Tishk International University
Faculty of Science
Information technology Department

Chapter Three
Scalar and Vector Quantities

1

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GENERAL PHYSICS I (CMPE 171)
Week 2
Fall Semester
Date

Outline

- **Coordinate systems**
  - Cartesian coordinate systems (2D)
  - Plane polar coordinate system (2D)
  - Conversion between the two systems
- **Math review**
  - Trigonometry
  - Pythagorean theorem
  - Cosine law
  - Sines law
- **Scalar and vector quantities**
  - Scalar quantities
  - Vector quantities
  - Components of a vector
  - Unit vectors
- **Vector applications**
  - Adding vectors
  - Subtracting vectors
  - Vector multiplication

Objectives

- Learning to apply Cartesian and Plane polar coordinate systems.
- Acquiring the capability to employ Pythagorean's Theorem, the Law of Sines, and the Law of Cosines to solve mathematical challenges.
- Gaining a thorough understanding of scalar and vector quantities, including the skill to decompose vectors into their respective components.
- Exploring the mathematical applications of vector quantities, including multiplication of vector quantities.

Coordinating systems

COORDINATE SYSTEMS

- A coordinate system is **used to determine** each point uniquely in a **plane.**

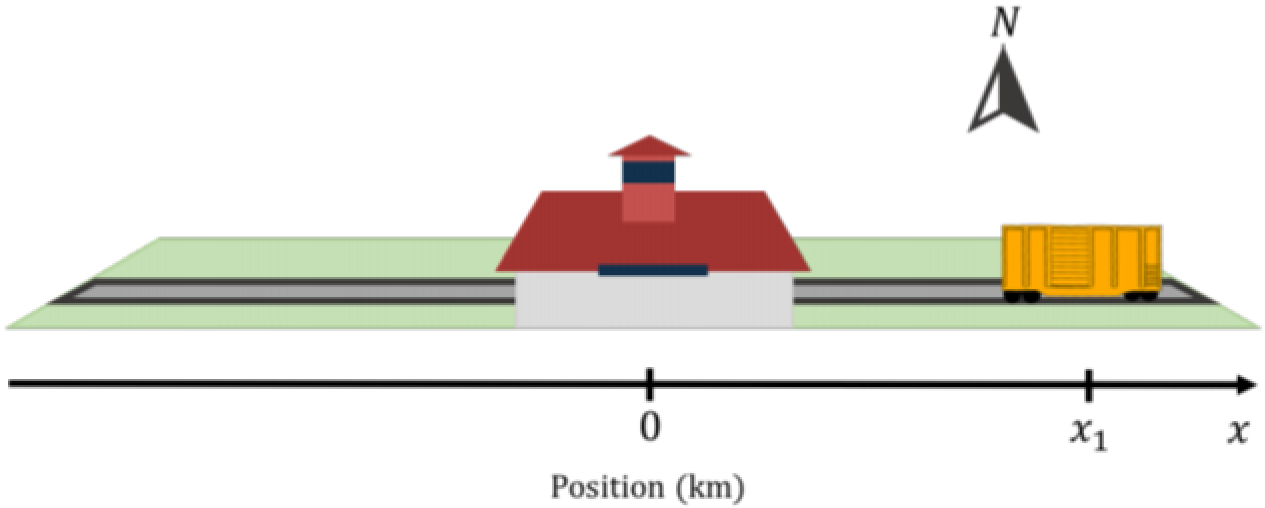
- A coordinate system is an artificial mathematical tool that we construct in order to describe the **position of a real object**.

- Coordinate system (**frame**) consists of:
  - a fixed **reference** point called the origin.
  - specific axes with scales and **labels.**
  - instructions on how to label a point relative to the origin and the axes.

- **Coordinating system can be**:
  - **1D Coordinate systems**
  - **2D Coordinate systems**
  - **3D Coordinate systems**

1D Coordinate systems

- The easiest coordinate system to construct is one that we can use to describe the location of objects in one dimensional space.
- For example, we may wish to describe the location of a train along a straight section of track that runs in the East-West direction.
- First define an “origin”, (x=0) which is the reference point of our coordinate system.
- We can describe the position of the train by specifying how far it is from the train station (the origin), using a single real number, say X-direction.



2D Coordinate systems

- To **describe** the position of an object in **two dimensions**

- We need to specify two numbers to define two axes, x and y, whose origin and direction we must define.

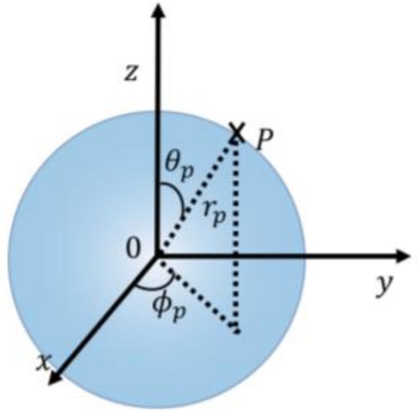
- **Examples of 2D** coordinating system:
  - **‘’Cartesian” coordinate system, and**
  - **“Polar” coordinate system**

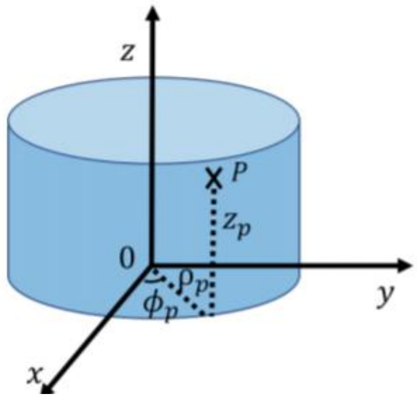
3D Coordinate systems

- In three dimensions, we need to specify three numbers to describe the position of an object (e.g. **a bird flying** in the air).

- In a three dimensional Cartesian coordinate system, we simply add a third axis, **z,** that is mutually perpendicular to both **x and y**.

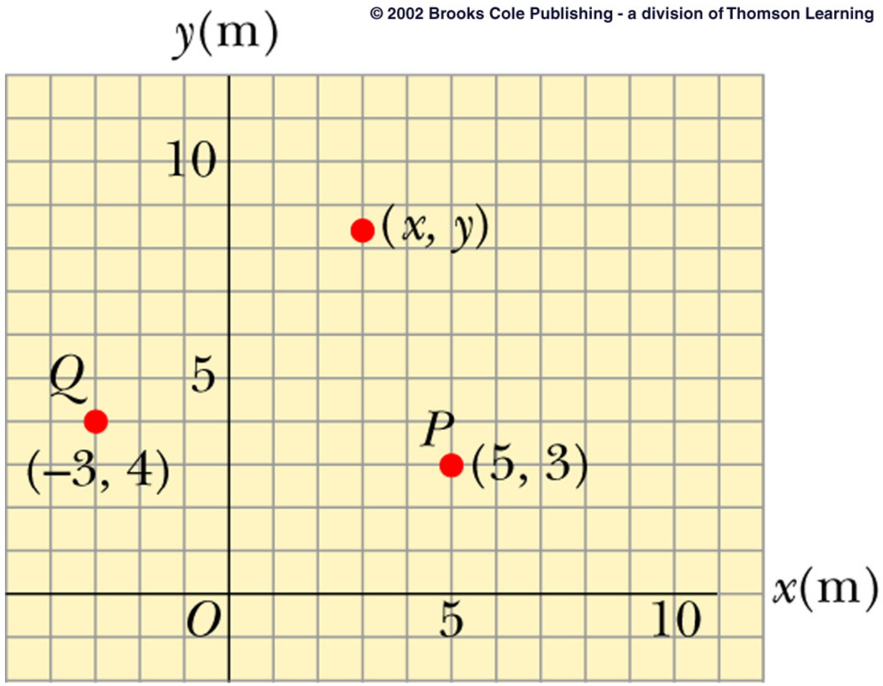
- **Examples of three dimensions:**
  - **“cylindrical” coordinates system and,**
  - **“spherical” coordinates system**





Cartesian Coordinate Systems (2D)

