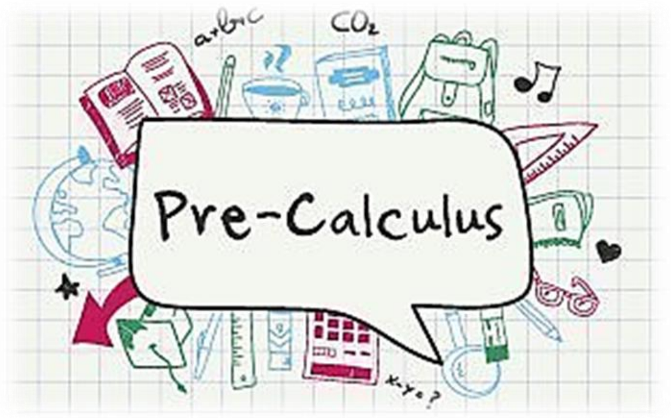
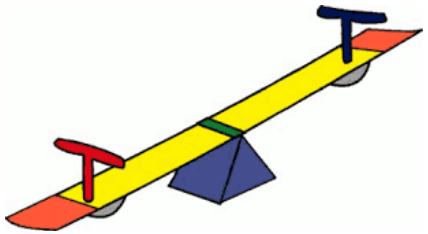
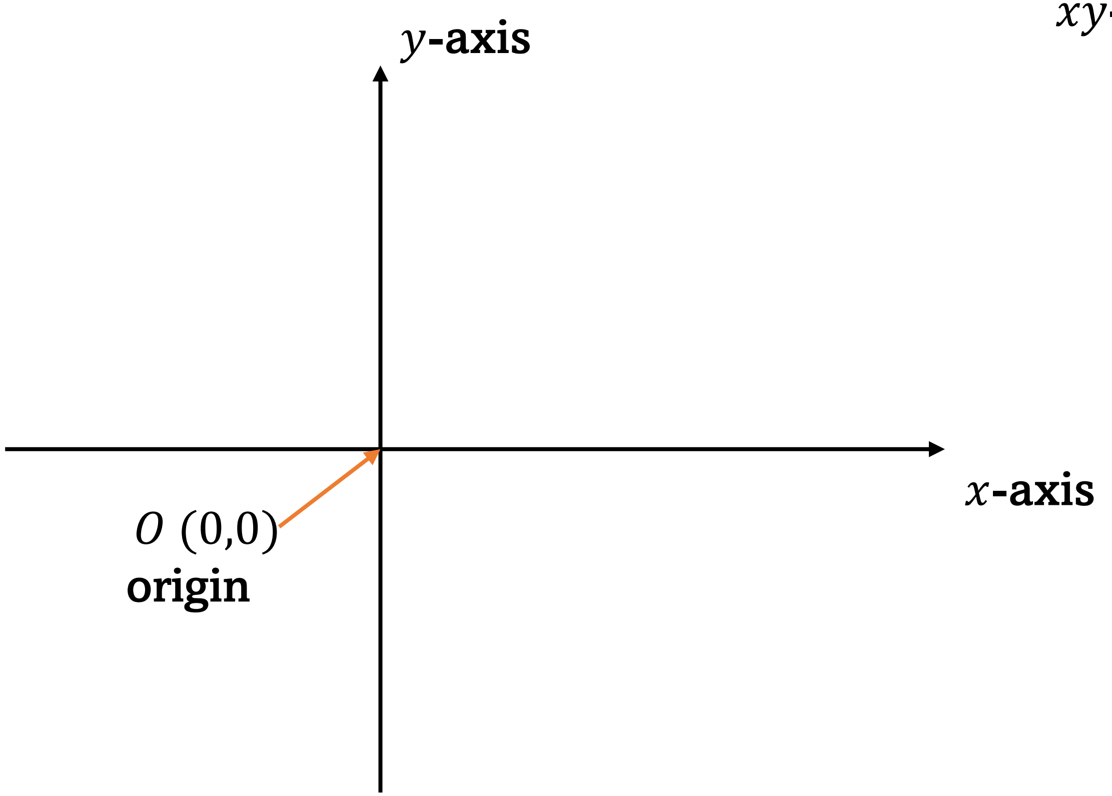


