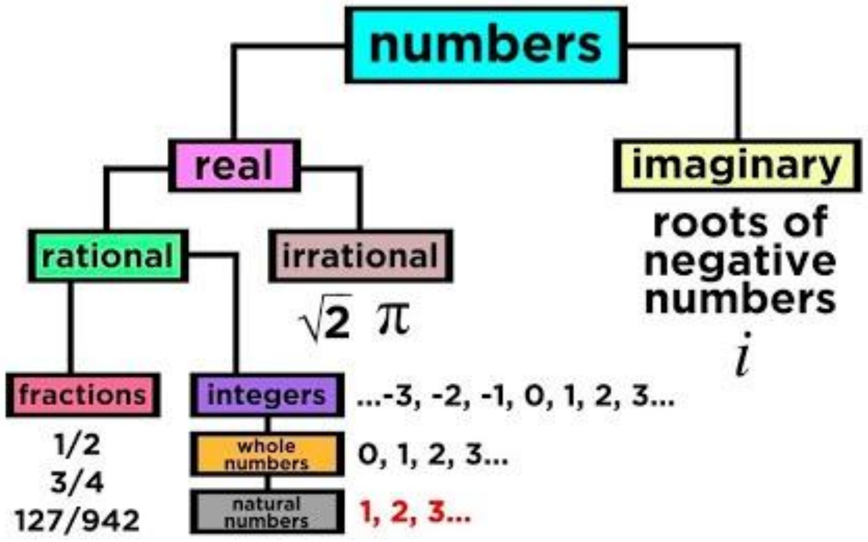
Lecture 1:

- Numbers
- Logical Operators
- Significant Figures
- Scientific Notation
- Factorial

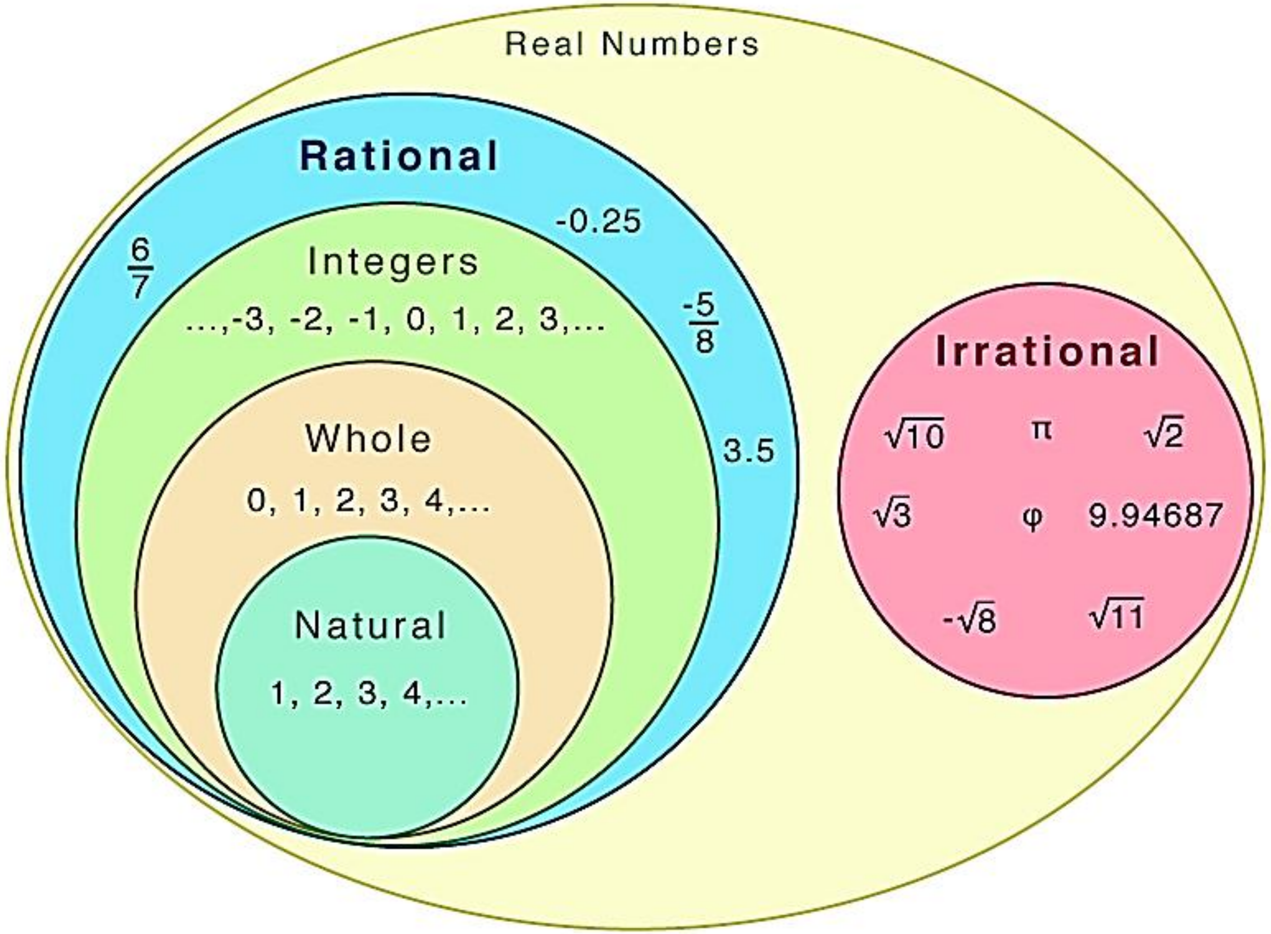
Number Classifications

- ✓ $N \ = \ \{1,2,3,\ldots\}$ – natural numbers/counting numbers
- ✓ $W \ = \{0, 1, 2, 3, \ldots\}$ – whole numbers
- ✓ $Z \ = \ \{0, \pm 1, \pm 2, \pm 3, \ldots\}$ – integers
  - → $Z^+ = \{0, 1, 2, 3, \ldots\}$ – non-negative integers
  - → $Z^- = \{0, -1, -2, -3, \ldots\}$ – non-positive integers