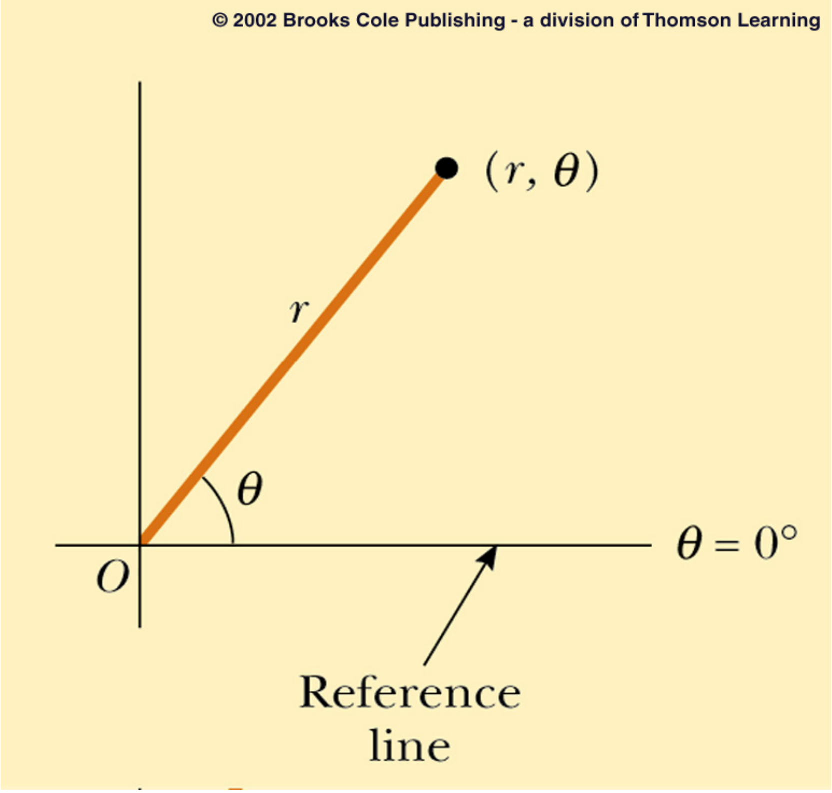
- It is also called rectangular coordinate system
- x- and y- axes
- Points are labeled as (x,y)



Plane polar coordinate system (2D)

Here:

- The origin and reference line are noted.
- The point **(r, θ)** is a distance (r) from the origin in the direction of angle θ.
- The points are labeled as (r, θ)



Conversion between the two systems

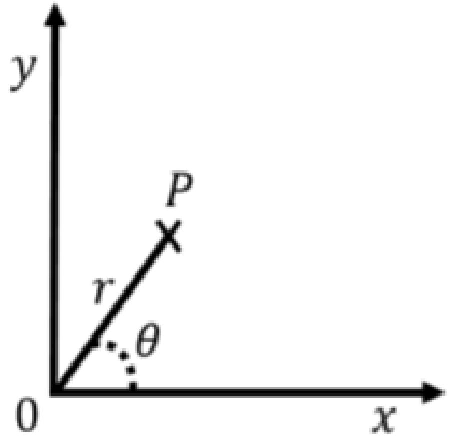
- On a polar coordinating system, the values of r of which corresponded to the x and y-axis can be measured as following:

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

- If x, and y given, the value of r is determined as : $r = \sqrt{x^2 + y^2}$
- The **angle** which the vector r makes with an original axis is measured as below:

$$\tan(\theta) = \frac{y}{x}$$

- Other approaches can contribute to find out one of the four parameters (x, y, r, and θ) on the below diagram.



- These approaches are:

1) **Sine, cosine and tan function or**
2) **Pythagorean Theorem**

- **Note**: These principles are mostly applied to solve mathematical problems of physics.

Math Review
I. Trigonometry

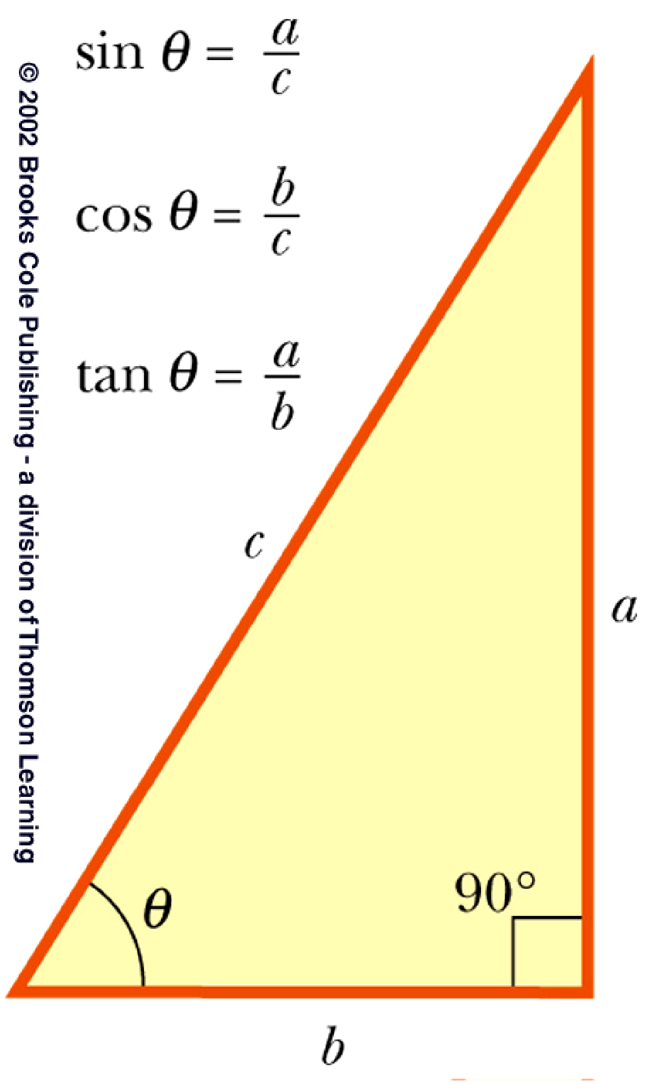
$$\sin\theta = \frac{\textit{opposite side}}{\textit{hypotenuse}}$$

$$\cos\theta = \frac{\textit{adjacent side}}{\textit{hypotenuse}}$$

$$\tan\theta = \frac{\textit{opposite side}}{\textit{adjacent side}}$$

- **Pythagorean Theorem**

$$c^2 = a^2 + b^2$$



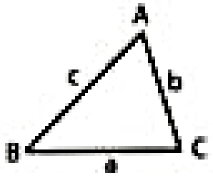
II. Cosine law

The Law of Cosines is useful for finding:

•The third side of a triangle when we know **two sides and the angle between** them (like the example above)

•The angles of a triangle when we know **all three sides**

Law of Cosines



$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

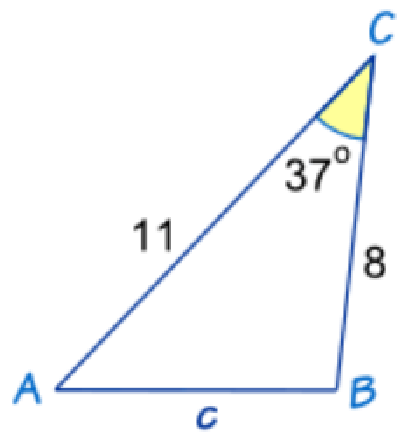
The formula can be rearranged to:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Example 1: determine the value of C between AB



We know angle C = 37º, and sides a = 8 and b = 11

The Law of Cosines says: $c^2 = a^2 + b^2 - 2ab\cos(C)$

Put in the values we know: $c^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos(37^\circ)$

Do some calculations: $c^2 = 64 + 121 - 176 \times 0.798...$

More calculations: $c^2 = 44.44...$

Take the square root: $c = \sqrt{44.44} =$ **6.67** to 2 decimal places

EXAMPLE 2

Find Angle "C" Using The Law of Cosines.

In this triangle we know the three sides:

- a = 8,
- b = 6 and
- c = 7.