Lecture 3

- Distance and Midpoint
- Absolute Value: equation & inequality
- Line Equation
- Systems of Equations



The Cartesian Plane



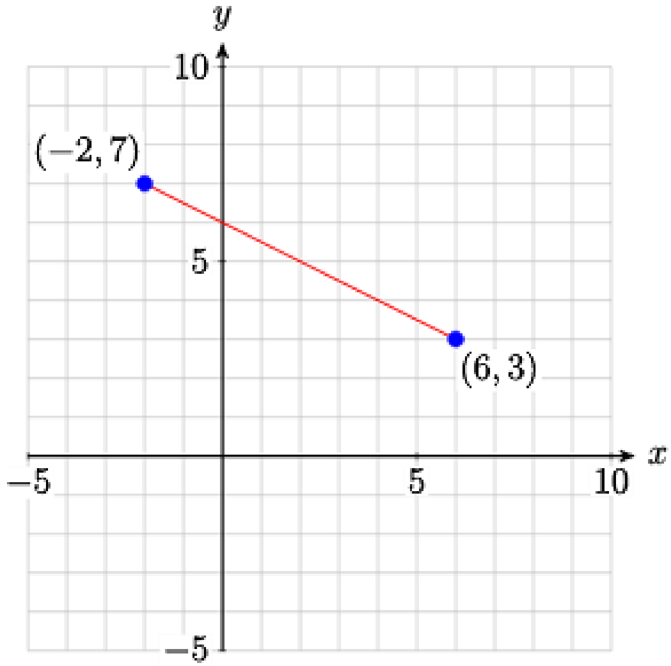
xy-plane – Cartesian plane

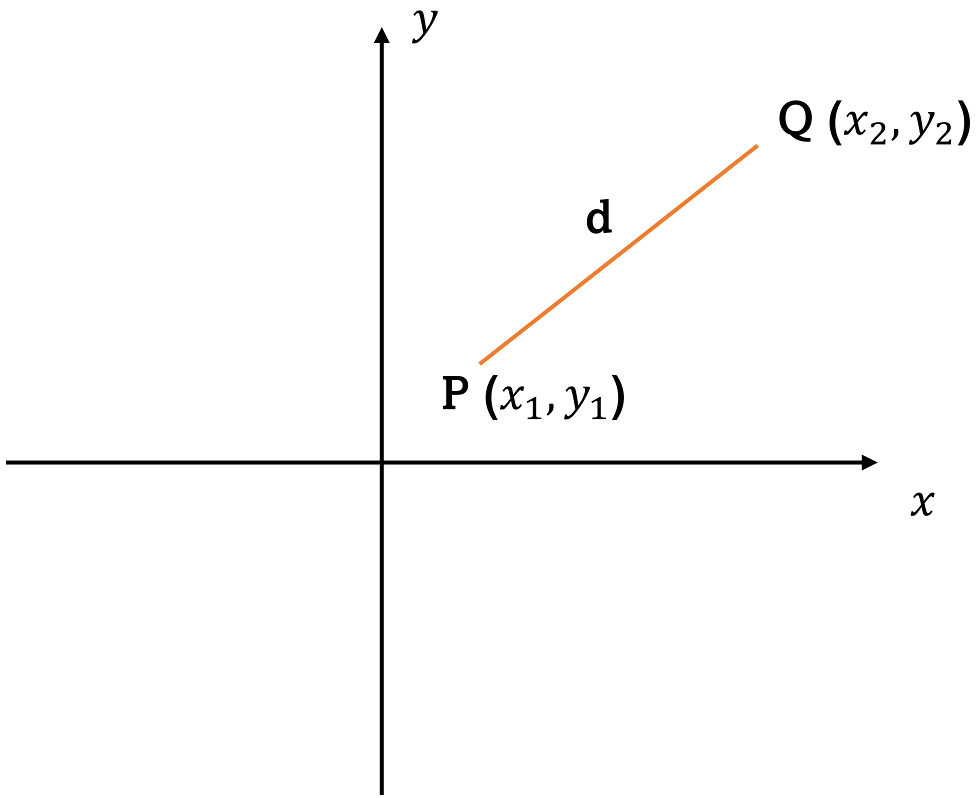
Plot points on a graph:

$P\ (2, 1)$
$Q\ (6, 4)$
$R\ (6, 1)$

Distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

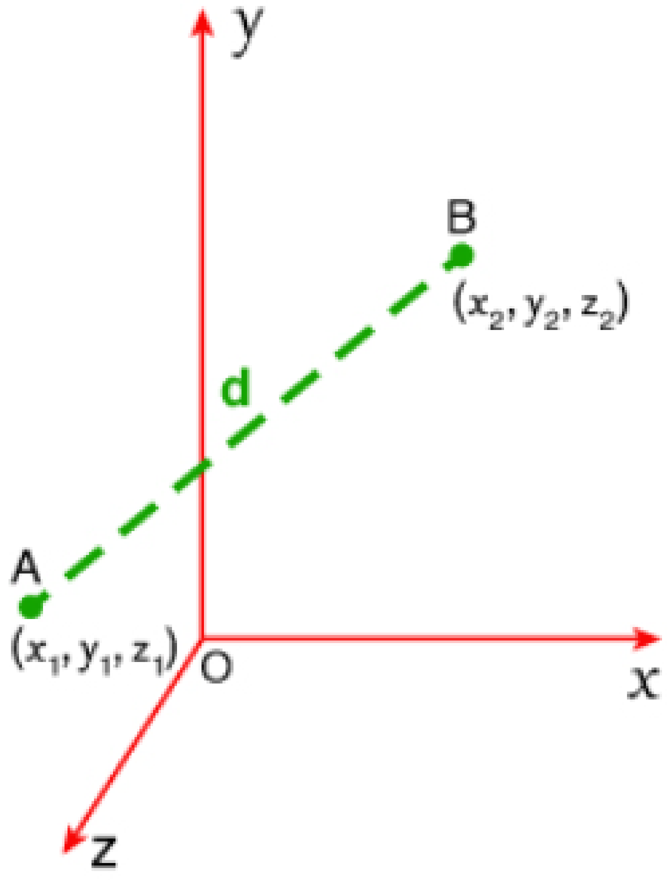




Distance between Two Points in a 3D Plane

the distance formula is also used to calculate their distance in a three-dimensional (3-D) plane. If we consider two points A (x_1, y_1, z_1) and

B (x_2, y_2, z_2) in a three-dimensional plane, then the distance between the points is given by



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Generalizing this to *n*-dimensional Euclidean space

the distance $d(x, y)$ between two points $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_n - y_n)^2}$$

$$= \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

Distance between Point and Line

The distance d between the **point** with coordinates (x_0, y_0) ,and
the **line** written in the general form $ax + by + c = 0$ is calculated as follows.

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

Example : Find the distance between the point $(0, 0)$ and the line $3x + 4y + 10 = 0$.

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

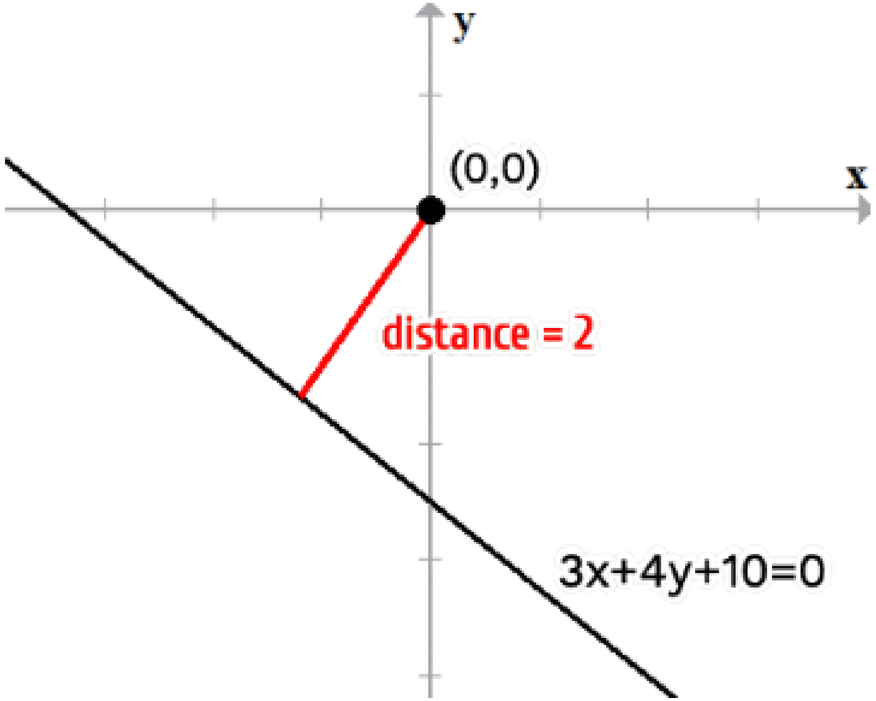
$$= \frac{|3(0) + 4(0) + 10|}{\sqrt{3^2 + 4^2}}$$