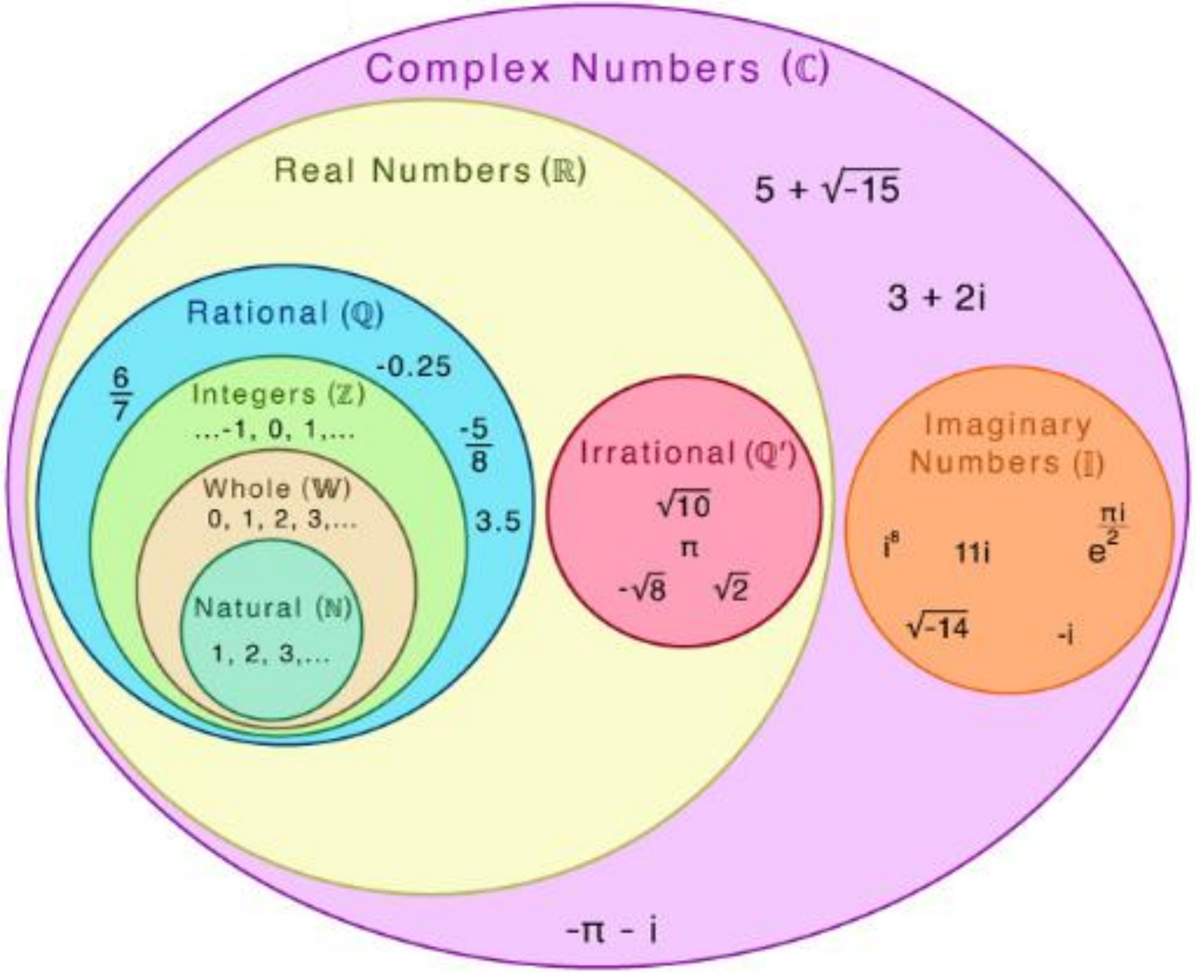


- ✓ $Q \ = \left\{\frac{1}{2}, \frac{3}{4}, 2, 5, 0, -\frac{7}{11}, -12, \ldots\right\}$ – rational numbers $\left\{\frac{a}{b} \middle| \, a, b \in Z, b \ \neq 0\right\}$
- ✓ $\sqrt{2}$, π, $\sqrt{3}$, … - irrational numbers
- ➢ Real numbers - set of irrational numbers & rational numbers
- ➢ Complex number $\quad z = a + ib$

Rational Numbers :decimals terminating (finite) or (repeating patterns)
Irrational Numbers: neither finite nor repeating decimals, π = 3.1415926535…



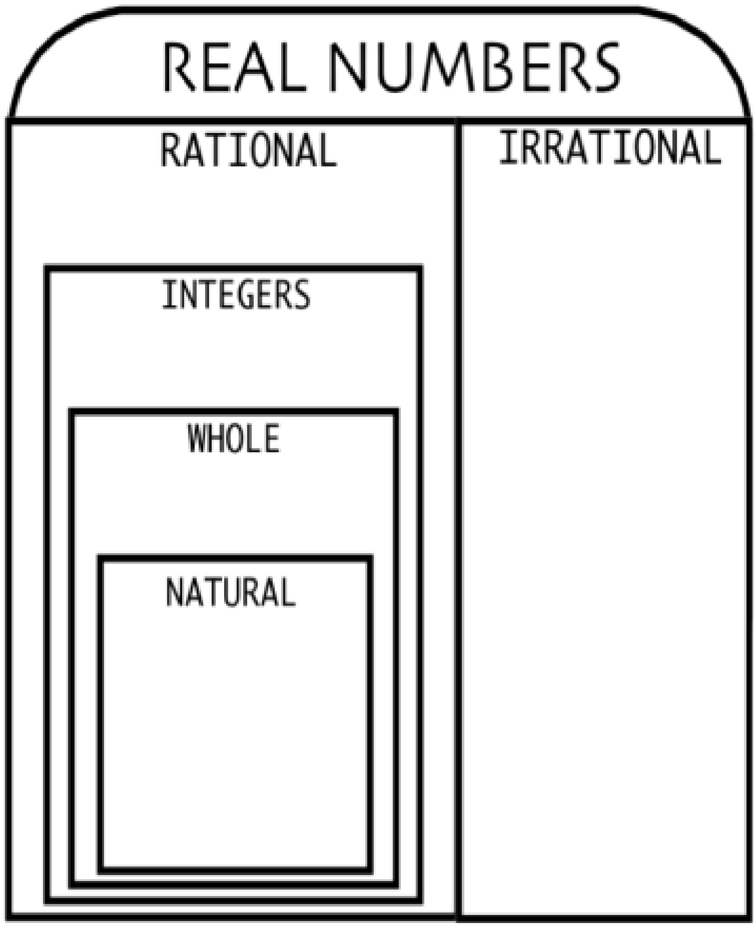
Real Numbers
Rational
$\frac{6}{7}$
-0.25
Integers
...,-3, -2, -1, 0, 1, 2, 3,...
$\frac{-5}{8}$
Whole
0, 1, 2, 3, 4,...
3.5
Natural
1, 2, 3, 4,...
Irrational
$\sqrt{10}$
π
$\sqrt{2}$
$\sqrt{3}$
φ
9.94687
$-\sqrt{8}$
$\sqrt{11}$



Complex Numbers (ℂ)
Real Numbers (ℝ)
$5 + \sqrt{-15}$
3 + 2i
Rational (ℚ)
-0.25
$\frac{6}{7}$
Integers (ℤ)
...-1, 0, 1,...
$\frac{-5}{8}$
Whole (𝕎)
0, 1, 2, 3,...
3.5
Natural (ℕ)
1, 2, 3,...
Irrational (ℚ')
$\sqrt{10}$
π
$-\sqrt{8}$
$\sqrt{2}$
Imaginary Numbers (𝕀)
i^8
11i
$e^{\frac{\pi i}{2}}$
$\sqrt{-14}$
-i
-π - i

1) **Re–write each number in the Venn Diagram where it belongs.**

-19	$1.\overline{2}$	0	3
$\sqrt{10}$	$\sqrt{81}$	3.456	$-{}^{6}/_{11}$
$-1.48298.....$		$\pi + 3$	-44