Use The Law of Cosines (angle version) to find angle **C** :

$$\cos C = (a^2 + b^2 - c^2)/2ab$$

$$= (8^2 + 6^2 - 7^2)/2\times8\times6$$

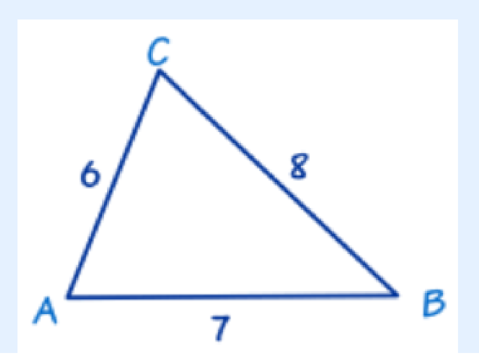
$$= (64 + 36 - 49)/96$$

$$= 51/96$$

$$= 0.53125$$

$$C = \cos^{-1}(0.53125)$$

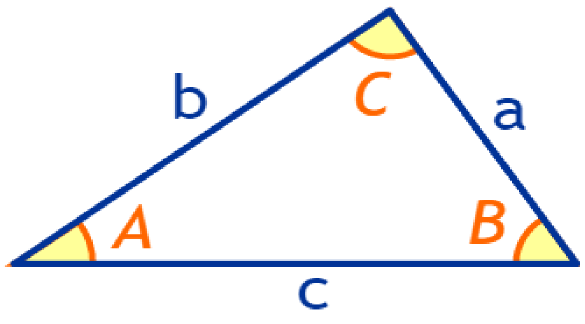
= **57.9°** to one decimal place



III. Sines Law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

It works for any triangle:



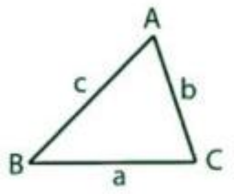
a, **b** and **c** are sides.

A, **B** and **C** are angles.

(Side a faces angle A, side b faces angle B and side c faces angle C).

And it says that:

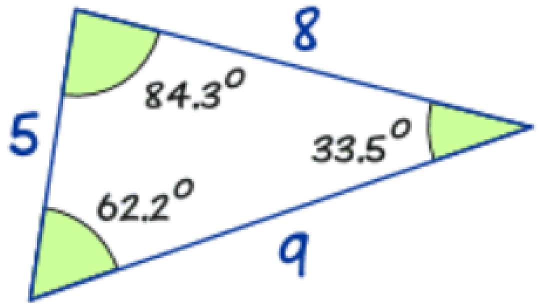
When we **divide side a by the sine of angle A**
it is equal to **side b divided by the sine of angle B**,
and also equal to **side c divided by the sine of angle C**



Law of sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Example 3: Prove sines law from the below diagram



$$\frac{a}{\sin A} = \frac{8}{\sin(62.2°)} = \frac{8}{0.885...} = \mathbf{9.04...}$$

$$\frac{b}{\sin B} = \frac{5}{\sin(33.5°)} = \frac{5}{0.552...} = \mathbf{9.06...}$$

$$\frac{c}{\sin C} = \frac{9}{\sin(84.3°)} = \frac{9}{0.995...} = \mathbf{9.04...}$$

The answers are **almost the same!**

*(They would be **exactly** the same if we used perfect accuracy).*

So now you can see that:

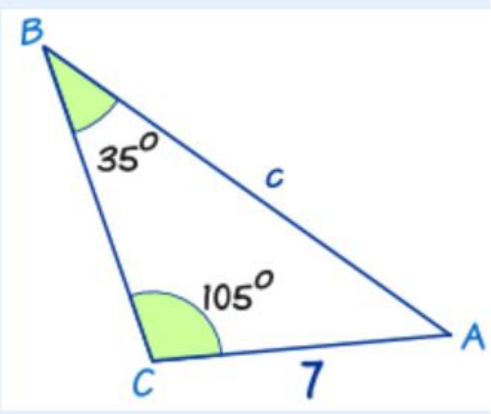
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

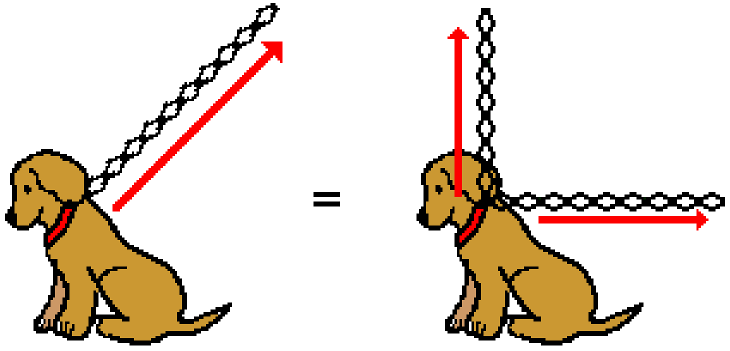
Example 4: Calculate side "c"

Law of Sines:	a/sin A = b/sin B = c/sin C
Put in the values we know:	a/sin A = 7/sin(35°) = c/sin(105°)
Ignore a/sin A (not useful to us):	7/sin(35°) = c/sin(105°)

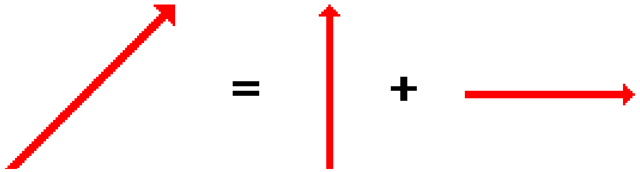
Now we use our algebra skills to rearrange and solve:

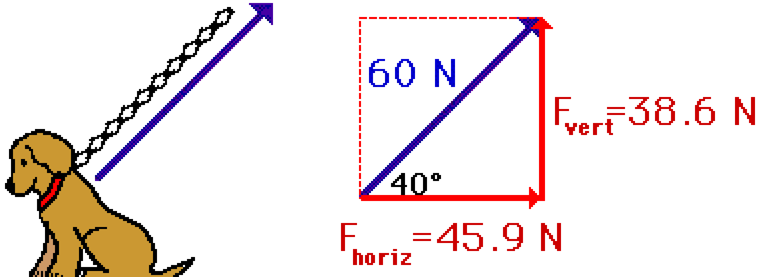
Swap sides:	c/sin(105°) = 7/sin(35°)
Multiply both sides by sin(105°):	c = (7 / sin(35°)) × sin(105°)
Calculate:	c = (7 / 0.574...) × 0.966...
	c = **11.8** (to 1 decimal place)





The upward and rightward force of the chain is equivalent to an upward force and a rightward force by two chains.

