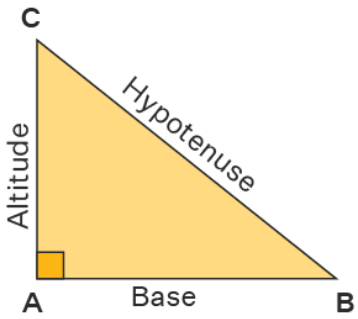
Lecture 2



$BC^2 = AB^2 + AC^2$

- Perfect Square Trinomials
- Factoring
- Theorem of Pythagoras
- Rationalizing the denominator
- Solving Quadratic Equations By Factorizing
- Inequalities

Quick Recap

Factor Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a+b)(a+b) = (a+b)^2$$

- Square of first term: a^2
- Twice the product of first and last term: $2ab$
- Square of last term: b^2
- First term: a
- Last term: b

$$a^2 - 2ab + b^2 = (a-b)(a-b) = (a-b)^2$$

Take note of the signs

Factorise:

1) $x^2 - 25$

2) $x^2 - 36$

3) $x^2 - 49$

4) $y^2 - 64$

5) $y^2 - 81$

6) $81 - y^2$

Factorise fully:

1) $16x^2 - 25$

2) $16x^2 - 25y^2$

3) $32x^2 - 50y^2$

4) $64x^2 - 100y^2$

5) $64x^3 - 100y^2x$

6) $49x^3 - 81y^2x$

7) $49x^4 - 81y^4$

8) $49x^5 - 81y^4x$

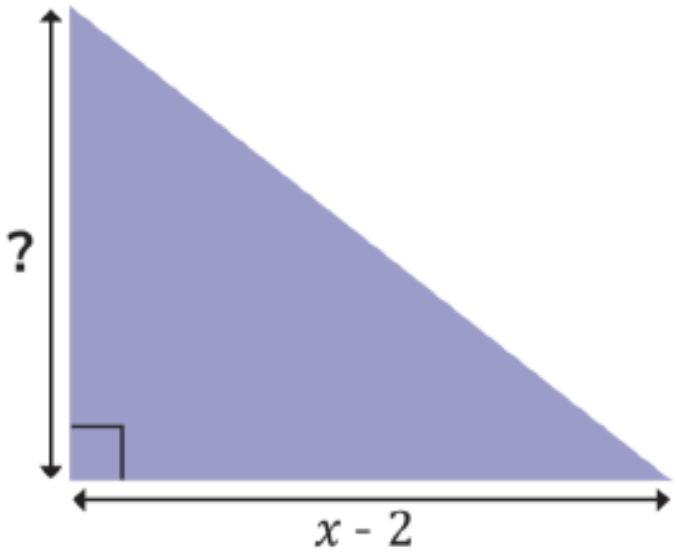
9) $49x^6 - 81y^6$

Applied

1. Solve $88^2 - 87^2$ without using a calculator.

2. The area of the triangle is equal to $\frac{1}{2}x^2 - 2$.

   Write an expression for the height of the shape.

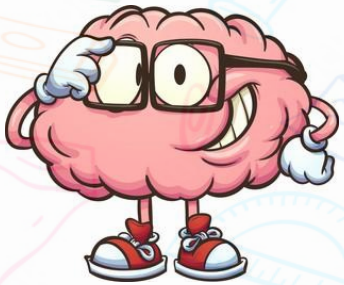


- $64x^2 - 9$
- $x^2 - 121$
- $3x^2 - 9$
- $3x^2 - 75$
- $25x^2 - 50$

- $144x^2 - 169$
- $25x^2 - 9$
- $25x^2 - 10$
- $14x^2 - 7$
- $12x^2 - 27$

- $(4x - 5)^2$
- $5(4p + 5)^2$
- $4(5p - 1)^2$
- $(4n - 3)^2$
- $5(5x - 4)^2$

- $(3k + 1)^2$
- $3(3x + 1)^2$
- $2(x - 4)^2$



Factor by grouping

Factor by grouping is writing the polynomial as a product of its factors.