2) **List all classifications of the number.**

a) $\sqrt{10}$ ______________________________

b) -44 ______________________________

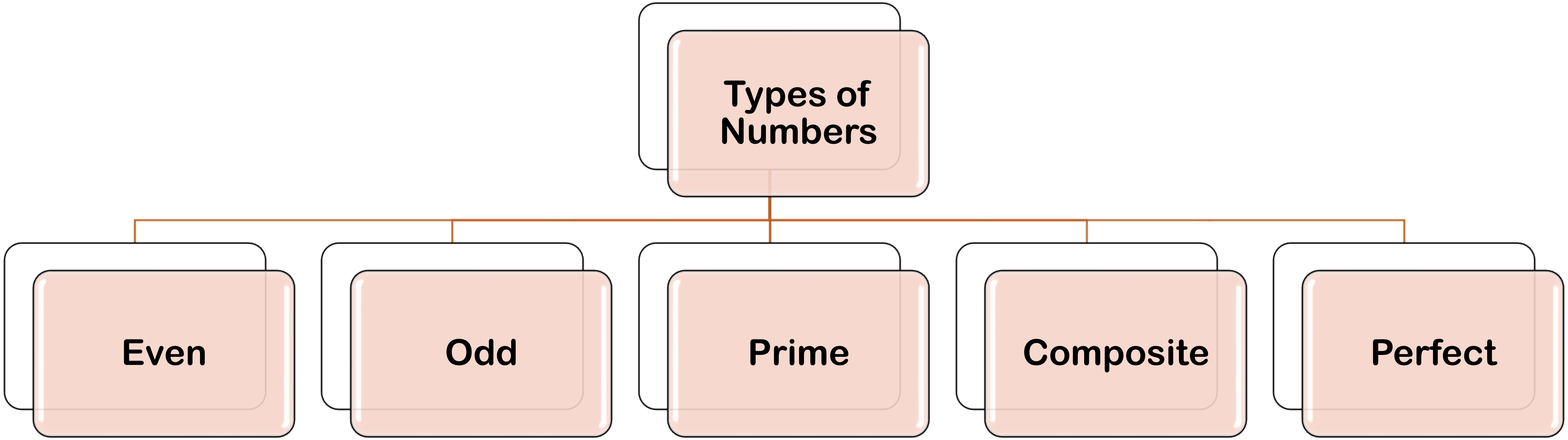
c) 3 ______________________________

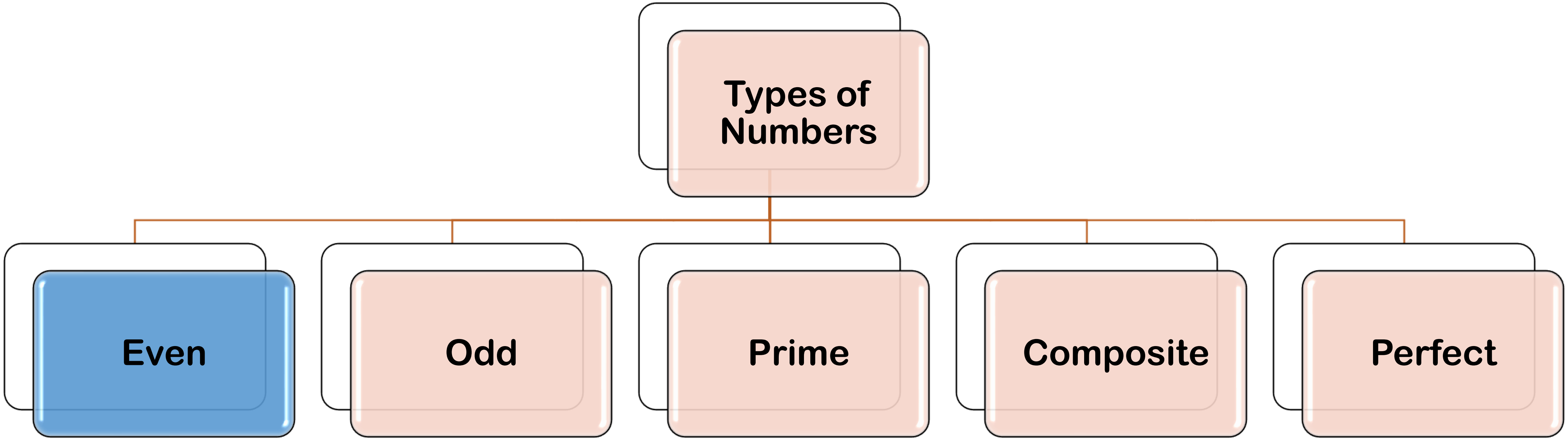
d) $-6/11$ ______________________________

3) **Check all boxes that apply to the number.**

		Natural	Whole	Integer	Rational	Irrational	Real
a)	$\sqrt{81}$						
b)	$1.\overline{2}$						
c)	0						
d)	13						

practice makes progress


Types of Numbers
Even
Odd
Prime
Composite
Perfect



- A number that can be exactly divided by 2.
- Even numbers always end up with the last digit as 0, 2, 4, 6 or 8.
- The general form of even numbers is given by $2k$, where $k \in Z$

- Ahmad has 30 pencils. He distributed 14 of those among his friends. Will he have an even number of pencils left? How do you know?

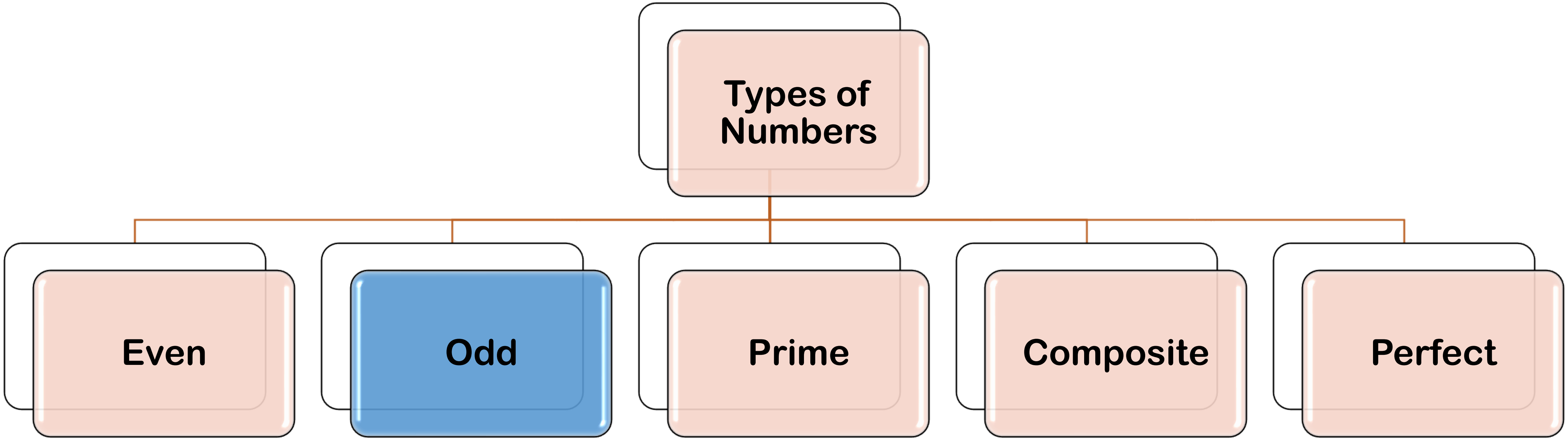
- State true or false: 0 is an even number.

