 0$
- - $\sqrt{a}$ is possible
- $\sqrt{-a}$ is an imaginary number

Squares and Differences of Squares

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a+b)(a-b)$

$$(a+b)^2 = a^2 + 2ab + b^2$$

- $(2x^2+5)^2 = (2x^2)^2 + 2\times(2x^2)\times(5) + 5^2 =$ $4x^4 + 20x^2 + 25$

$$(a-b)^2 = a^2 - 2ab + b^2$$

- $(3y^2 - 7)^2 = (3y^2)^2 - 2 \times (3y^2) \times (7) + 7^2 =$
  $9y^4 - 42y^2 + 49$

$$a^2 - b^2 = (a+b)(a-b)$$

- $4z^2 - 81 = (2z+9)(2z-9)$

$2z \times 2z$ $\qquad\qquad$ 9×9

$a = 2z$ $\qquad\qquad$ $b = 9$

- $54x^2 - 6y^2 = 6(3x+y)(3x-y)$

Quick Recap

Factor Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a+b)(a+b) = (a+b)^2$$

Square of first term

Twice the product of first and last term

Square of last term

First term

Last term

$$a^2 - 2ab + b^2 = (a-b)(a-b) = (a-b)^2$$

Take note of the signs

Match Column A with its factor in Column B by joining its line.

Column A	Column B
1. $a^2 + 8a + 16$	$(2a + 1)^2$
2. $a^2 - 10a + 25$	$(a + 4)^2$
3. $4a^2 - 4a + 1$	$(2a - 1)^2$
4. $4a^2 - 12a + 9$	$(a - 5)^2$
5. $4a^2 + 4a + 1$	$(2a - 3)^2$
6. $9a^2 + 24a + 16$	$(3a + 4)^2$
7. $a^2 + 12a + 36$	$(a - 6)^2$
8. $a^2 - 12a + 36$	$(a + 6)^2$
9. $a^2 - 8a + 16$	$(a + 5)^2$
10. $a^2 + 10a + 25$	$(a - 4)^2$

Complete the following expression to make a perfect square trinomial.

1) $a^2 + 8a +$ ________
2) $a^2 - 10a +$ ________
3) $a^2 + 4a +$ ________
4) $a^2 - 24a +$ ________
5) $a^2 - 4a +$ ________
6) $a^2 + 24a +$ ________
7) $a^2 + 12a +$ ________
8) $a^2 - 12a +$ ________
9) $a^2 - 6a +$ ________
10) $a^2 + 10a +$ ________

Practice

- $64n^2 - 32n + 4$
- $10 - 140n + 490n^2$
- $9 + 18p + 9p^2$
- $x^3y - 4xy^3$
- $288b^2 + 672b + 392$
- $96v^2 + 48v + 6$
- $640v^2 + 1120v + 490$
- $49x^2 + 84x + 36$
- $25x^2 + 9$
- $2y^5 - 162y$
- $9x^2 - 12x + 4$
- $81x^2 - 180x + 100$
- $36x^2 + 132x + 121$
- $x^2 - \frac{9}{64}$

$64x^2 - 9$	$144x^2 - 169$	$(4x-5)^2$	$(3k+1)^2$
$x^2 - 121$	$25x^2 - 9$	$5(4p+5)^2$	$3(3x+1)^2$
$3x^2 - 9$	$25x^2 - 10$	$4(5p-1)^2$	$2(x-4)^2$
$3x^2 - 75$	$14x^2 - 7$	$(4n-3)^2$	
$25x^2 - 50$	$12x^2 - 27$	$5(5x-4)^2$	

$80p^2 + 200p + 125$

$m^2 + 10m + 25$

$16n^2 - 24n + 9$

$4b^2 + 20b + 25$

$27x^2 + 18x + 3$

$100n^2 - 80n + 16$

$100p^2 - 40p + 4$

YOU can DO IT!