Example: Factor the following expression

$$x^2 - 8x - 2x + 16$$

$$= \left(x^2 - 8x\right) + (-2x + 16)$$

$$= x(x - 8) + -2(x - 8)$$

$$= (x - 8)(x - 2)$$

Factor the expressions

1) $12b^3 - 9b^2 + 4b - 3$

2) $2y + 10 + 5y^3 + 25y^2$

3) $5x^3 - 15x^2y - 4xy + 12y^2$

Factoring g Trinomials by Trial and Error

Example: Factor the trinomial $x^2 + 7x + 12$ as a product of two binomials

Examples: **Factor the following expressions**

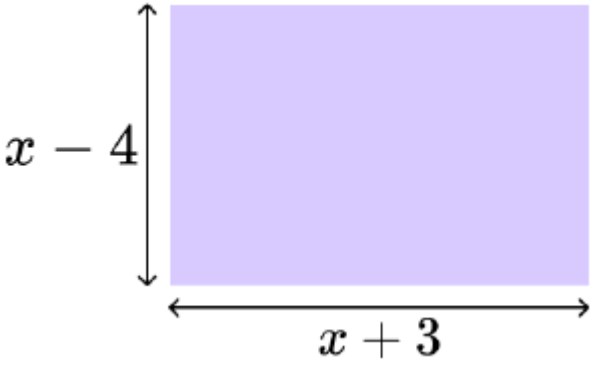
1) $x^2 + 5x - 24$

2) $5x^2 + 16x + 3$

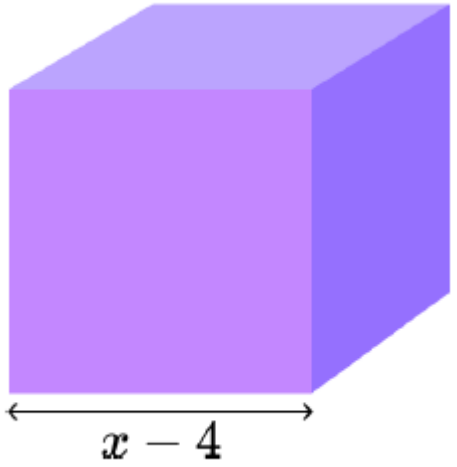
3) $\mathbf{4}x^2 - 12x + 5$

Applied

1) The length of the rectangle is $x + 3$ and the width is $x - 4$.
Write an expression for the area of the rectangle in expanded form.

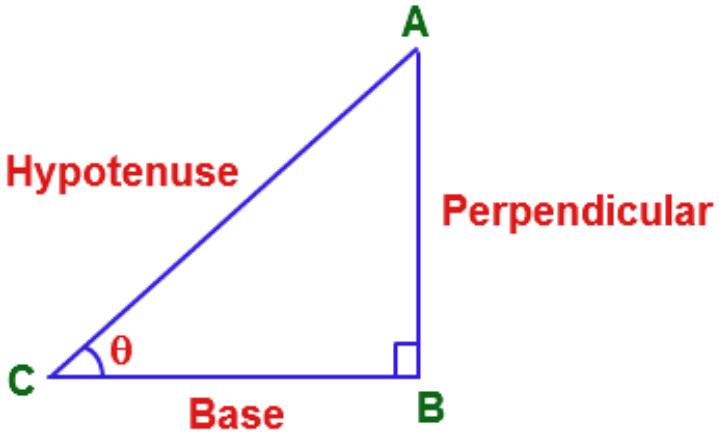


2) The base of the cube is equal to $x - 4$.
Write an expression for the volume of the cube in expanded form.



Pythagoras theorem

Pythagoras theorem states that the square of the longest side of a right angled triangle (called the hypotenuse) is equal to the sum of the squares of the other two sides.