- Select the pair of <u>consecutive</u> even numbers from the following:

  a) 24 and 28
  b) 91 and 93
  c) 84 and 86
  d) 39 and 42

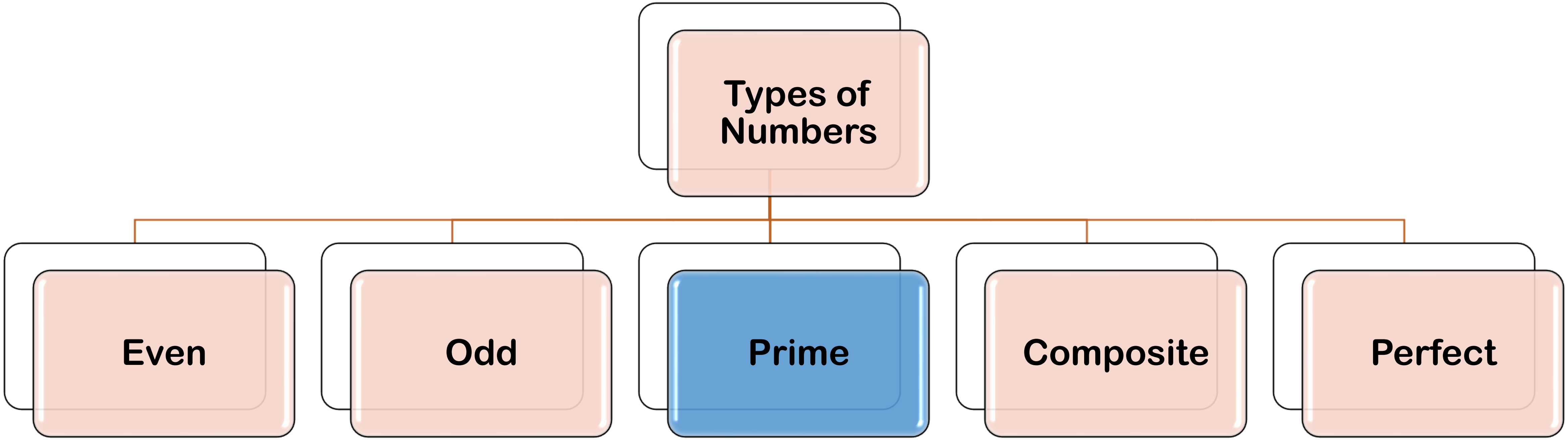
- When you buy a dozen bananas, are you getting an even number or an odd number of bananas?

- Select the even numbers from the following:
  a.) 778
  b.) 912
  c.) 223



- A number which is not divisible by 2.
- An odd number always ends in 1, 3, 5, 7, or 9.
- The general form of odd numbers is given by $2k + 1$, where $k \in Z$

- Determine whether 135 is an odd number or not.
- Is 350 an odd number or an even number?
- Will the sum of 23 + 35 result in an odd number?
- Answer the following questions with reference to odd numbers:
  a.) 1 is odd or even?
  b.) Which is the smallest 4 digit odd number?
  c.) What is the sum of any two odd numbers?
  d.) Is 2 an odd number?
- State true or false with respect to odd numbers.
  a.) The sum of two odd numbers is always an even number.
  b.) The smallest odd number is 5.
  c.) 9 is an odd number.



- **A natural number that are divisible by only 1 and the number itself.**
- **Ex: 2, 3, 5, 7, 11, 13, …**

1 is **not a prime number:** prime numbers must have *exactly two distinct factors* (1 and themselves), but 1 only has one factor: itself.

Which of the two numbers is a prime number, 13 or 15?

Why is 20 not a prime number?

State true or false with respect to prime numbers.

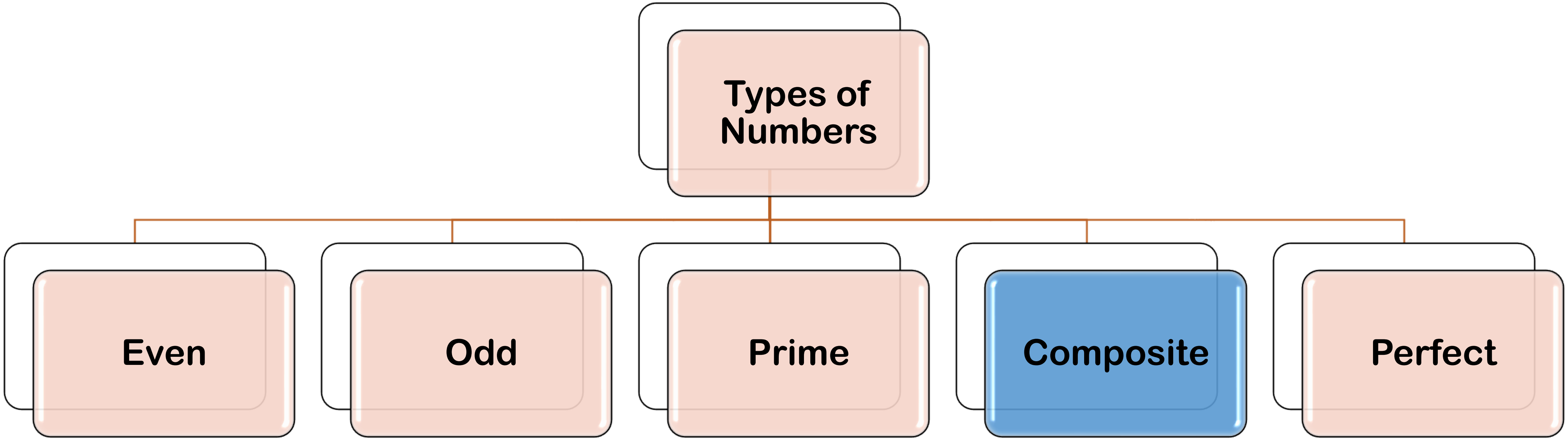
a.) 1 is a prime number.
b.) The only even prime number is 2.
c.) The first five prime numbers are 2, 3, 5, 7, and 9.
d.) All prime numbers are odd.

Which of the following numbers is a prime number?

a) 4
b) 10
c) 33
d) 43

Choose true/false against each statement.

	True	False
2 is the only even prime number.	○	○
3 is the smallest prime number.	○	○
97 is the largest prime number.	○	○
All prime numbers are odd.	○	○



- A natural number or a positive integer which has more than two factors.
- Ex: 15 has factors 1, 3, 5 and 15.

Always remember that 1 is neither prime nor composite

Which of the following is a composite number?

a) 34
b) 31
c) 39

Fill in the blanks:

a.) The smallest composite number is __.
b.) The smallest odd composite number is __.