$2x^2 - 16x + 32$

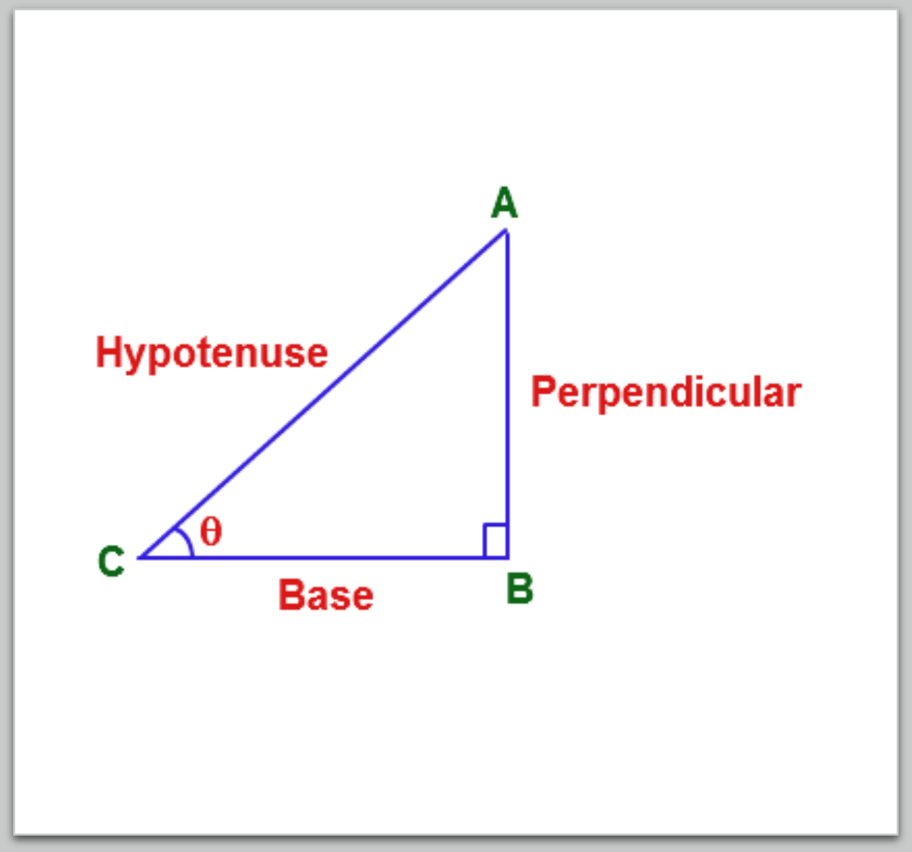
$125x^2 - 200x + 80$

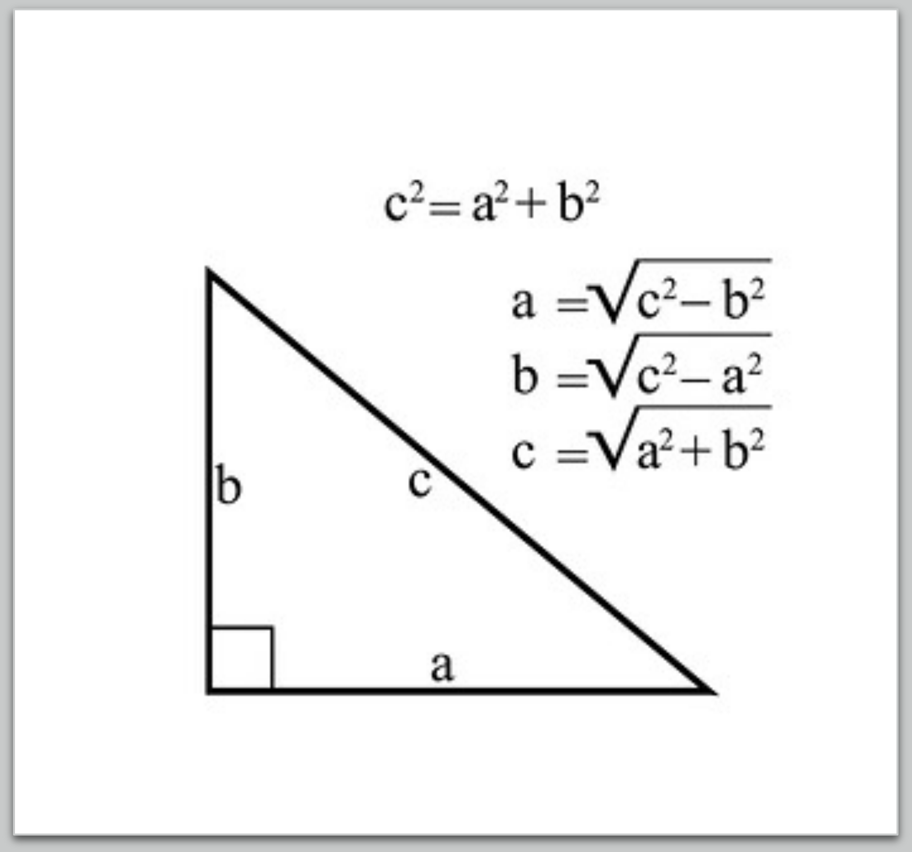
$16x^2 - 40x + 25$

$9k^2 + 6k + 1$

$4x^2 - 8x + 4$

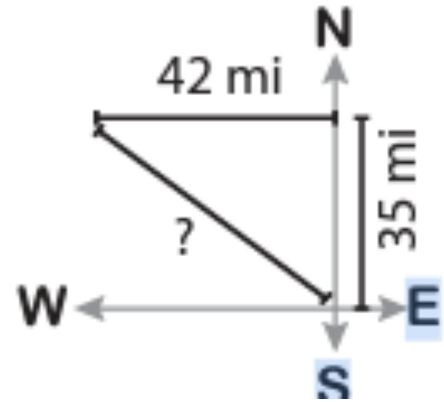
PYTHAGORAS' THEOREM



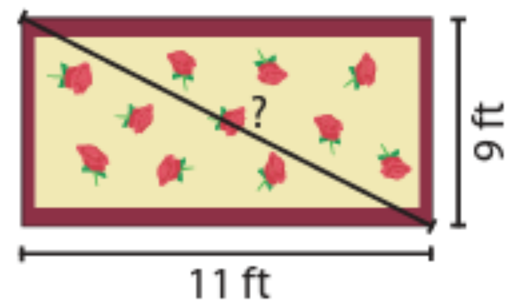


Practice

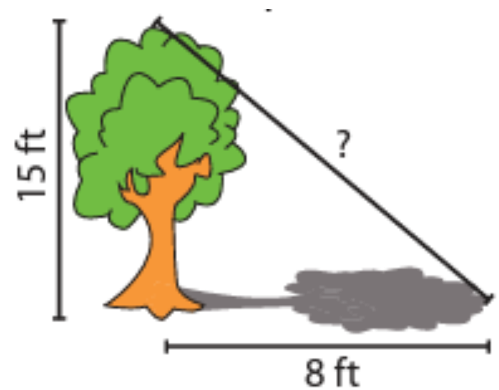
Adam is on his way home from work. He drives 35 miles due North and then 42 miles due West. Find the shortest distance he can cover to reach home early.