$$= \frac{|0 + 0 + 10|}{\sqrt{9 + 16}}$$

$$= \frac{|10|}{\sqrt{25}}$$

$$= \frac{10}{5}$$

$$d = 2$$



Example 2: Find the distance between the point $(3, -4)$ and the line $6x - 8y = 5$.

Distance Between Two Parallel Lines

If the equations of parallel lines are given:

$$ax + by + c_1 = 0$$

$$ax + by + c_2 = 0$$

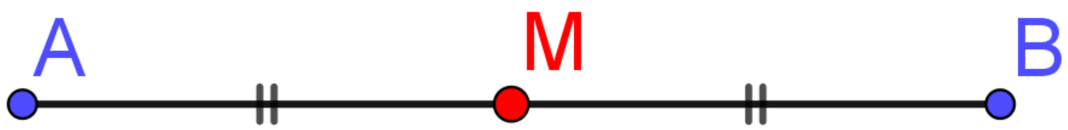
The distance between these parallel lines can be calculated by

$$d = \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$$

Example: What will be the distance between two lines 5x + 3y + 9 = 0 and 5x + 3y – 9 = 0? Find this by using the distance between two lines formula.

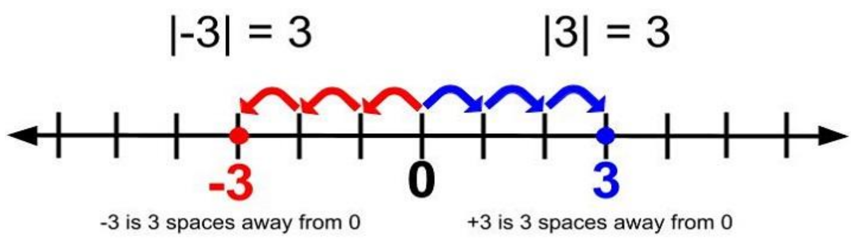
Midpoint

m $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



Find midpoint of a segment PQ, where $P\ (-2,4)$ and $Q\ (4,-2)$.

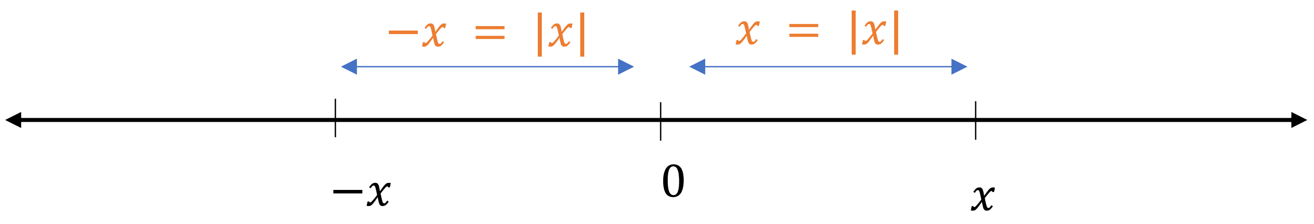
Distance and Absolute Value



- Let $x \in \mathbb{R}$
- Define the absolute value or magnitude of x to be

$$|x| = \begin{cases} x, if\ x \geq 0 \\ x, if\ x < 0 \end{cases}$$

= distance between x and 0 on the real line.