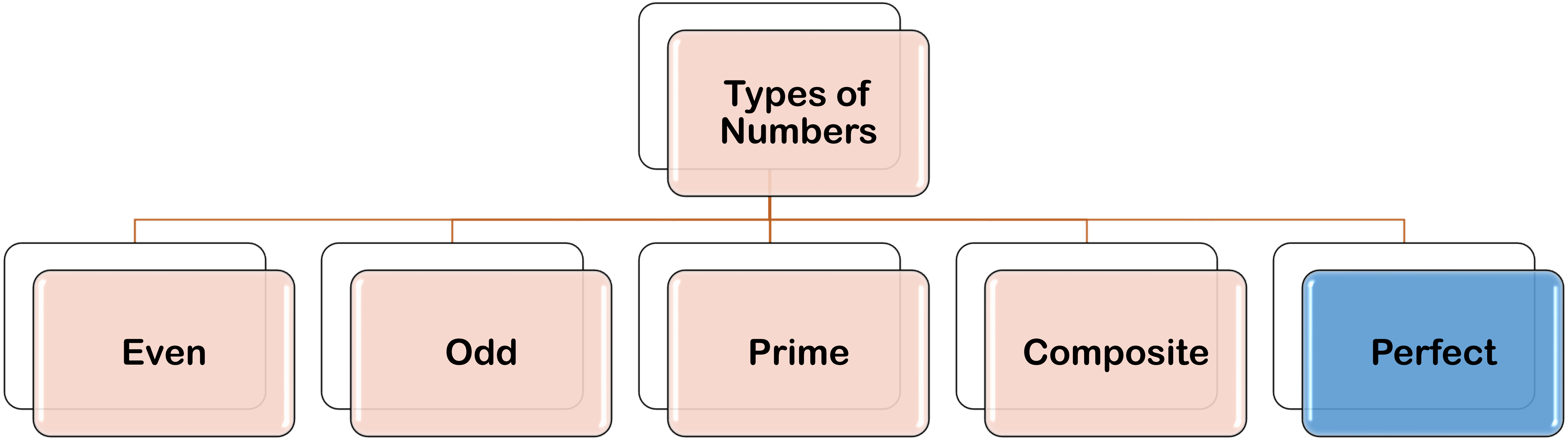
State true or false with respect to composite numbers.
a.) All even numbers are composite numbers.
b.) 1 is a composite number.

Aya is listing all the composite numbers between 3 and 10. Can you help her choose the correct option?

a) 4, 6, 8, 9
b) 4, 9
c) 4, 5, 6, 7, 8, 9
d) 4, 8, 9

The smallest composite number is 2.

a) True
b) False



- A positive integer that is equal to the sum of its positive factors, excluding the number itself.

- Ex: 6, 28, 496, 8128, 33550336, …

- All the perfect numbers are also complete numbers.

Perfect Number	Positive Factors	Sum of all factors excluding itself
6	1, 2, 3, 6	6
28	1, 2, 4, 7, 14, 28	28
496	1, 2, 4, 8, 16, 31, 62, 124, 248, 496	496
8,128	1, 2, 4, 8, 16, 32, 64, 127, 254, 508, 1016, 2032, 4064, 8128	8,128

- Is 28 a perfect number?

- Select the perfect numbers from the following.

  a) 5
  b) 6
  c) 32
  d) 28
  e) 9

- State true or false:
  a.) Perfect numbers are the positive integers that are equal to the sum of its factors except for the number itself.
  b.) All the perfect numbers are odd numbers.

- Check whether the given numbers are perfect numbers or not by finding the sum of their factors:
  a.) 8
  b.) 25

Even + Even = Even

Odd + Odd = Even

Even + Odd = Odd

Even × Even = Even

Even × Odd = Even

Odd × Odd = Odd.

Perfect Square Numbers

- Perfect squares are the squares of a whole number (when a number is multiplied by itself two times).

Perfect Square Formula

$$N = X^2$$

$1^2 = 1$	$11^2 = 121$	$21^2 = 441$
$2^2 = 4$	$12^2 = 144$	$22^2 = 484$
$3^2 = 9$	$13^2 = 169$	$23^2 = 529$
$4^2 = 16$	$14^2 = 196$	$24^2 = 576$
$5^2 = 25$	$15^2 = 225$	$25^2 = 625$
$6^2 = 36$	$16^2 = 256$	$26^2 = 676$
$7^2 = 49$	$17^2 = 289$	$27^2 = 729$
$8^2 = 64$	$18^2 = 324$	$28^2 = 784$
$9^2 = 81$	$19^2 = 361$	$29^2 = 841$
$10^2 = 100$	$20^2 = 400$	$30^2 = 900$

- Is 100 a perfect square number?

- In an auditorium, the number of rows is the same as the number of columns. If there are 60 chairs in a row, how many chairs are there in the auditorium?

- What smallest whole number is to be added to 75 to make it a perfect square?

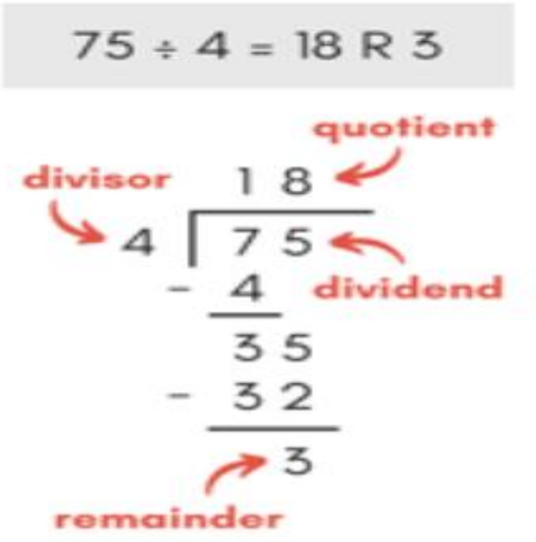
- Which of the following is not a perfect square?

  a) 900
  b) 800
  c) 400
  d) 100

- What will be the area of a square having a side of 16 meters?

Modulo Operator

- Mod is also known as modulus or modulo.
- It gives the remainder after dividing one number by another number.
- Modulus of any real number x will always give positive value as it's output.



$$\frac{A}{B} = Q, remainder R$$

A (mod B) = R

A = Dividend

B = Divisor

Q = Quotient

R = Remainder

$$A = B \times Q + R$$

Example

14 ÷ 3 = 4, remainder 2 ⇒ 14 (mod 3) = 2

Another way: $A \equiv R \ (mod\ B)$

$$a : b \ (a \div b) \quad \text{where} \quad a, b, q, r \in Z, \ b \neq 0, \quad 0 \leq r \leq |b|$$

$$a = b \cdot q + r$$

$$27 \ mod \ 4 = 27 : 4 \Longrightarrow \quad 27 = 4 \cdot 6 + 3$$

$$-15 : 4 \Longrightarrow -3 \cdot 4 + (-3) = -15$$

$$0 \leq r \leq |b|$$

$$-15 : 4 \Longrightarrow 4 \cdot (-4) + 1 = -15$$

$$113 : (-3) \Longrightarrow 113 = -(3) \cdot (-37) + 2$$

-5 mod 9 $\Longrightarrow$ -5 = 9 . (-1) + 4

-19 mod 5 $\Longrightarrow$ -19 = 5 . (-4) + 1

$$3 \bmod 10 = 3$$
$$13 \bmod 10 = 3$$
$$23 \bmod 10 = 3$$
$$33 \bmod 10 = 3$$

-9 mod 9 = 0

-8 mod 9 = 1

-7 mod 9 = 2

-4 mod 9 = 5

-2 mod 9 = 7

-1 mod 9 = 8

What is -6 mod 18 ? = 12

What is -4 mod 9? = 5

What is -9 mod 6? = 3

What is -13 mod 1? = 0

What is 17 mod 7? = 3

What is -49 mod 5? = 1

What is -14 mod 2? = 0

What is -29 mod 4? = 3

What is -29 mod 3? = 1

What is 6 mod 18? = 6

What is 9 mod -6? = 3

What is 4 mod 9? = 4

What is -6 mod 18? = 12

What is 7 mod 6? = 1

1) What is the remainder value:

- 108 is divided by 3.
- 129 is divided by 7.