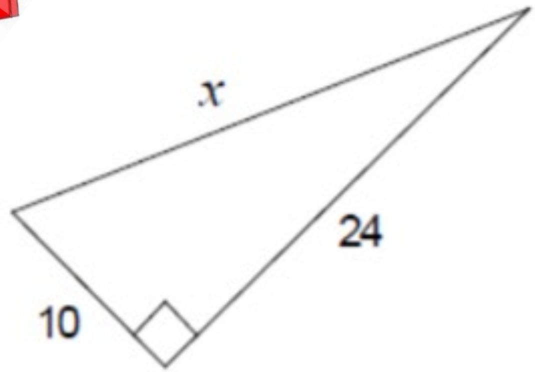


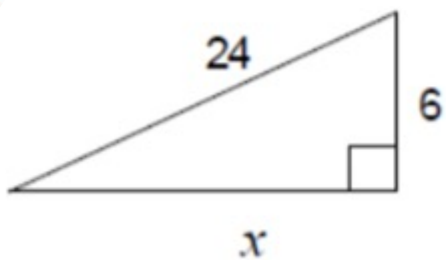
Rachel bought a rug for her apartment. The rug is 11 feet long and 9 feet wide. Find the diagonal length of the rug.

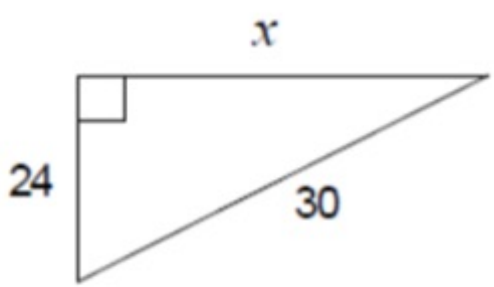


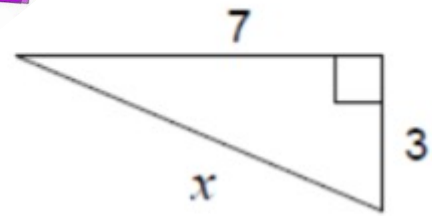
A 15 feet tree casts a shadow that is 8 feet long. What is the distance from the tip of the tree to the tip of its shadow?