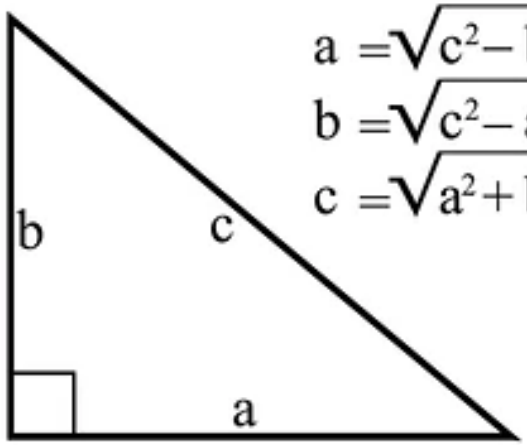


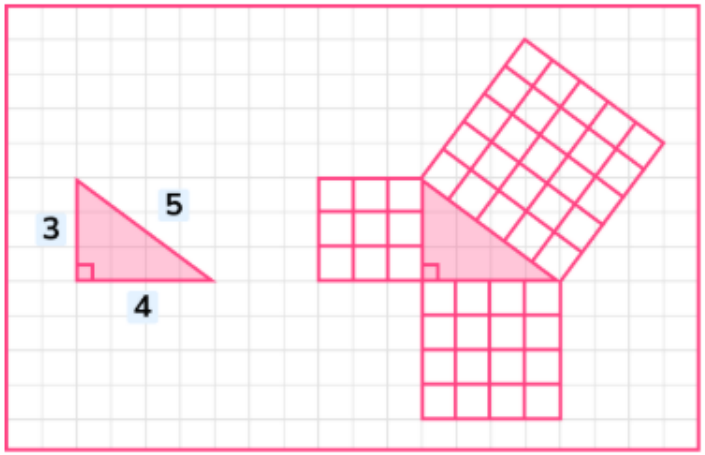
$$c^2 = a^2 + b^2$$

$$a = \sqrt{c^2 - b^2}$$

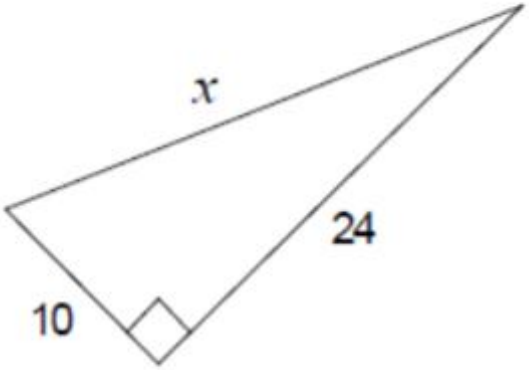
$$b = \sqrt{c^2 - a^2}$$

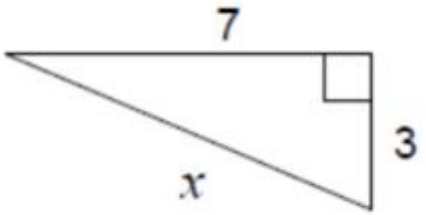
$$c = \sqrt{a^2 + b^2}$$

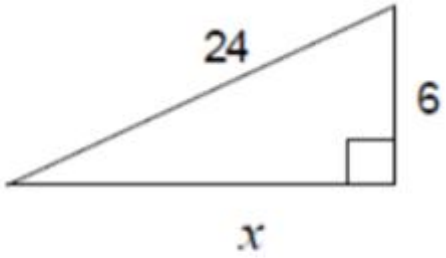


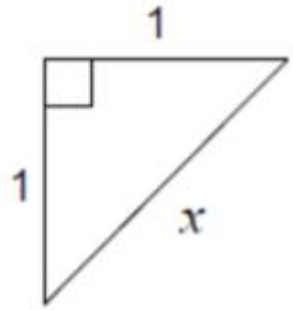


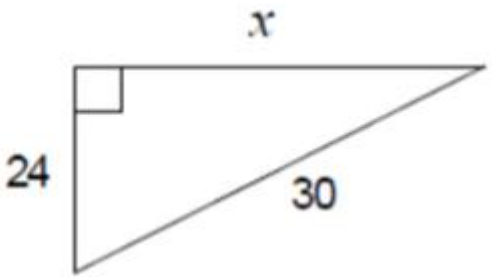
Find the value of x and y for the given triangles