- Find the product $23 \cdot 43$ modulo 8
- Find 11 mod 8.
- Find -3 mod 8.
- Find 49 mod 5.

2) Perform the modular arithmetic operation

What is 13 mod 1 =

What is -17 mod 7 =

What is - 4 mod 9 =

What is 4 mod 9 =

What is -7 mod 6 =

What is 49 mod 5 =

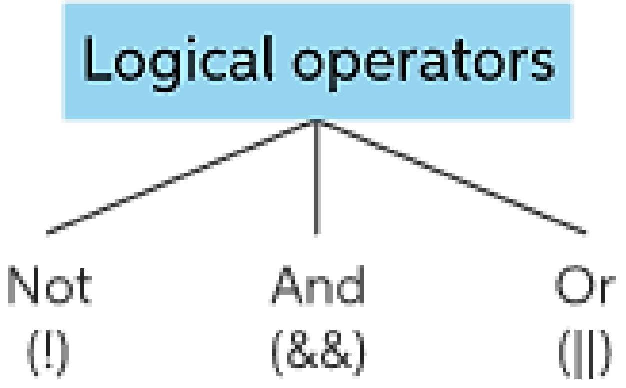
What is -49 mod 5 =

What is 25+37 mod 12 =

Logical Operators

- Logical operators are useful when we want to test multiple conditions.
- There are 3 types of logical operators and they work the same way as the boolean AND, OR and NOT operators.
- && - Logical AND
  - All the conditions must be true for the whole expression to be true.
  - Example: if (a == 10 && b == 9 && d == 1)

    means the *if* statement is only true when *a* == 10 **and** *b* == 9 **and** *d* == 1.



Logical Operators

- || - Logical OR
  - The truth of one condition is enough to make the whole expression true.
  - Example: if (a == 10 || b == 9 || d == 1)

    means the *if* statement is true when **either one** of *a*, *b* or *d* has the right value.
- ! - Logical NOT (also called logical negation)
  - Reverse the meaning of a condition
  - Example: if (!(points > 90))

    means if points not bigger than 90.

Logical Operator

<table>
<tr><th>Expression</th><th>Expression Equivalent</th></tr>
<tr><td>!(a == b)</td><td>a != b</td></tr>
<tr><td>!(a == b || a == c)</td><td>a != b && a != c</td></tr>
<tr><td>!(a == b && c > d)</td><td>a != b || c <= d</td></tr>
</table>

Answer the following questions as **True** or **False**:

If $x = -2, y = 5, z = 0$, and $t = -4$, what is the value of each of the following expressions:

1. $x + y < z + 1$
2. $x - 2 * y + y < z * 2/3$
3. $t > 5 || z < (y + 5) \&\& y < 3$
4. $!(4 + 5 * y >= z - 4) \&\& (z - 2 < 7)$

If x = -2, y = 5, z = 0 , and t = -4, what is the value of each of the following logical expressions?

1. x + y < z + 1
2. x - 2 * y + y < z * 2 / 3
3. 3 * y / 4 < 8 && y >= 4
4. t > 5 || z < 2
5. x * y < 10 || y * z < 10
6. (y + 2) / 3 > 3 && t < 0
7. x * 3 > 0 || y + 5 / t < 2
8. !(x > 0)
9. !(x * t < 10) || y / x * 4 < y * 2
10. t > 5 || z < (y + 5) && y < 3
11. !(4 + 5 * y >= z - 4) && (z - 2 < 7)

Order of Operations

- ()
- *, /, % Multiplicative operators
- +, - Additive operators
- <, >, >=, <= Relational operators
- ==, != Then do any comparisons for equality and inequality
- && Logical and
- || Logical or
- = Assignment operator

Write syntactically correct logical expressions for the following conditions:

1. m is less than 100
2. n is positive and greater than m
3. m is between 5 and 10 (inclusive)
4. k is less than 1 or greater than 2
5. j and k are both negative
6. i is an even number

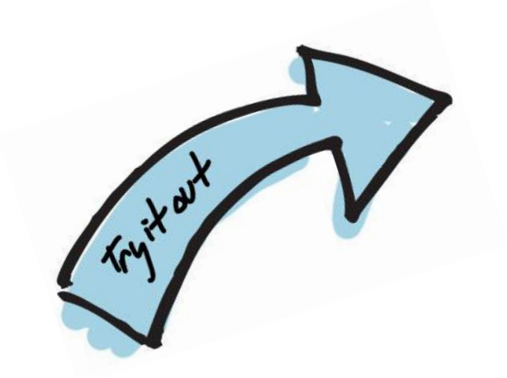
Given

int a = 5, b = 7, c = 17 ;

evaluate each expression as True or False.

1. c / b == 2
2. c % b <= a % b
3. b + c / a != c - a
4. (b < c) && (c == 7)
5. (c + 1 - b == 0) || (b = 5)

- Assume a=5, b=2, c=4, d=6, and e=3. Determine the value of each of the following expressions:
  - a > b
  - a != b
  - d % b == c % b
  - a * c != d * b
  - a % b * c



- 25 < 7 || 15 > 36
- 15 > 36 || 3 < 7
- 14 > 7 && 5 <= 5
- 4 > 3 && 17 <= 7
- ! false
- ! (13 != 7)
- 9 != 7 && ! 0
- 5 > 1 && 7

```
int x, y;
X = 0 ;
y = 1 ;
if ( x < y | |  y < 5  &&  x  = = 3 )
{
   printf ("True \n");
{
else
{
   printf("False \n ");
{
```

Which is printed?

SigFigs

Significant figures are important to show the precision of your answer. This is important in science and engineering because no measuring device can make a measurement with 100% precision. Using Significant figures allows the scientist to know how precise the answer is, or how much uncertainty there is.

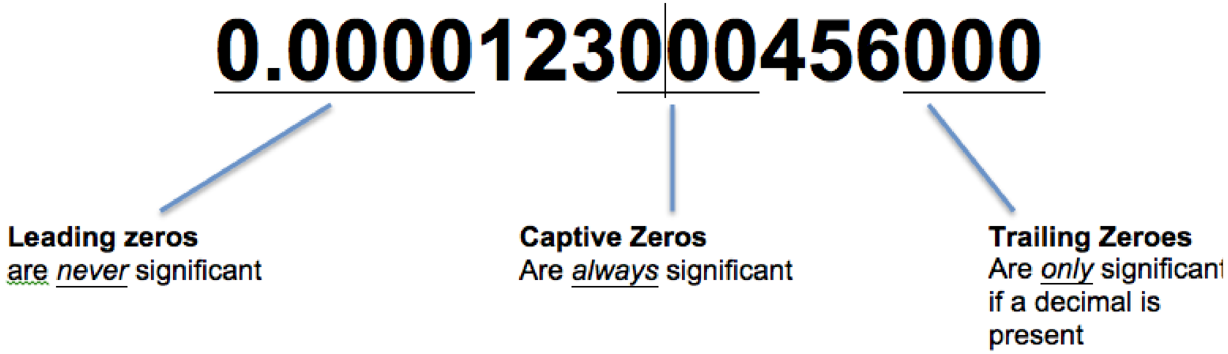
2002 has two significant zeroes, but 0.0103 has only 1 significant zero.