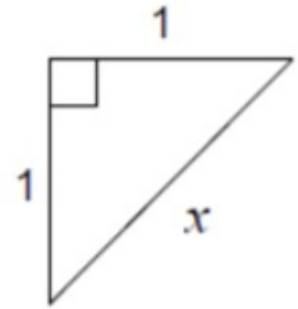


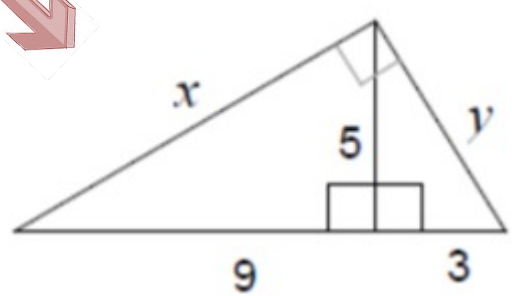

x
24
10


24
6
x


x
24
30


7
3
x

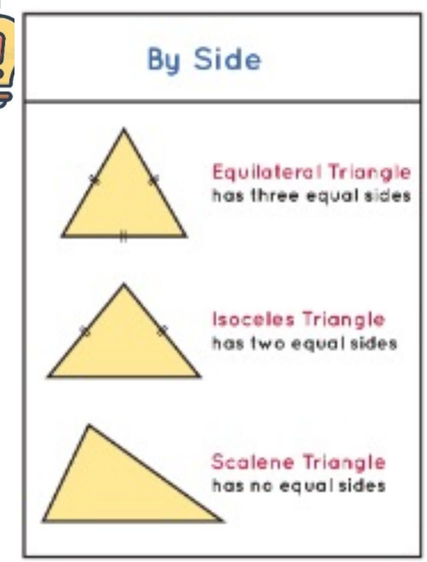

1
1
x


x
5
y
9
3

1. Ms. Green tells you that a right triangle has a hypotenuse of 13 and a leg of 5. She asks you to find the other leg of the triangle. What is your answer?

2. Two joggers run 8 miles north and then 5 miles west. What is the shortest distance, to the *nearest tenth* of a mile, they must travel to return to their starting point?

3. Oscar's dog house is shaped like a tent. The slanted sides are both 5 feet long and the bottom of the house is 6 feet across. What is the height of his dog house, in feet, at its tallest point?



4. To get from point A to point B you must avoid walking through a pond. To avoid the pond, you must walk 34 meters south and 41 meters east. To the *nearest meter*, how many meters would be saved if it were possible to walk through the pond?

Rationalizing the denominator

- $\frac{1}{a\pm\sqrt{b}} = \left(\frac{1}{a\pm\sqrt{b}}\right)\left(\frac{a\mp\sqrt{b}}{a\mp\sqrt{b}}\right) = \frac{a\mp\sqrt{b}}{a^2-b}$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

Practice

- $\frac{2}{\sqrt{2}}$
- $\frac{1}{3-\sqrt{5}}$
- $\frac{1}{\sqrt{9}-\sqrt{8}}$
- $\frac{y+1}{5+2\sqrt{11}}$
- $\frac{5}{\sqrt{5}}$
- $\frac{2}{4+\sqrt{3}}$
- $\frac{\sqrt{8}}{\sqrt{24}}$
- $\frac{x-2}{6-7\sqrt{2}}$
- $\frac{\sqrt{5}}{\sqrt{45}}$
- $\frac{6}{5-\sqrt{2}}$
- $\frac{x}{4-3\sqrt{7}}$
- $\frac{1}{\sqrt{x}-\sqrt{y}}$

Factorization of Zero

$$(x-1)(x+3)=0$$

$$\swarrow \qquad\qquad \searrow$$

$x-1=0$ or $x+3=0$

$$x=1 \qquad\qquad x=-3$$

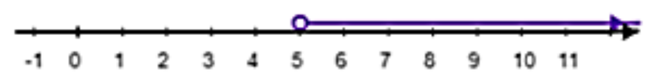
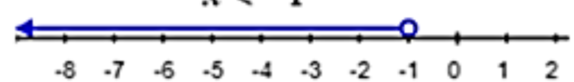
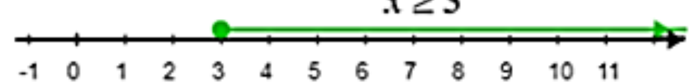
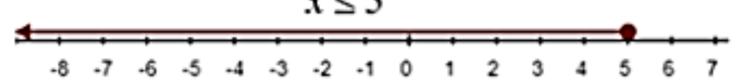
Practice

- $(2x+1)(3x-4)$
- $(-\frac{2}{5}x+1)(4x+4)$
- $3(\frac{4x}{5}-\frac{1}{7})(1\frac{1}{3}-\frac{3x}{10})$
- $9(\frac{x}{11}-2\frac{2}{5})(\frac{2}{3}-\frac{3x}{12})$
- $(2\sqrt{3}y-4)(4y+7)$
- $(9\sqrt{5}z-13)(24z+43)$
- $(7-3\sqrt{5}y)(3y+\frac{2}{3})(\frac{15y}{17}+\frac{34}{55})$