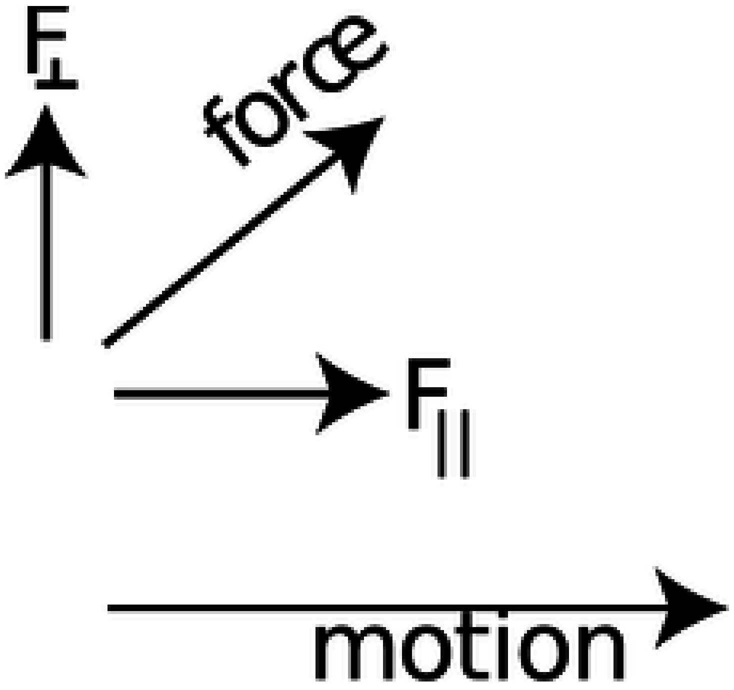




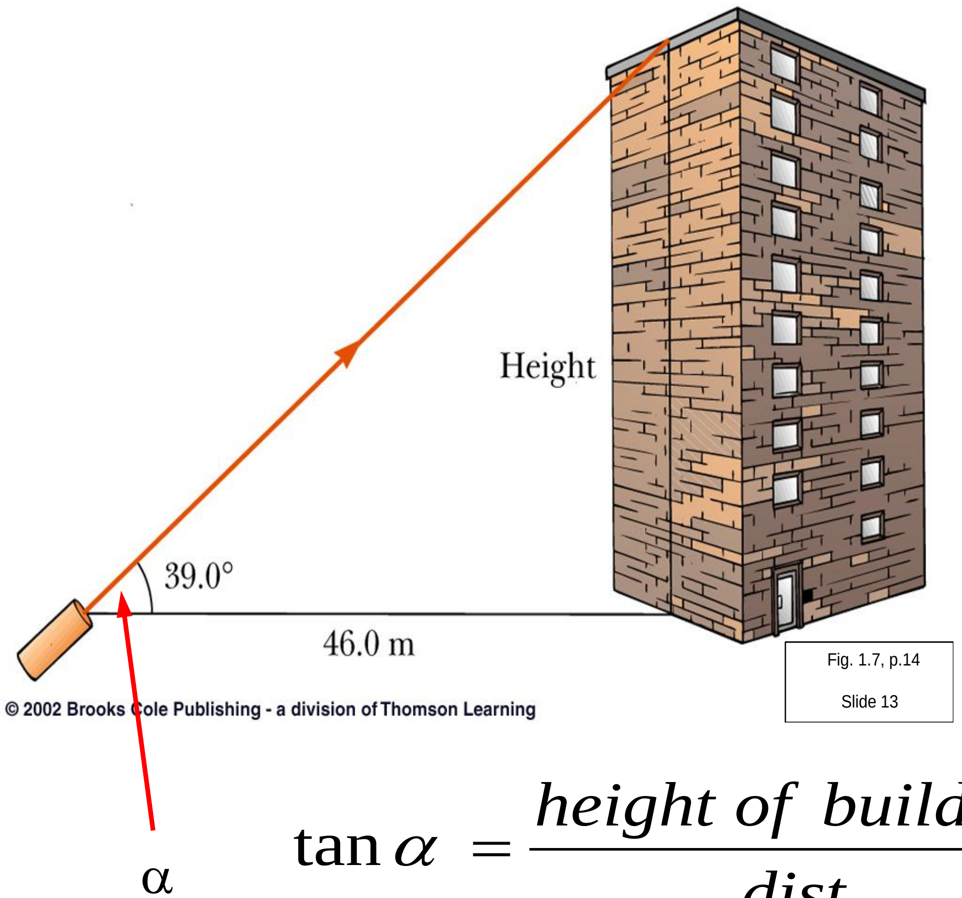
$$\sin 40° = \frac{F_{vert}}{60\ N} \qquad \cos 40° = \frac{F_{horiz}}{60\ N}$$

$$F_{vert} = 60\ N \times \sin 40° \qquad F_{horiz} = 60\ N \times \cos 40°$$

$$F_{vert} = 38.6\ N \qquad F_{horiz} = 45.9\ N$$



Example 5: How high is the building?



Known: angle and one side
Find: another side

Key: tangent is defined via two sides!

$$\tan\alpha = \frac{height\ of\ building}{dist.},$$

$$height = dist. \times \tan\alpha = (\tan 39.0^\circ)(46.0\ m) = 37.3\ m$$

Example 6: Finding Polar Coordinates

If the rectangular coordinates of a point are given by (**3**, **y**) and its polar coordinates are (**r**, **60^o**), determine **y** and **r**.

Note : sin 30 = cos 60 = ½ sin 60 = cos 30 =√3 / 2

Solution:

Y= r sinɵ, x = r cosɵ, x=3

Thus: 3 = r cos60, r = 3/cos60 = 3*2 = 6 m

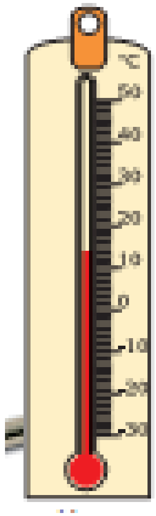
Y = r sin 60 = 6* √3 /2=3 √3 m

Scalar and vector quantities

I. SCALAR QUANTITIES

- **<u>A SCALAR</u>** is a quantity of physics that has MAGNITUDE only, however, direction is not associated with it.
- Magnitude – A numerical value with units.

- Some examples of scalar quantities



Scalar Example	Magnitude
Speed	20 m/s
Distance	10 m
Age	15 years
Heat	1000 calories

II. VECTOR QUANTITIES

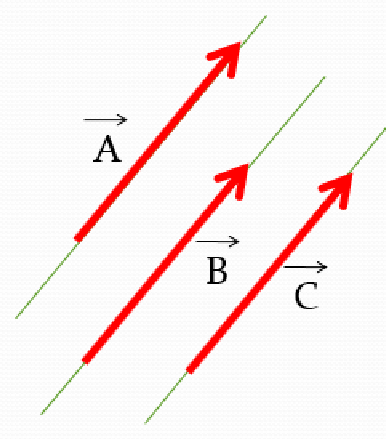
- A **VECTOR** is a quantity which has both MAGNITUDE and DIRECTION.
- Examples: force, displacement, velocity….

Vector	Magnitude & Direction
Velocity	20 m/s, North
Acceleration	10 m/s/s, East
Force	5 N, West

Vectors are typically illustrated by drawing an ARROW above the symbol. The arrow is used to convey direction and magnitude.

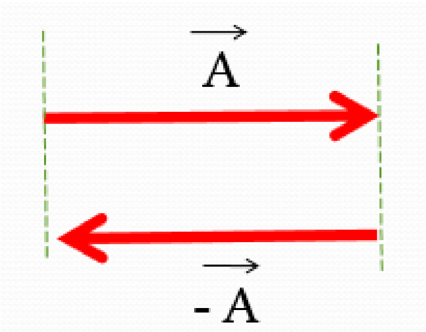
III. Some properties of Vectors

- **Equality of Two Vectors**
  - Two vectors are **equal** if they have **the same magnitude and the same direction.**



$$\vec{A} = \vec{B} = \vec{C}$$

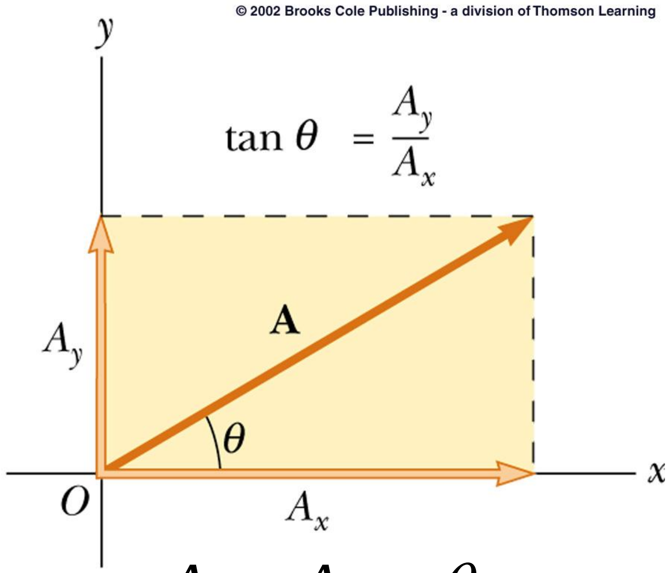
- **Negative Vectors**
  - Two vectors are **negative** if they have the same magnitude but are 180° apart (opposite directions)



IV. COMPONENTS OF A VECTOR

- The projections of a vector on the x and y axis are called the components of the vector.

$$A_y = A\sin\theta$$



$$A_x = A\cos\theta$$

- *The y-component of a vector is the projection along the y-axis.*
- *The x-component of a vector is the projection along the x-axis.*