Absolute Value Equation

$$|5 - 2x| - 11 = 0$$

$$|5 - 2x| = 11$$

Isolate the absolute value

Split the equation up into two separate equations

$5 - 2x = 11$	$5 - 2x = -11$
$-2x = 6$	$-2x = -16$
$x = -3$	$x = 8$

Solve each of the equations

Absolute Value Inequalities

For $b > 0$

if $|a| < b$ then $-b < a < b$

if $|a| \le b$ then $-b \le a \le b$

if $|a| > b$ then $a < -b$ or $a > b$

if $|a| \ge b$ then $a \le -b$ or $a \ge b$

Absolute Value Inequality

$$|x - 4| < 7$$

$$x - 4 < 7$$

$$x - 4 + 4 < 7 + 4$$

$$x < 11$$

$$|9 - 4| = |5| = 5$$

$$5 < 7$$

$$x - 4 > -7$$

$$x - 4 + 4 > -7 + 4$$

$$x > -3$$

$$|-2 - 4| = |-6| = 6$$

$$6 < 7$$

$$-3 < x < 11$$

IMPORTANT

$|2x+1| \geq -5$

All real numbers. The absolute value will always be greater than zero.

$|8-x| \leq -3$

No solution. The absolute value will never be less than zero. Just like absolute value cannot be = to a negative number.

Example:

$$|x+3| < -2$$

$$|5+3| < -2$$

$$|8| < -2$$

$$8 < -2$$ False

If an absolute value equation is equal to zero,

$$|x+5| = 0$$

there is one solution.

$$|x| = -3$$

NO SOLUTION

An absolute value can never equal a negative number

Two Solutions	One Solution	No Solutions
$\|x\| = 6$	$\|x\| = 0$	$\|x\| = -6$
$\|2x-5\| = 8$	$\|2x-5\| = 0$	$\|2x-5\| = -8$
$\left\|\frac{2}{3}x-7\right\| = 23$	$\left\|\frac{2}{3}x-7\right\| = 0$	$\left\|\frac{2}{3}x-7\right\| = -23$

Solve the equation.

$\frac{1}{4}\,|2x-6|+1=2$

$-3|x-1|-6=3$

$|\,x-7|+2=2$

$|3x+2|=\,|x-6|$

$|x-4|=\,|4-x|$

Solve the inequality.

$2|x-9|+6\;>6$

$-4|3x-1|\;\geq 8$

$-5|2x+2|-3\;\geq\;-3$

$-10+\frac{1}{2}\,|x-4|\;\geq\;-10$

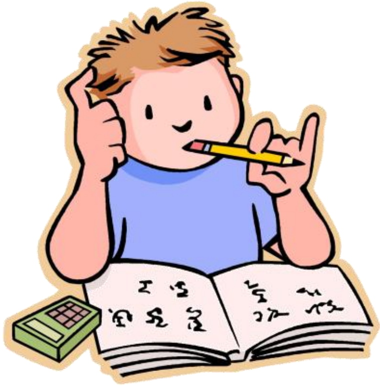
$3\left|\frac{1}{2}\,x+2\right|+6\;<15$

$\frac{1}{4}\|2x-6\|+1=2$	$\{1,5\}$
$-3\|x-1\|-6=3$	No Solution
$\|x-7\|+2=2$	$\{7\}$
$\|3x+2\|=\|x-6\|$	$\{-4,1\}$
$\|x-4\|=\|4-x\|$	$\mathbb{R}$

$2\|x-9\|+6>6$	$(-\infty, 9) \cup (9, \infty)$
$-4\|3x-1\| \geq 8$	No Solution
$-5\|2x+2\|-3 \geq -3$	$\{-1\}$
$-10+\frac{1}{2}\|x-4\| \geq -10$	$\mathbb{R}$
$3\left\|\frac{1}{2}x+2\right\|+6<15$	$(-10, 2)$