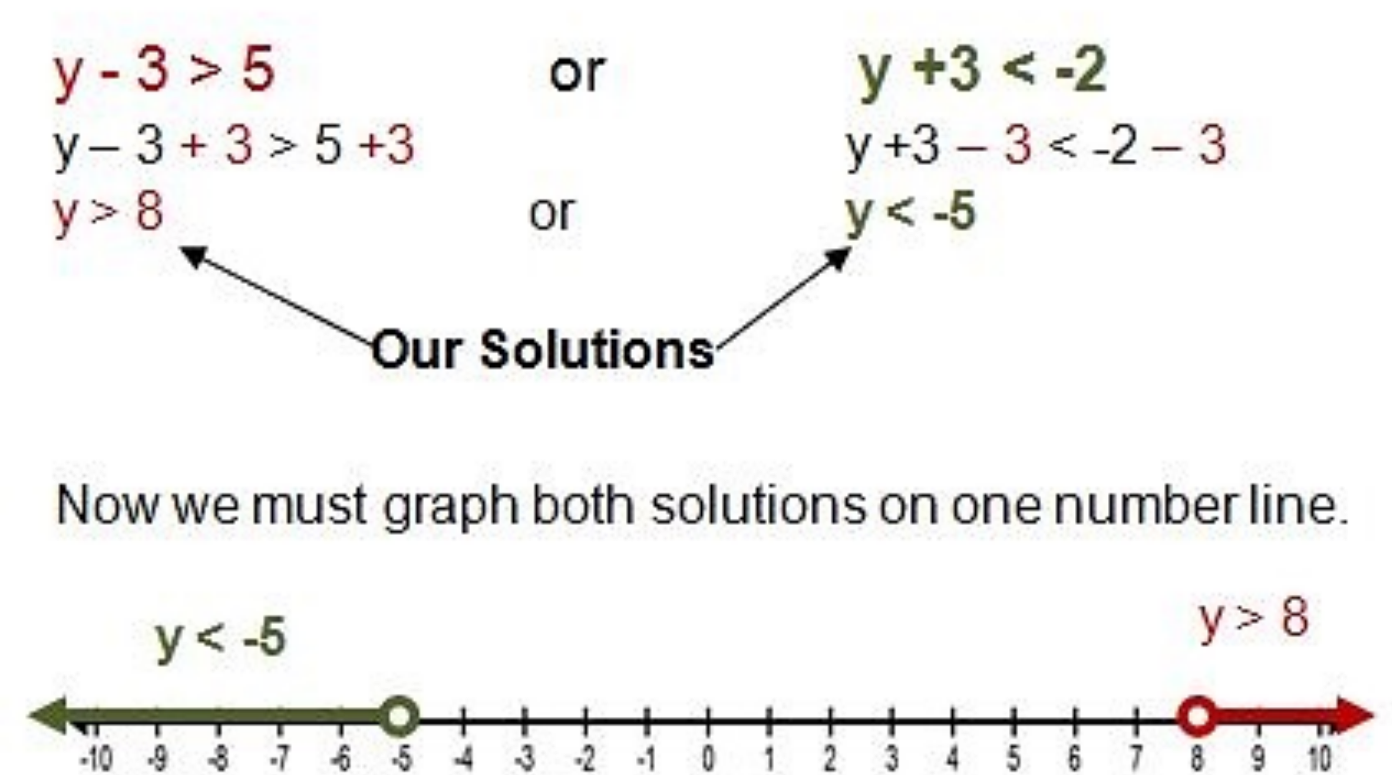
Inequalities

We can say that a *set* is a collection of objects, and the objects in a set are called the elements of a set. The notation is $S = \{x\colon \text{statement about } x\}$.

For instance, $S = \{x\colon\ x - 1 > 0\}$.

Symbol	Words	Example
>	Greater than	$x > 5$ 
<	Less than	$x < -1$ 
≥	Greater than or equal to	$x \geq 3$ 
≤	Less than or equal to	$x \leq 5$ 

Inequalities



An *interval* is simply a segment on the real line. If its endpoints are the numbers a and b, then the interval consists of all numbers that lie between a and b.

Set Notation	Set Definition	Name
$[a, b]$	$\{x \mid a \leq x \leq b\}$	closed interval
(a, b)	$\{x \mid a < x < b\}$	open interval
$[a, b)$	$\{x \mid a \leq x < b\}$	left-closed, right-open
$(a, b]$	$\{x \mid a < x \leq b\}$	left-open, right-closed
(a, ∞)	$\{x \mid x > a\}$	left-open, unbounded
$[a, \infty)$	$\{x \mid x \geq a\}$	left-closed,unbounded
$(-\infty, a)$	$\{x \mid x < a\}$	unbounded, right-open
$(-\infty, a]$	$\{x \mid x \leq a\}$	unbounded, right-closed
$(-\infty, \infty)$	$R = \{x \mid -\infty < x < \infty\}$	Set of real numbers

Remark: The notation (a, b), where $a < b$, has two different meanings. It denotes an ordered pair as well as an interval. To avoid ambiguity, some authors use $]a, b[$ to denote the open interval $\{x \in \mathbb{R} : \ a < x < b\}$.