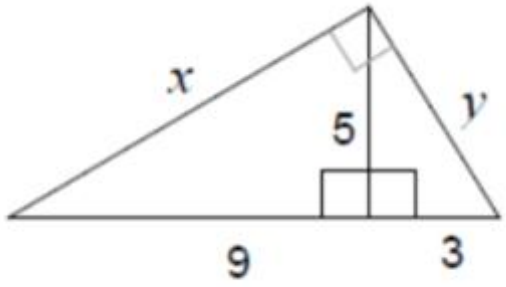






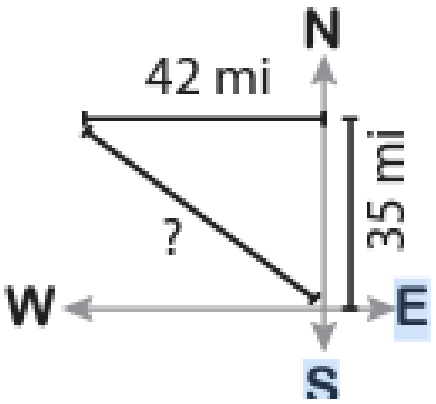




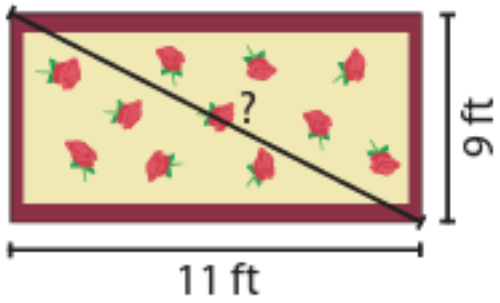


Practice

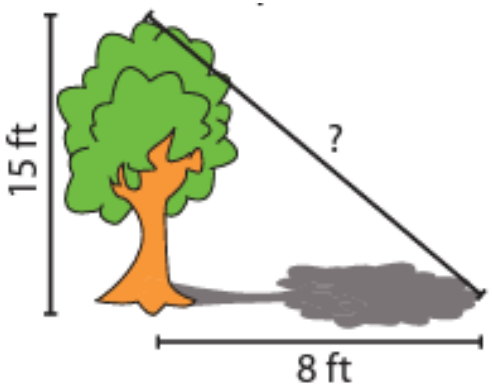
Adam is on his way home from work. He drives 35 miles due North and then 42 miles due West. Find the shortest distance he can cover to reach home early.



Rachel bought a rug for her apartment. The rug is 11 feet long and 9 feet wide. Find the diagonal length of the rug.



A 15 feet tree casts a shadow that is 8 feet long. What is the distance from the tip of the tree to the tip of its shadow?



Rationalizing the denominator

Rationalizing the denominator is an algebraic technique used to eliminate radical expressions (like square roots, cube roots, etc.) from the denominator of a fraction. The goal is to rewrite the fraction in an equivalent form where the denominator is a rational number (an integer or a fraction without radicals).

For example,

$$\frac{8}{\sqrt{2}} = \frac{8 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

$$\frac{1}{7+\sqrt{5}} = \frac{1}{7+\sqrt{5}} \times \frac{7-\sqrt{5}}{7-\sqrt{5}}$$

$$= \frac{7-\sqrt{5}}{(7)^2 - (\sqrt{5})^2}$$

$$= \frac{7-\sqrt{5}}{49-5}$$

$$= \frac{7-\sqrt{5}}{44}$$

Practice

- $\frac{2}{\sqrt{2}}$
- $\frac{1}{3-\sqrt{5}}$
- $\frac{1}{\sqrt{9}-\sqrt{8}}$
- $\frac{y+1}{5+2\sqrt{11}}$
- $\frac{5}{\sqrt{5}}$
- $\frac{2}{4+\sqrt{3}}$
- $\frac{\sqrt{8}}{\sqrt{24}}$
- $\frac{x-2}{6-7\sqrt{2}}$
- $\frac{\sqrt{5}}{\sqrt{45}}$
- $\frac{6}{5-\sqrt{2}}$
- $\frac{x}{4-3\sqrt{7}}$
- $\frac{1}{\sqrt{x}-\sqrt{y}}$

Solving Quadratic Equations By Factorizing

Solve $x^2 - 8x + 15 = 0$

$$(x - 3)(x - 5) = 0$$

$$x - 3 = 0 \qquad x - 5 = 0$$

$$x = 3 \qquad x = 5$$

Solve by factorizing:

1) $x^2 + 9x + 20 = 0$

2) $x^2 - 21x + 20 = 0$

3) $x^2 - 12x + 20 = 0$

4) $x^2 + 8x - 20 = 0$

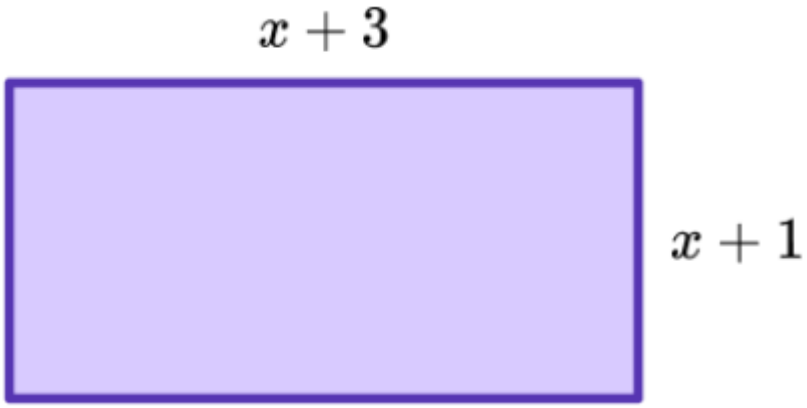
5) $2\,x^2 + 11x + 15 = 0$

6) $2\,x^2 - 11x + 15 = 0$

7) $3\,x^2 + 12x + 12 = 0$

Applied

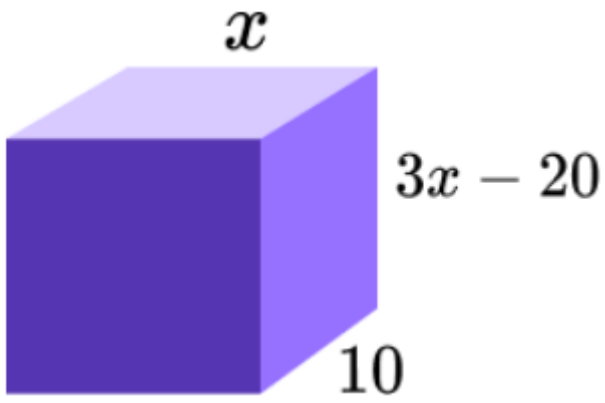
1) The area of the rectangle is $15cm$.



a) Form an equation for the area of this rectangle.

b) Determine the value of x.

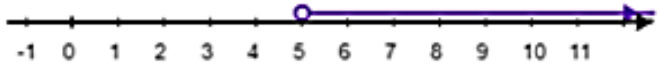
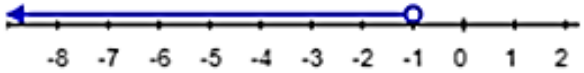
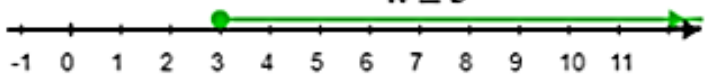
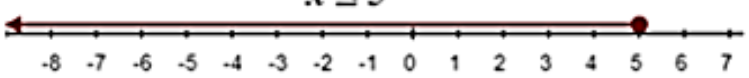
2) The volume of the cuboid is 1000cm^3



a) Show that $3x^2 - 20x - 100 = 0$

b) Solve $3x^2 - 20x - 100 = 0$. To find x , the length of the cuboid.

Inequalities

Symbol	Words	Example
>	Greater than	$x > 5$ 
<	Less than	$x < -1$ 
≥	Greater than or equal to	$x \geq 3$ 
≤	Less than or equal to	$x \leq 5$ 

Solving the inequalities

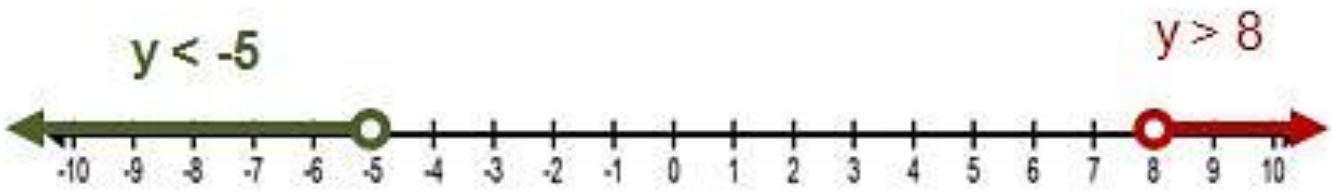
$y - 3 > 5$ or $y + 3 < -2$

$y - 3 + 3 > 5 + 3$ or $y + 3 - 3 < -2 - 3$

$y > 8$ or $y < -5$

Our Solutions

Now we must graph both solutions on one number line.



Solve the inequalities below:

1) $3x + 9 \leq 5x + 3$

2) $10 + 5x \geq 22 + x$

3) $\frac{x+2}{7} \geq 6 - x$

4) $\frac{x+1}{3} \leq \frac{x-1}{2}$

Applied

1) **(a)** Solve the linear inequality $3 < 2x + 1 \leq 9$

(b) Show the solution on a number line.

(c) List the integer values that satisfy the inequality.

2) **(a)** Solve the linear inequality $1 - 2x < 11$

(b) Show the solution on a number line.

(c) What is the smallest integer that satisfies the inequality?

Practice

a) If 5 times a number is increased by 4, the result is at least 19. Find the least possible number that satisfies these conditions.

b) The sum of twice a number and 5 is at most 15. What are the possible values for the number?

c) Three times a number increased by 8 is no more than the number decreased by 4. Find the number.