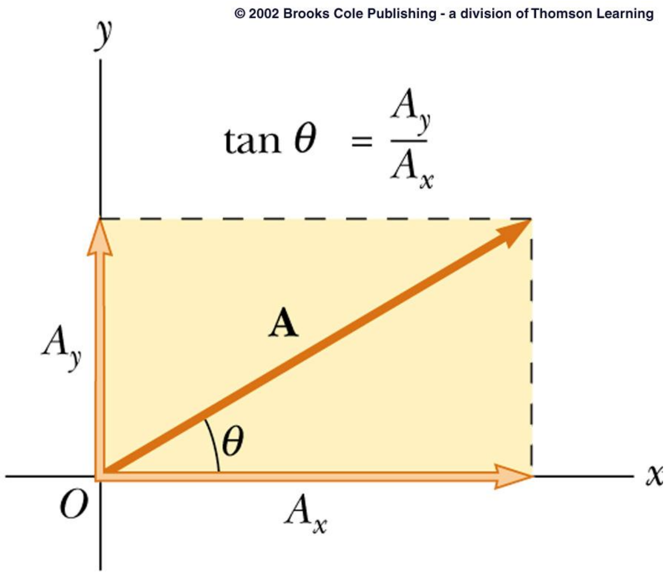
$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

Vectors with two and three components

- Vectors with two components

$$\vec{\mathbf{A}} = A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}}$$

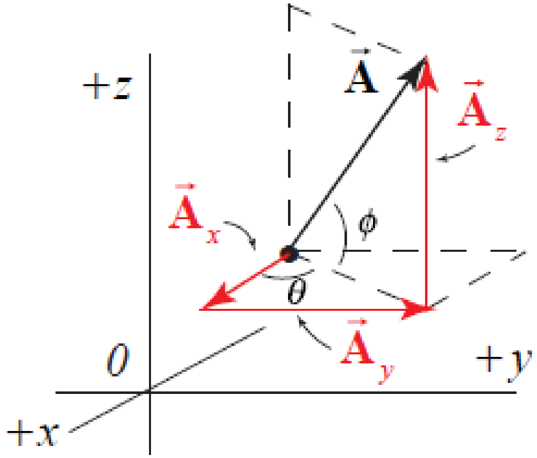
$$A = \sqrt{A_x^2 + A_y^2}$$



- Vectors with three components

$$\vec{\mathbf{A}} = A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}} + A_z\,\hat{\mathbf{k}}\,.$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



TWO-DIMENSIONAL SYSTEMS

Rectangular Components

- Perpendicular projections are also called *orthogonal* projections.

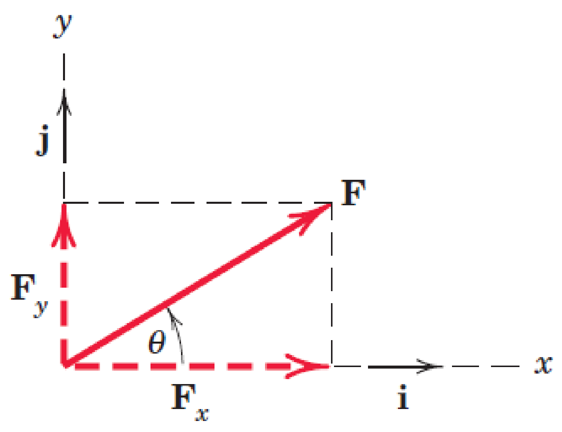
$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \qquad (2/2)$$

where the scalars F_x and F_y are the x and y *scalar components* of the vector **F**.

The scalar components can be positive or negative, depending on the quadrant into which **F** points. For the force vector of Fig. 2/5, the x and y scalar components are both positive and are related to the magnitude and direction of **F** by

$$F_x = F\cos\theta \qquad F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x} \qquad (2/3)$$



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THREE-DIMENSIONAL SYSTEMS

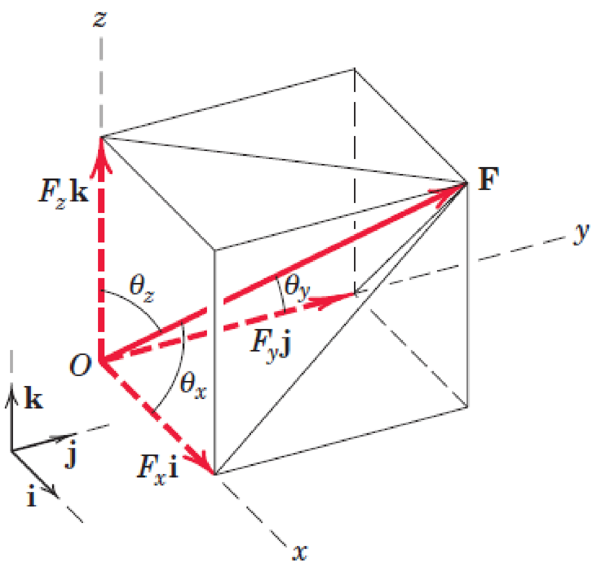


Figure 2/16

Many problems in mechanics require analysis in three dimensions, and for such problems it is often necessary to resolve a force into its three mutually perpendicular components. The force **F** acting at point O in Fig. 2/16 has the *rectangular components* F_x, F_y, F_z, where

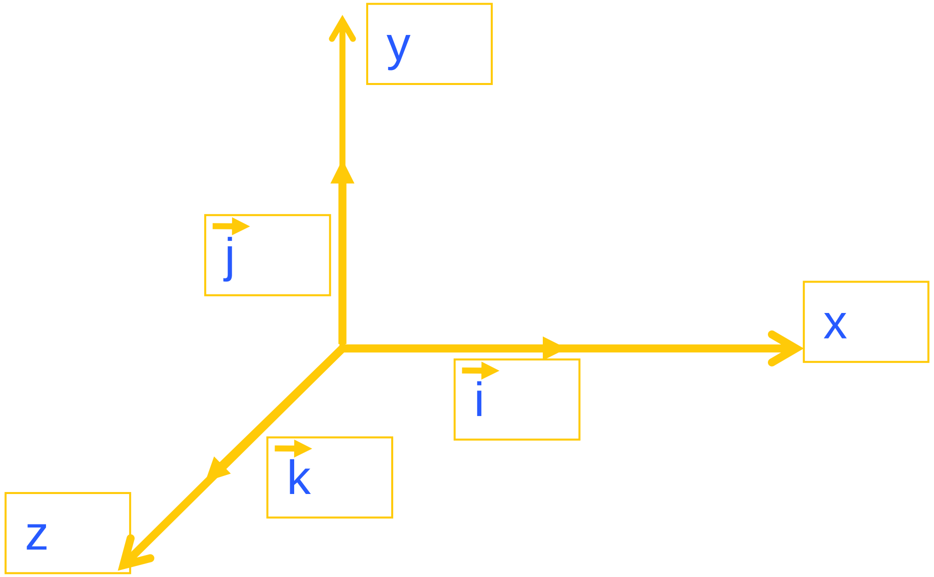
$$F_x = F\cos\theta_x \qquad F = \sqrt{F_x{}^2 + F_y{}^2 + F_z{}^2}$$
$$F_y = F\cos\theta_y \qquad \mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \qquad (2/11)$$
$$F_z = F\cos\theta_z \qquad \mathbf{F} = F(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z)$$

The unit vectors **i**, **j**, and **k** are in the x-, y-, and z-directions, respectively. Using the direction cosines of **F**, which are $l = \cos\theta_x$, $m = \cos\theta_y$, and $n = \cos\theta_z$, where $l^2 + m^2 + n^2 = 1$, we may write the force as

$$\mathbf{F} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k}) \qquad (2/12)$$

V. Unit Vectors

- Vector quantities are often expressed in terms of unit vector.
- A unit vector is a dimensionless vector having a magnitude of exactly one.
- Unit vectors are used to specify a given direction and have no other physical significance.
- Symbols **i, j and k** are used for unit vectors and pointing in the positive x, y and z direction respectively.



VECTOR APPLICATIONS

VECTOR APPLICATIONS

You will study:

1. **Adding vectors**
2. **Subtracting vectors**
3. **Vector multiplication**

Note: There are two approaches of vector applications:

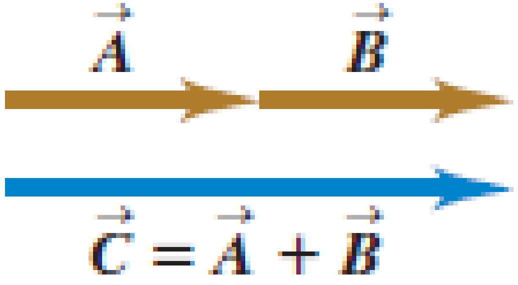
1. Graphical Methods: the vectors are plotted.
   - Use scale drawings.