Practice

If 5 times a number is increased by 4, the result is at least 19. Find the least possible number that satisfies these conditions.

The sum of twice a number and 5 is at most 15. What are the possible values for the number?

Three times a number increased by 8 is no more than the number decreased by 4. Find the number.

The cost of a gallon of orange juice is $3.50. What is the maximum number of containers you can buy for $15?

Question 7: Find the smallest integer that satisfies each inequality below.

(a) $2x - 5 \geq 12$ (b) $4x > 9$ (c) $\frac{x+9}{3} \geq 7$

(d) $7x + 1 > 60$ (e) $10x - 16 \geq 76$ (f) $9x + 4 > 7x + 15$

Question 8: Solve each of the inequalities below

(a) $6 < x + 3 < 10$ (b) $4 \leq 2x \leq 7$ (c) $1 \leq 3x < 9$

(d) $4 < \frac{x}{5} < 6$ (e) $9 \leq 2x + 3 \leq 25$ (f) $-3 \leq \frac{x}{4} - 1 < 0$

Question 9: Find the integers that satisfy each of the inequalities below

(a) $5 < x < 9$ (b) $-3 < x \leq 1$ (c) $4 \leq 2x \leq 8$

(d) $16 \leq 5x + 1 < 31$ (e) $0 \leq \frac{x-6}{2} < 2$ (f) $-9 < \frac{x}{4} - 1 < -8$

Practice Time

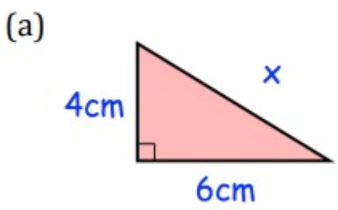
Perfect Square Trinomials & Difference of two squares

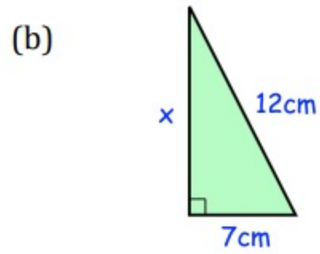
- $(4m^3 + n^3)(4m^3 - n^3)$
- $144x^2 + 264x + 121$
- $(3x + 1)(3x - 1)$
- $36m^2 - 121$
- $6(7r + 5)(7r - 5)$
- $x^2 + 18x + 81$
- $36x^2 + 132x + 121$
- $(2y + 5)^2$
- $(5x^2 - 9)^2$

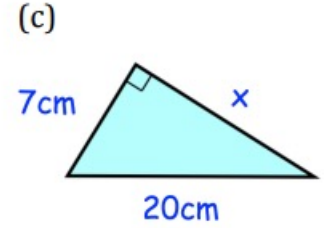
- $18z^2 - 96z + 128$
- $45a^2 - 240ay + 320y^2$
- $5(5m^2 + 2n^2)(5m^2 - 2n^2)$
- $x^2 - 9y^2$
- $36v^2 - 132v + 121$
- $(3x + 2)^2$
- $4a^2 - 81b^2$
- $12x^2 - 75$
- $a^2b - b^3$

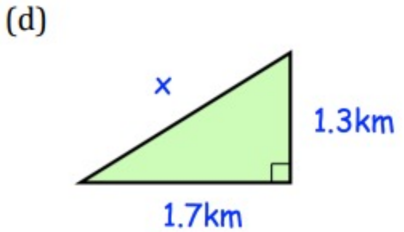
- $2k^2 - 32km + 128m^2$
- $9x^4 - 4$
- $121n^2 - 110n + 25$
- $50k^2 - 160k + 128$
- $6(2x + 3)(2x - 3)$
- $-x^2 + 16$
- $(5p + 3)^2$
- $(8y - 2)^2$
- $(4z^2 + 8)^2$