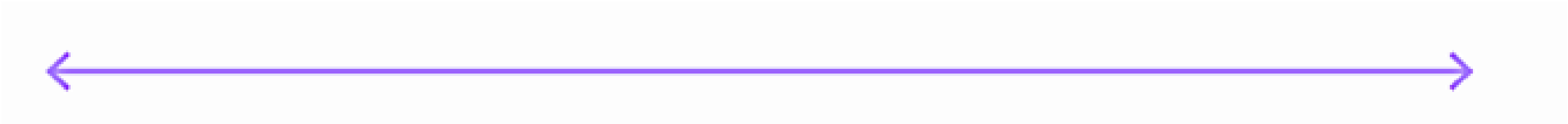
- $|6x + 4| = 8x + 10$
- $|3x - 1| + 5 = 3$
- $|x - 2| = 2x - 3$
- $|12 - 6x| = 42$
- $|2b - 4| = 2b - 4$
- $|2a + 1| = a + 5$
- $-3|x + 5| + 1 = 7|x + 5| + 8$
- $5|c - 2| = 30$

- $|x - 5| > 7$
- $|2x + 3| \leq 12$
- $2|x - 6| \geq 24$
- $|4x - 1| - 11 < 20$
- $-3|x + 2| \leq -9$
- $\frac{|3x-3|}{-5} > -12$
- $8 + |4v - 7| \geq 17$



Lines

- A line is a one-dimensional figure, which has length but no width.
- A line is a set of collinear points with no curves and extends limitlessly in opposite directions is called a Straight line.

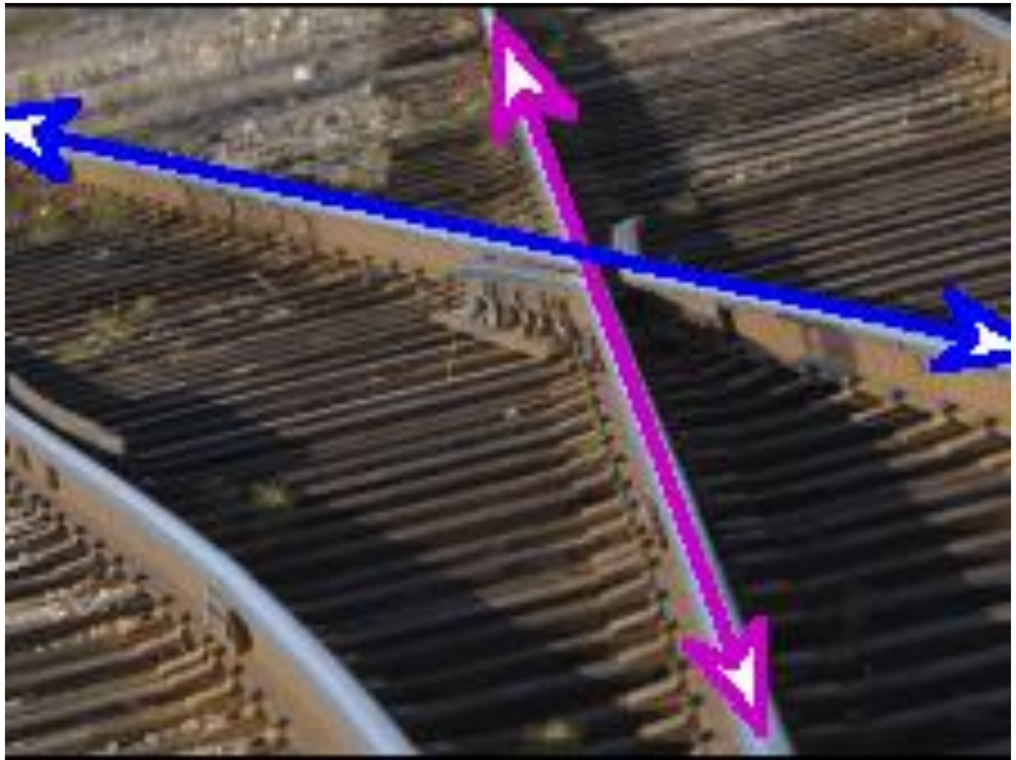


- General equation:

$$ax + by = c \qquad a, b, c \ - constants$$

variables




How do we find the equation of a line?