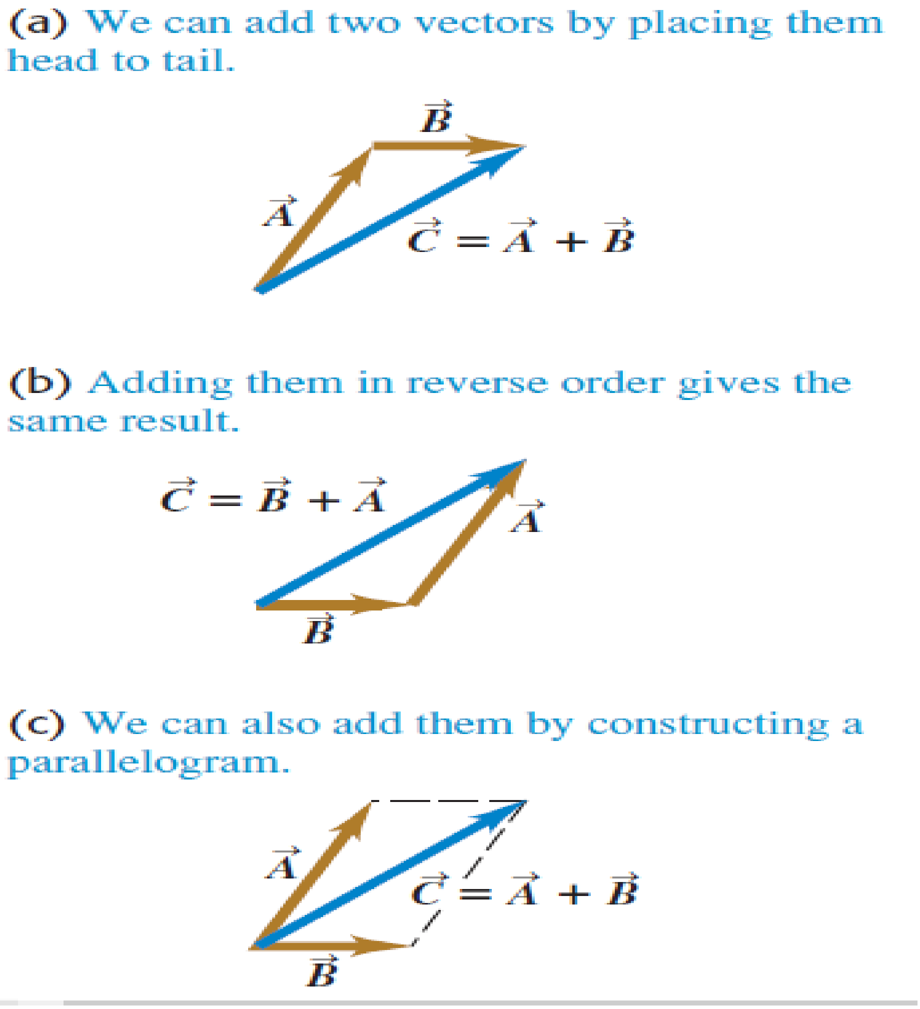
2. Algebraic Methods: mathematics is used.
   - More convenient

Vectors application graphically

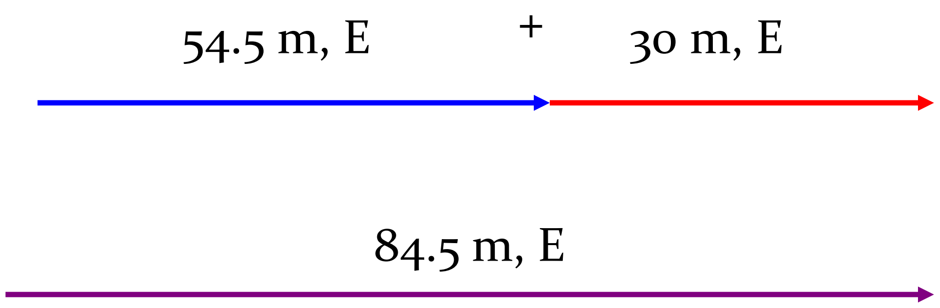
I. Adding Vectors graphically

- When adding vectors, their **directions must be taken into consideration.**
- Units must be the same.





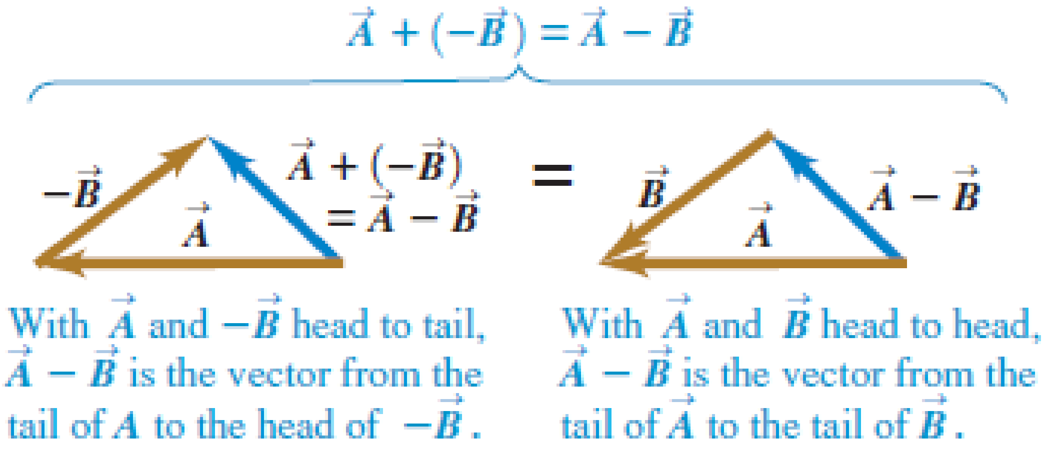
- **Example 7**: **A man walks 54.5 meters east, then another 30 meters east. Calculate his displacement relative to where he started?**



Notice that the SIZE of the arrow conveys MAGNITUDE and the way it was drawn conveys DIRECTION.

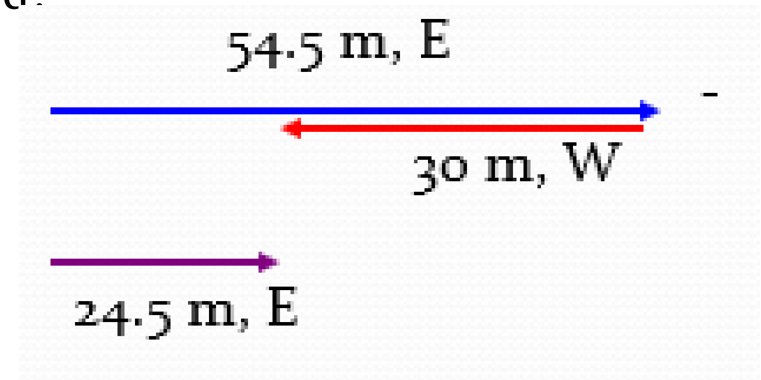
II. Subtracting vectors graphically

- The vectors **A** & **–A** have the same magnitude but point in opposite directions. Therefore when we add negative vectors we get zero.



- **Example 8:** A man walks 54.5 meters east, then 30 meters west. Calculate his displacement relative to where he started?