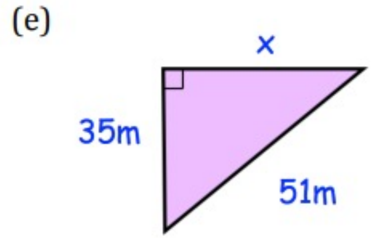
Pythagorean theorem

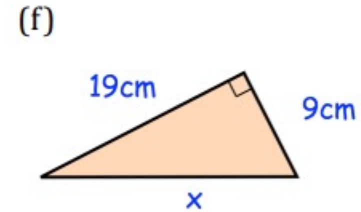


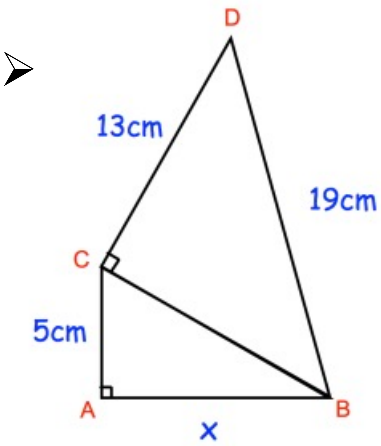


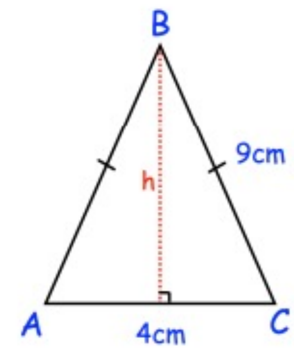






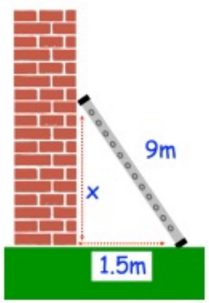


- 

- 

- To wash a window that is 8 meters off the ground, Aisha leans a 10 meter ladder against the side of the building. To reach the window, how far from the building should Aisha place the base of the ladder?

- A 9m ladder is placed against a wall.
  The foot of the ladder is 1.5m from the foot of the wall.
  How far up the wall does the ladder reach?



- A rectangle is 20cm long and 8cm wide.
  Find the length of the diagonal of the rectangle.

Rationalizing the Denominator & Factorization of Zero

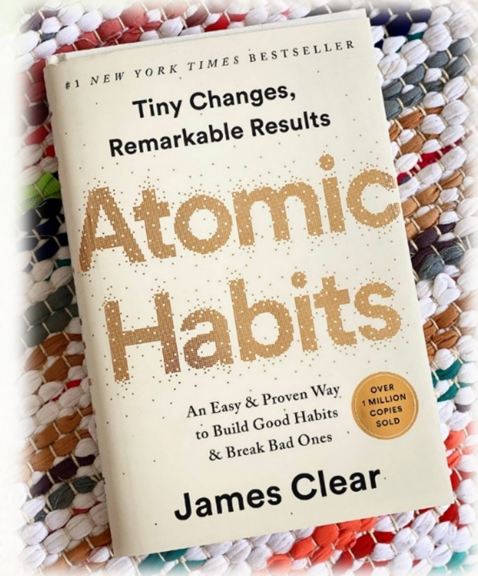
- $\dfrac{33}{4-\sqrt{5}}$
- $\dfrac{\sqrt{5}-7}{\sqrt{5}+1}$
- $\dfrac{20-\sqrt{50}}{3\sqrt{2}-5}$
- $\dfrac{\sqrt{2}+5}{\sqrt{3}-\sqrt{5}}$
- $\dfrac{17\sqrt{3}+5\sqrt{5}}{2\sqrt{3}-\sqrt{5}}$
- $\dfrac{3\sqrt{x}}{2\sqrt{x}+\sqrt{y}}$
- $\dfrac{\sqrt{7}}{\sqrt{45}}$
- $\dfrac{8}{3\sqrt{x}}$

- $x(x+2)(x-2)(3x^2-4)$
- $4(\frac{3y}{12}+2\,\frac{3}{7})(\frac{5}{9}-\frac{3y}{15})$
- $(6\sqrt{3}x-18)(16x+70)$
- $x(2x-1)(x-1)(x+1)$
- $(13-2\sqrt{8}y)(5y+\frac{6}{13})(\frac{14y}{23}-\frac{46}{77})$