- $ax + by = c$
- $by = -ax + c$
- $y = \frac{-ax+c}{b}, \qquad b \neq 0$
- $y = \left(-\frac{a}{b}\right)x + \frac{c}{b}$

$$y = mx + k$$

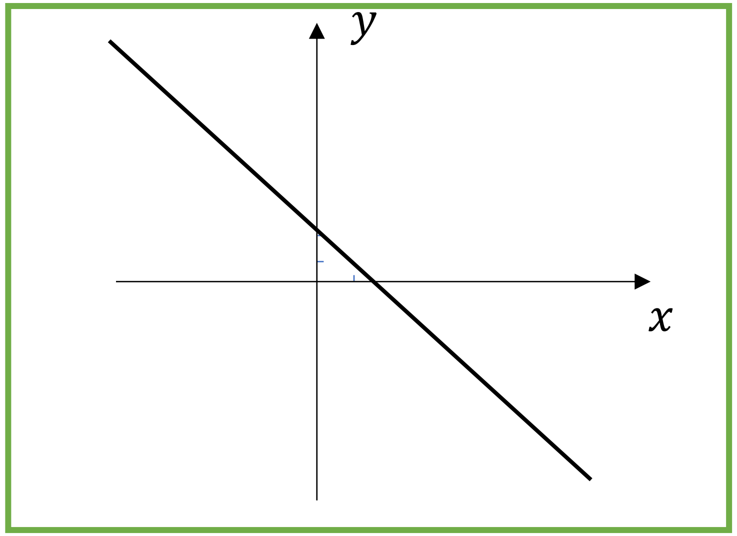
Lines

- Sketch the line.

$$2x + 3y = 6$$

$$3y = 6 - 2x$$

$$y = -\frac{2}{3}x + 2$$



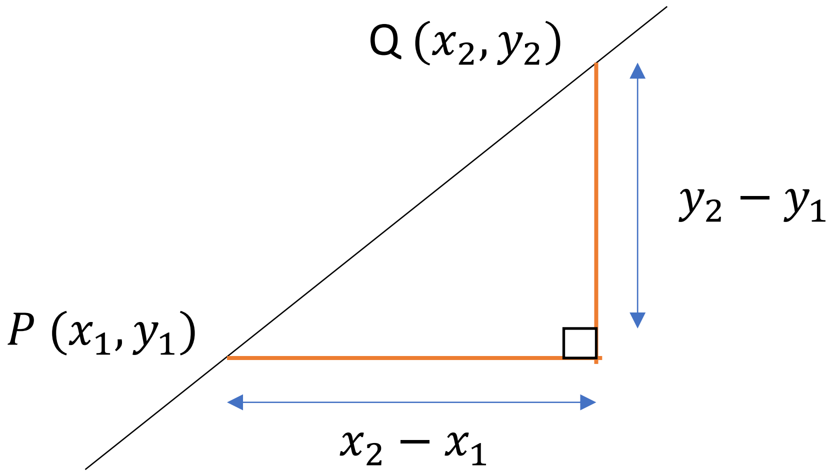
➢ A line is determined by two points.

x	y
0	2
1	$\frac{4}{3}$

Slope

The slope or gradient of a line is a number that describes both the direction and the steepness of the line.