Solution

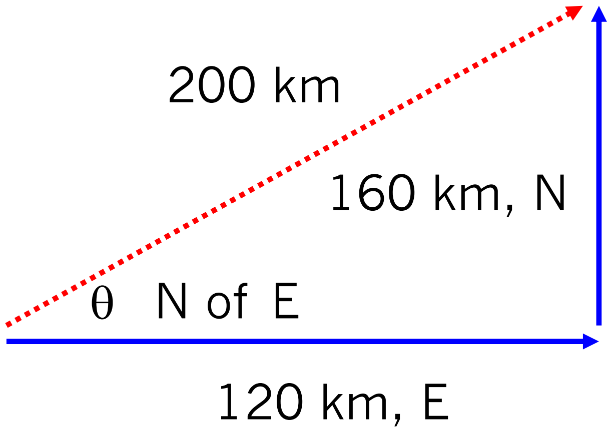


Perpendicular vectors graphically

- When two vectors are perpendicular, you can use the Pythagorean theorem if the angle is not given.
- Just putting N of E is not good enough (how far north of east ?).
- We need to find a numeric value for the direction.

- To find the value of the angle we use a Trig function called TANGENT.

Example 9: From the below diagram, find the angle between the resultant and east components.



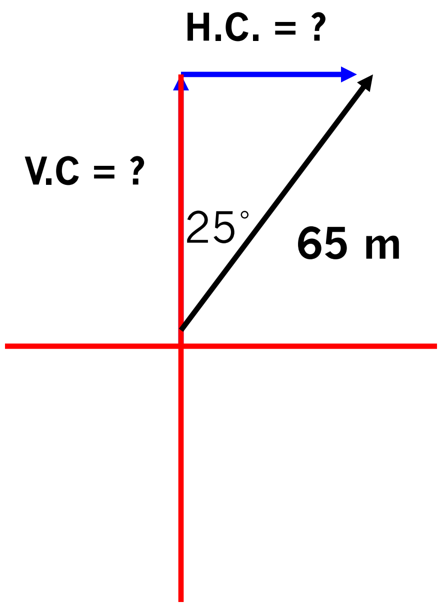
$$Tan\,\theta = \frac{opposite\ side}{adjacent\ side} = \frac{160}{120} = 1.333$$

$$\theta = Tan^{-1}(1.333) = 53.1^{o}$$

So the COMPLETE final answer is : **200 km, 53.1 degrees North of East**

Perpendicular vectors graphically

Example 10: Suppose a person walked 65 m, 25 degrees East of North. What were his horizontal and vertical components?



The goal: **ALWAYS MAKE A RIGHT TRIANGLE!**

To solve for components, we often use the trig functions sine and cosine.

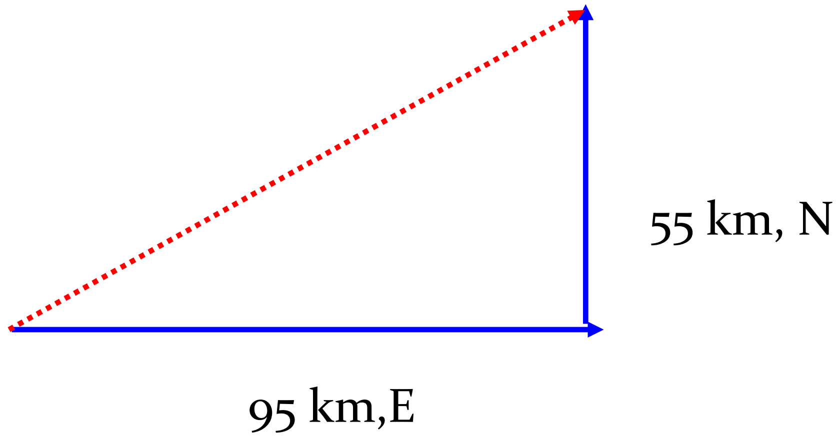
$$\text{cosine}\,\theta = \frac{adjacent\ side}{hypotenuse} \qquad \text{sine}\,\theta = \frac{opposite\ side}{hypotenuse}$$

$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$adj = V.C. = 65\cos 25 = 58.91m, N$$

$$opp = H.C. = 65\sin 25 = 27.47m, E$$

Example 11: A man walks 95 km East then 55 km north. Calculate his resultant displacement.

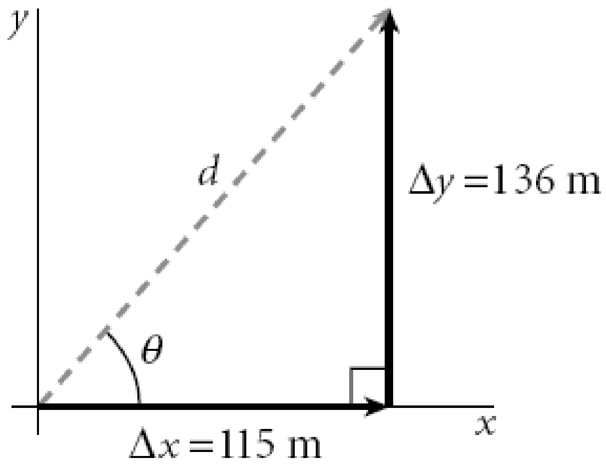


$$c^2 = a^2 + b^2 \rightarrow c = \sqrt{a^2 + b^2}$$

$$c = \text{Resultant} = \sqrt{95^2 + 55^2}$$

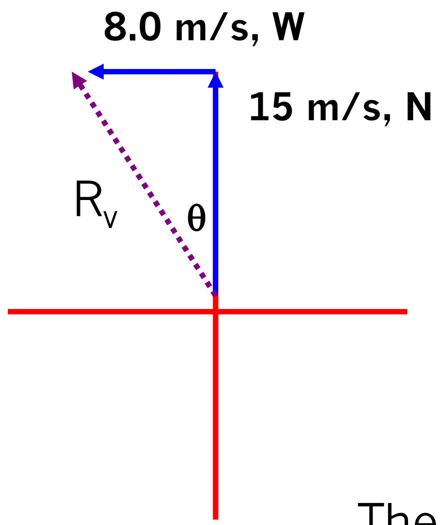
$$c = \sqrt{12050} = 109.8\ km$$

Example 12: (HW) From the diagram below, determine the value of d.



Example 13

A boat moves with a velocity of 15 m/s, N in a river which flows with a velocity of 8.0 m/s, west. Calculate the boat's resultant velocity with respect to due north.



$$R_v = \sqrt{8^2 + 15^2} = 17\, m/s$$

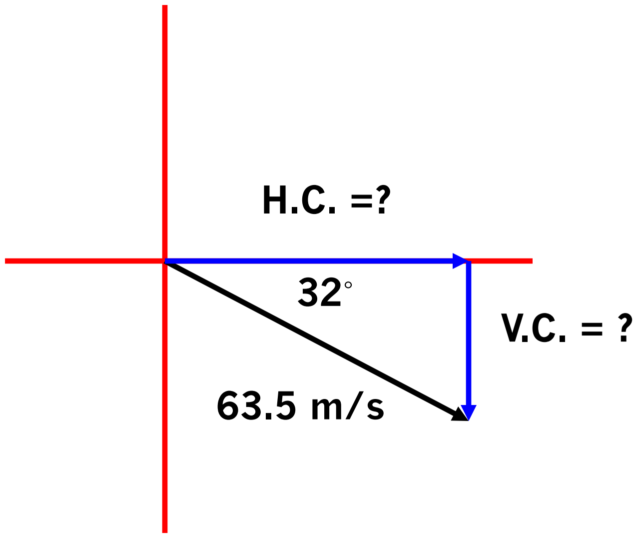
$$Tan\theta = \frac{8}{15} = 0.5333$$

$$\theta = Tan^{-1}(0.5333) = 28.1^{\circ}$$

The Final Answer : **17 m/s, @ 28.1 degrees West of North**

Example 14

A plane moves with a velocity of 63.5 m/s at 32 degrees South of East. Calculate the plane's horizontal and vertical velocity components.



$$\text{cosine}\,\theta = \frac{adjacent\ side}{hypotenuse} \qquad \text{sine}\,\theta = \frac{opposite\ side}{hypotenuse}$$

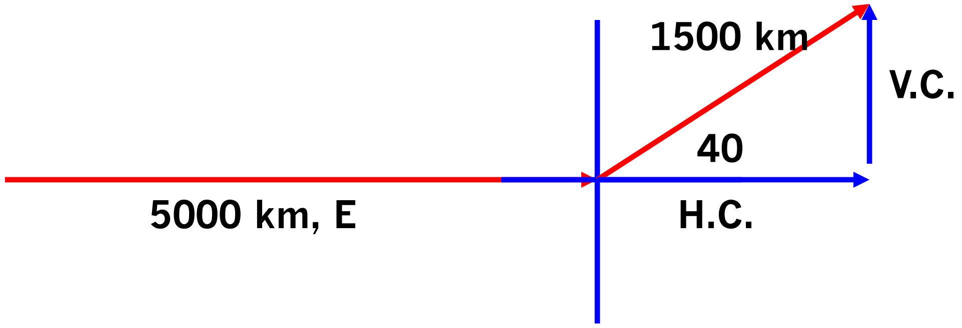
$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$adj = H.C. = 63.5\cos 32 = 53.85\, m/s, E$$

$$opp = V.C. = 63.5\sin 32 = 33.64\, m/s, S$$

Example 15

A storm system moves 5000 km due east, then shifts course at 40 degrees North of East for 1500 km. Calculate the storm's resultant displacement.