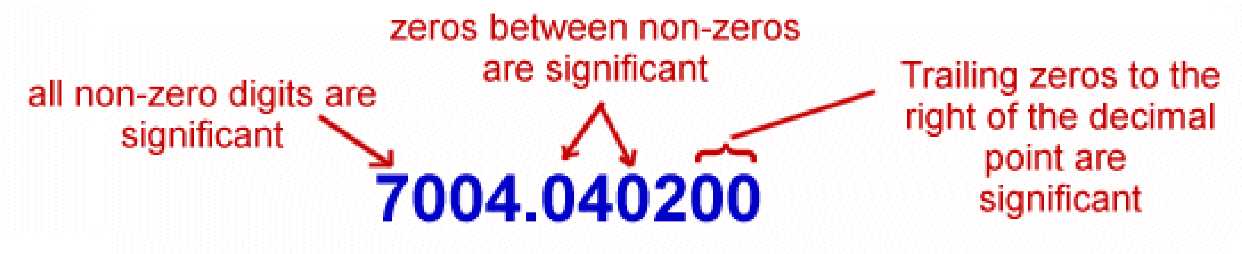
Significant Figures

- The number of digits counted to the right from the leftmost positive digit is called the *number of significant figures*. For example,

26.103 5 significant figures , 202.000 6 significant figures

0.00304 3 significant figures, 0.003040 4 significant figures





Significant Figures Rules:

- All non-zero digits **DO** count.
  - 24 = 2
  - 3.56 = 3
- Leading zeros **DON'T** count.
  - (zeros in front of numbers)
  - 0.0025 = 2
- Captive Zeros **DO** count.
  - (zeros between non-zero numbers)
  - 1502 = 4 1.008 = 4
- Trailing Zeros **DO** count **IF** the number contains a **DECIMAL**.
  - (zeros at the end of numbers)
  - 100 = 1 2306.0 = 5 $1.00 \times 10^3 = 3$

- 1000 (with no decimal point):

  → 1 significant figure

  (Only the digit 1 is clearly significant; the zeros may just be placeholders.)
- 1000. (with a decimal point):

  → 4 significant figures

  (The decimal point shows that all zeros are significant.)
- 1.00×10^3:

  → 3 significant figures
- 1.000×10^3:

  → 4 significant figures

Sometimes, you'll be asked to round with significant figures. Significant figures have to do with the number of digits known with a degree of certainty. Keeping track of this number is important when gathering data from an experiment because it minimizes error. Here are the rules for significant figures:

1. All nonzero digits are significant.
2. All zeroes between nonzero digits are significant.
3. Trailing zeroes to the right of a decimal point are significant.
4. Leading zeroes to the left of the first non-zero number are not significant.

Here are some examples. Can you see which rule applies?