Inequalities

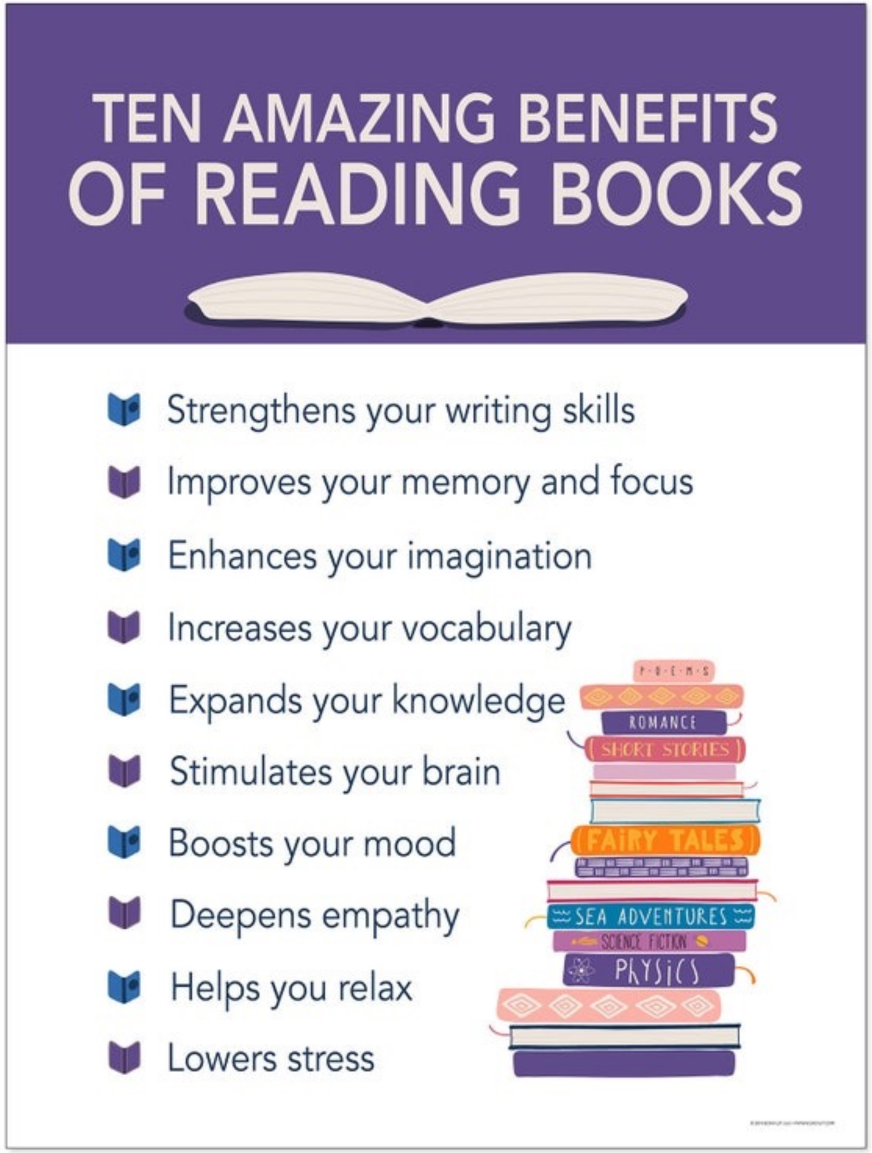
- $8\frac{1}{2} - x < x + 4\frac{5}{6}$
- $9 \leq 2x + 3 \leq 25$
- $\frac{x+3}{2} \geq 5$
- $4(x+1) < 2x + 3$
- $2(2x - 9) \geq 22$
- $\frac{x}{2} + 1 \leq 5$
- $-6 + 1\frac{4}{5}x \leq -1\frac{1}{3}x + 1\frac{4}{5}x$
- $-3\frac{2}{9}n + 1\frac{1}{2} < -2\frac{13}{18} + n$
- $4x + 8 < 32$
- $9 - 2x \geq -7x - 4x$
- $\frac{x+3}{2} \geq 5$
- $1 - \frac{3}{2}x \geq x - 4$
- $13x - 12 < 3x + 13$
- $r + 1\frac{5}{8} \leq -1\frac{3}{4}r - 1\frac{1}{8}$
- $a + 1\frac{7}{10} > 2a - 1\frac{1}{20}$
- $2(4x - 1) < 38$
- $0 \leq \frac{x-6}{2} < 2$
- $\frac{x-5}{4} > 2$
- $-9 - \frac{x}{2} \geq \frac{x}{2} + 1$
- $6(x+2) < 42$
- $n - \frac{31}{9} - 9\frac{3}{10} + 13\frac{89}{180} \leq 2n + 1\frac{3}{4}$

Atomic Habits is the definitive guide to **breaking bad behaviors** and **adopting good ones** in four steps, showing you how small, incremental, everyday routines compound into massive, positive change over time.



Why must we read?

Reading is good for you because it **improves** your **focus**, **memory**, **empathy**, and **communication skills**. It can reduce stress, improve your mental health, and help you live longer. Reading also allows you to learn new things to help you succeed in your *work* and *relationships*.


TEN AMAZING BENEFITS
OF READING BOOKS
Strengthens your writing skills
Improves your memory and focus
Enhances your imagination
Increases your vocabulary
Expands your knowledge
Stimulates your brain
Boosts your mood
Deepens empathy
Helps you relax
Lowers stress
POEMS
ROMANCE
SHORT STORIES
FAIRY TALES
SEA ADVENTURES
SCIENCE FICTION
PHYSICS