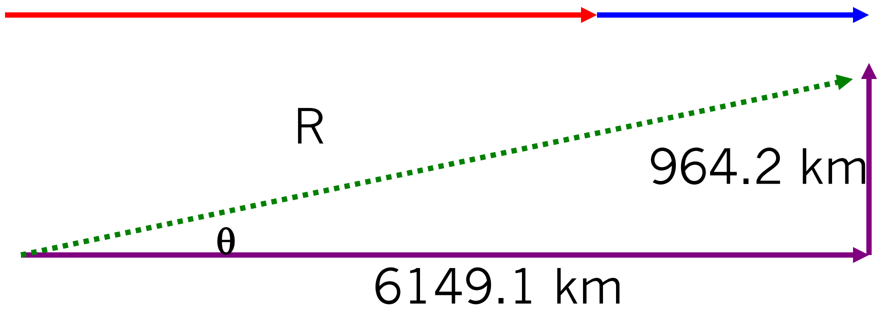
$$\text{cosine}\,\theta = \frac{adjacent\ side}{hypotenuse} \qquad \text{sine}\,\theta = \frac{opposite\ side}{hypotenuse}$$

$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$adj = H.C. = 1500\cos 40 = 1149.1\ km, E$$

$$opp = V.C. = 1500\sin 40 = 964.2\ km, N$$

5000 km + 1149.1 km = 6149.1 km



$$R = \sqrt{6149.1^2 + 964.2^2} = 6224.2\,km$$

$$Tan\,\theta = \frac{964.2}{6149.1} = 0.157$$

$$\theta = Tan^{-1}(0.157) = 8.92^{o}$$

Vector application (algebraic method)

I. Add and subtracts vectors

Example 16: Using the two vectors below, determine: A+B and B+A, A-B and B-A.

Let A = (3i - 4j - 5k)
Let B = (2i + 7j + 3k)

II. Vector multiplication (algebraic method)

- **Vector multiplications can be:**

1. **Dot product**
2. **Cross product**

- **Notes**:
  - Dot product (**A.B**) multiplication results a **scalar quantity.**
  - Cross product (**A** x **B**) multiplication results a **vector quantity.**
  - When vectors are added, subtracted or multiplied, the unit vectors have to be considered.
  - As **two vectors added or subtracted**, the similar components (**i with i, j with j and k with k**) are combined or subtracted together, only.
  - When two vectors multiplied with dot (.), non-similar components multiplications are equal to zero.
  - We can also **measure the angle between the two vectors** using the below formular:

$$\mathbf{A.B} = |A|\,|B| \cos\theta$$

SCALAR DOT PRODUCT

- Multiplying two vectors (for example A and B) sometimes gives you a SCALAR quantity which we call it the **SCALAR DOT PRODUCT**.

$$A = A_x i + A_y j + A_z k$$
$$B = B_x i + B_y j + B_z k$$

$$A \bullet B = A_x B_x + A_y B_y + A_z B_z$$

Example 17: Let A = (3i - 4j - 5k) calculate A.B.
Let B = (2i + 7j + 3k)

Therefore, A "dot" B = (3)(2) + (-4)(7) +(-5)(3) = -37

Cross product

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB\sin\theta)$$

$$\begin{aligned}\mathbf{A} \times \mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\ &= A_xB_x(\mathbf{i}\times\mathbf{i}) + A_xB_y(\mathbf{i}\times\mathbf{j}) + A_xB_z(\mathbf{i}\times\mathbf{k}) \\ &+ A_yB_x(\mathbf{j}\times\mathbf{i}) + A_yB_y(\mathbf{j}\times\mathbf{j}) + A_yB_z(\mathbf{j}\times\mathbf{k}) \\ &+ A_zB_x(\mathbf{k}\times\mathbf{i}) + A_zB_y(\mathbf{k}\times\mathbf{j}) + A_zB_z(\mathbf{k}\times\mathbf{k})\end{aligned}$$

Carrying out the cross-product operations and combining terms yields

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k} \quad (4\text{–}4)$$

This equation may also be written in a more compact determinant form as

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad (4\text{–}5)$$

$$\begin{array}{lll} \mathbf{i}\times\mathbf{j} = \mathbf{k} & \mathbf{i}\times\mathbf{k} = -\mathbf{j} & \mathbf{i}\times\mathbf{i} = 0 \\ \mathbf{j}\times\mathbf{k} = \mathbf{i} & \mathbf{j}\times\mathbf{i} = -\mathbf{k} & \mathbf{j}\times\mathbf{j} = 0 \\ \mathbf{k}\times\mathbf{i} = \mathbf{j} & \mathbf{k}\times\mathbf{j} = -\mathbf{i} & \mathbf{k}\times\mathbf{k} = 0 \end{array}$$

III. Multiplying or Dividing a Vector by a Scalar quantity

- A vector can be multiplied or divided by a quantity (such as a number).

- If the scalar is positive, the direction of the resultant is the same as of the original vector.

$\vec{A}$ $\qquad$ $5 \times \vec{A} = 5\vec{A}$ $\qquad$ $\frac{1}{2}\vec{A}$

- If the scalar is negative, the direction of the resultant is opposite that of the original vector.

$\vec{A}$ $\qquad$ $-2 \times \vec{A} = -2\vec{A}$

Resultants

Parallel Forces. For a system of parallel forces not all in the same plane, the magnitude of the parallel resultant force **R** is simply the magnitude of the algebraic sum of the given forces.

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

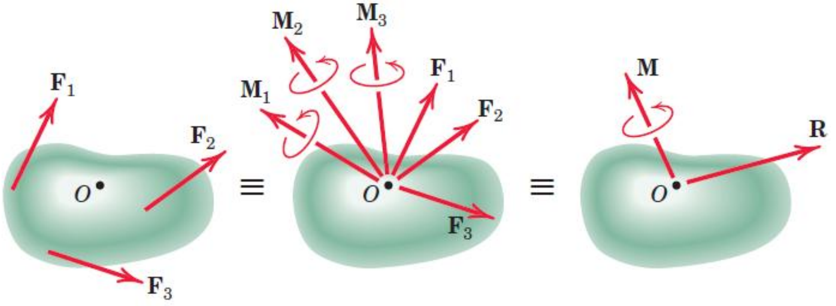
$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \cdots = \Sigma(\mathbf{r} \times \mathbf{F})$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

$$\mathbf{M}_x = \Sigma(\mathbf{r} \times \mathbf{F})_x \qquad \mathbf{M}_y = \Sigma(\mathbf{r} \times \mathbf{F})_y \qquad \mathbf{M}_z = \Sigma(\mathbf{r} \times \mathbf{F})_z$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$



Conclusion

- Coordinating systems are useful tools to find out the dimensions of vector quantities.
- Sine and cosine rules are mathematical approaches to find out dimensions of a regular shape.
- In physics, two different quantities are available, scalar and vector quantities.
- Vector quantities are added, subtracted and multiplies to find out the resultant vectors or unknown parameters.

References (in APA style)

- ✓ Young, H. D., Freedman, R. A., & Ford, A. L. (2014). University physics with modern physics (p. 822). New York: Pearson.
- ✓ Sommerfeld, A. (2016). *Mechanics: Lectures on theoretical physics, Vol. 1* (Vol. 1). Elsevier.