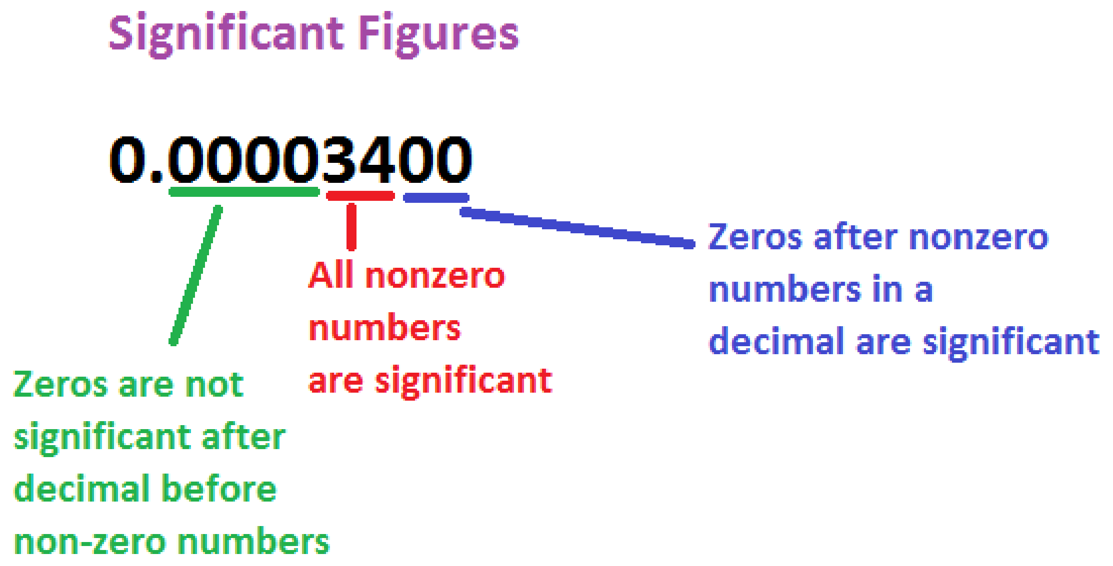
1.23 has 3 significant figures

1001 has 4 significant figures

2.03 has 3 significant figures

0.033 has 2 significant figures

0.20 has 2 only significant figures



1. Find the number of significant figure in each of the following:

(a) 7.3

(b) 162.5 m

(c) 306 g

(d) 3.57 m

(e) 7.005 kg

(f) 0.045 km

(g) 0.00234 l

(h) 82.030 mg

2. Round off each of the following correct up to 3 significant figures:

(a) 56.4517 g

(b) 5.20763 kg

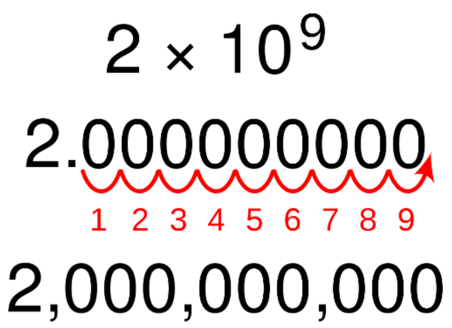
(c) 33.311 km

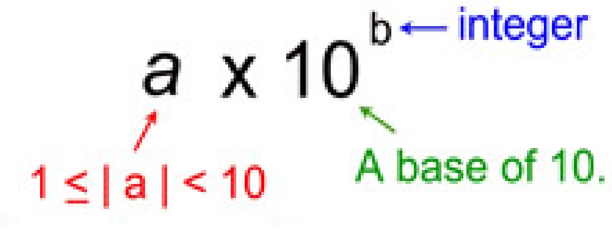
(d) 50.001 cm

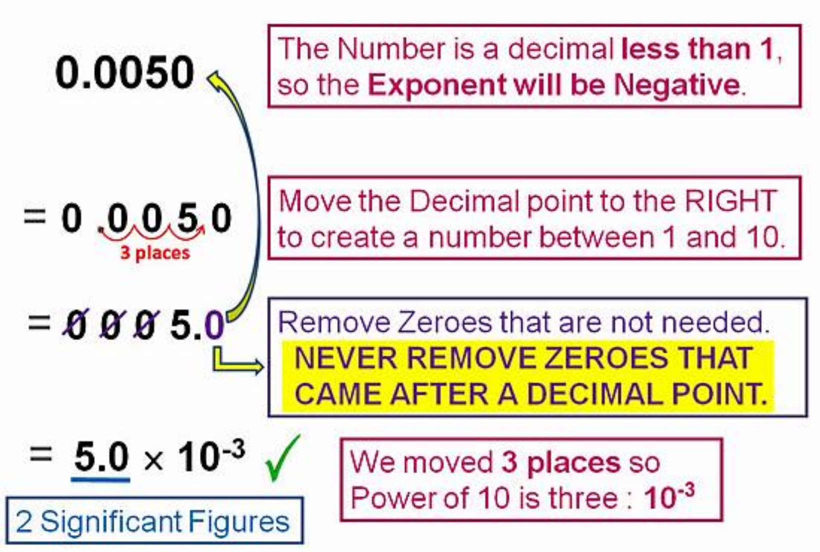
(e) 0.0012485 m

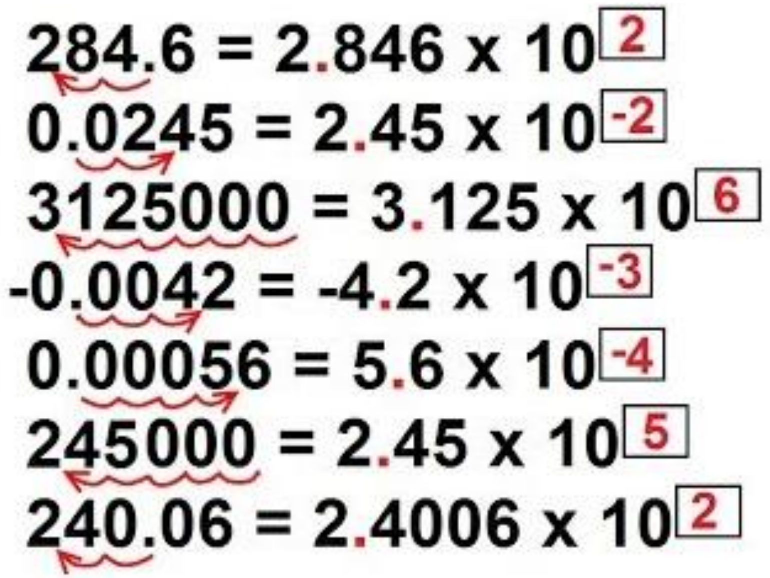
(f) 0.0013020 l

Scientific Notation









- $193.034 = 1.93034 \times 10^{2}$
- $0.003040 = 3.040 \times 10^{-3}$

Convert the following numbers into scientific notation:

1) 923 __ **9.23×10^{2}** ____________

2) 0.00425 __ **4.25×10^{-3}** ____________

3) 4523000 __ **4.523×10^{6}** ____________

4) 0.94300 __ **9.4300×10^{-1}** ____________

5) 6750. __ **6.750×10^{3}** ____________

6) 92.03 __ **9.203×10^{1}** ____________

7) 7.80 __ **7.80×10^{0}** ____________

8) 0.00000032 __ **3.2×10^{-7}** ____________

Convert the following numbers into standard notation:

9) 3.92400×10^{5} ____ **392400** ___________

10) 9.2×10^{6} ________ **9200000** _______

11) 4.391×10^{-3} _____ **0.004391** _________

12) 6.825×10^{-4} _____ **0.0006825** _________

13) 4.6978×10^{4} _____ **46978** __________

14) 8.36×10^{1} _______ **83.6** ________

15) 2.46×10^{-5} ______ **0.0000246** ________

16) 8.8×10^{2} ________ **880** _______

Factorial

!

exclamation mark

$$\mathbf{0! = 1}$$

$$1! = 1$$

$$2! = 2 \times 1 = 2$$

$$3! = 3 \times 2 \times 1 = 6$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Example 1.

Simplify this factorial expression.

$$3!$$

Solution.

- **Use this formula to calculate a factorial expression:**

$$n! = n \bullet (n-1) \bullet (n-2) \bullet \cdots \bullet 1$$

- **Calculate the factorial expression.**

$$3! = 3 \bullet 2 \bullet 1$$

$$= 6$$

It is commonly implemented using **iteration** to solve problems involving **permutations, combinations, and algorithm complexity analysis.**

4 books for four possible positions

Books:

MATH	PHYS	CHEM	HIST
Position 1	**Position 2**	**Position 3**	**Position 4**

$$4 \times 3 \times 2 \times 1 = 24$$

$$= 4!$$

24 possible ways!

practice MAKES progress

Match each expression on the left with an equivalent expression on the right.

A	$\frac{14!}{13!}$
B	$\frac{52!}{51!}$
C	$\frac{101!}{99!}$
D	$20 \times 19!$
E	$90 \times 8!$
F	$30 \times 4!$

Letter		
	1	10100
	2	6!
	3	52
	4	10!
	5	14
	6	20!

Determine the value for each expression. Simplify fully before using a calculator.

a) $\frac{10!}{5!}$ b) $\frac{21!}{14!}$ c) $\frac{9!}{3!6!}$ d) $\frac{12!}{8!4!}$ e) $\frac{7!}{2!5!}+\frac{7!}{4!3!}$

f) $\frac{15!}{9!6!}+\frac{15!}{10!5!}$ g) $2\times\frac{5!}{2!3!}$ h) $3\times\frac{11!}{7!4!}$

$$\frac{(n-1)!\cdot n!}{(n!)^2}$$

$$\frac{88!}{90!}$$

$$\frac{(4-1)!}{4!}$$

$$\frac{38!\cdot 3!}{39!}$$

$$\frac{(n+5)!}{(n+1)!}$$

$$\frac{(2\cdot 3)!}{3!}$$

$$\frac{77!\cdot 2!}{78!}$$

$$\frac{10!}{12!}$$

$$\frac{3!4!}{6!}$$

$$\frac{16\cdot 15\cdot 14\cdot 13}{20!}$$

$$\frac{(8!+7!)(6!+5!)}{(8!-7!)(6!-5!)}$$

1) $\frac{(6-2!)!}{4!}$

2) $6!+(-3\times 5!)$

3) $9-2!$

4) $(3!)!$

5) $\frac{18!}{16!}$

6) $-35+0!+7$

7) $25-5!-1!$

8) $10\times 3!$

9) $\frac{14!}{13!}\div\frac{7!}{6!}$

10) $4!\,2!+40$

11) $5!+16$

12) $\frac{22!}{19!\,8!}$

1) $4!$

2) $8!$

3) $7!$

4) $\frac{4!}{3!}$

5) $\frac{6!}{1!}$

6) $\frac{6!}{4!}$

7) $\frac{6!}{4!2!}$

8) $\frac{5!}{2!2!}$

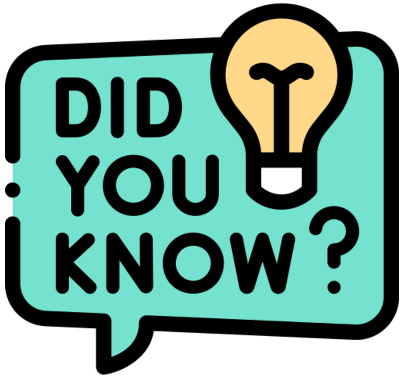
9) $\frac{7!}{3!2!}$

10) $\frac{6!}{(5-3)!3!}$

11) $\frac{7!}{(7-4)!4!}$

12) $\frac{4!}{(4-1)!1!}$

Answers: 1) 24 2) 40320 3) 5040 4) 4 5) 720 6) 30 7) 15 8) 30 9) 420 10) 60 11) 35 12) 4



What makes a good life?
Lessons from the longest study on happiness

Robert Waldinger

What keeps us healthy and happy as we go through life?

https://www.youtube.com/watch?v=8KkKuTCFvzI