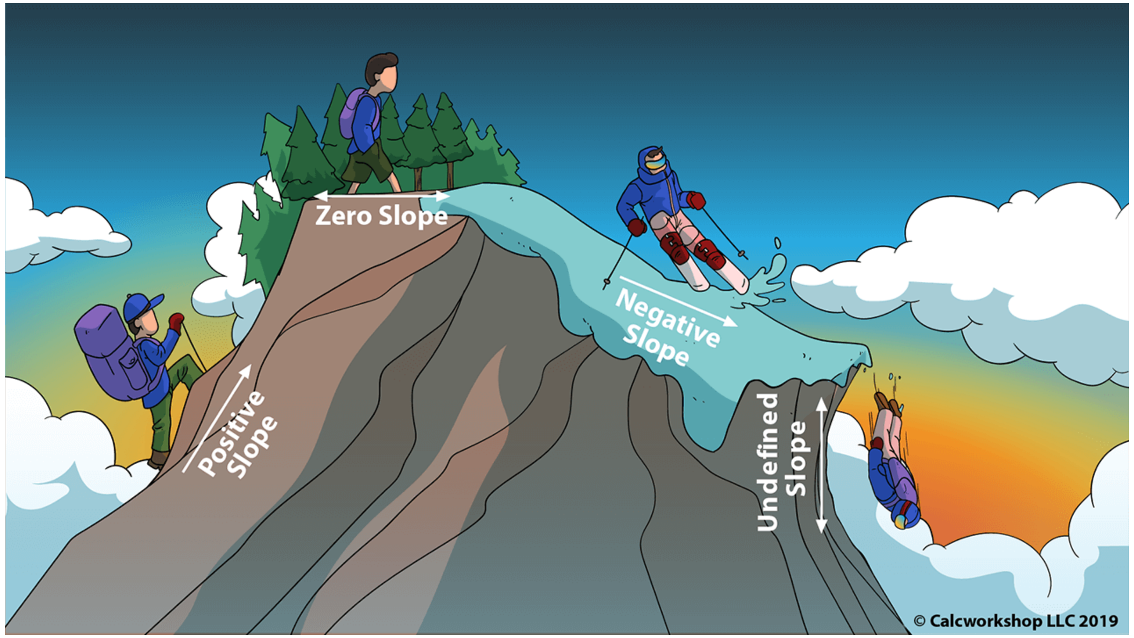
$$m = slope = \frac{vertical\ rise}{horizontal\ run}$$



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

In mathematics, the slope or gradient of a line is a **number that describes both the direction and the steepness of the line.**

Slope tells you how steep a line is, or how much y increases as x increases. The slope is constant (the same) anywhere on the line.



Some real life examples of slope include:

- ✓ in building roads one must figure out how steep the road will be.
- ✓ skiers/snowboarders need to consider the slopes of hills in order to judge the dangers, speeds, etc.
- ✓ when constructing wheelchair ramps, slope is a major consideration.

- Find the slope of the line passing through P (2, 2) and Q (5, 6)

P(x_1, y_1) Q (x_2, y_2)

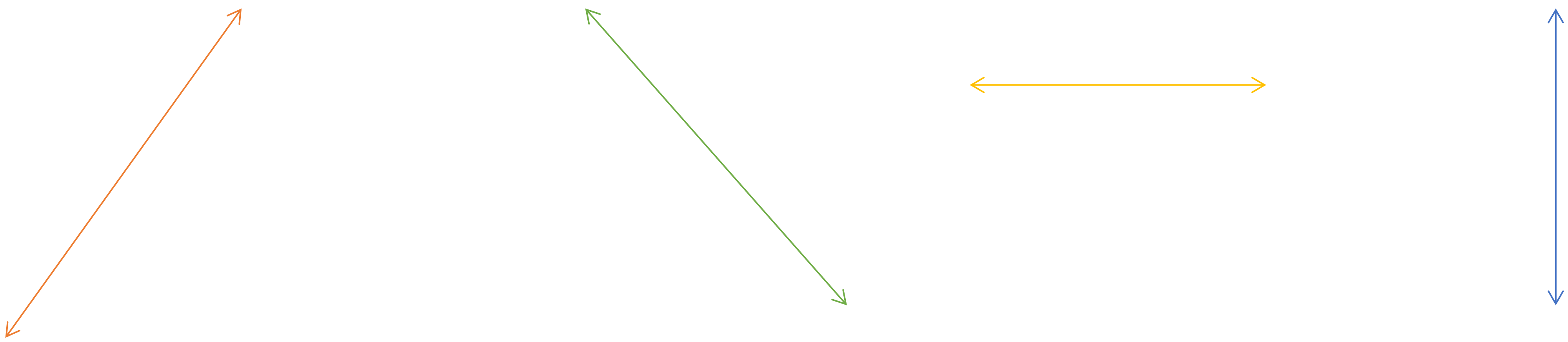
$$slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{5-2} = \frac{4}{3}$$

- Find the slope where P (-1, 2) and Q (5, -4)

TRY THIS!

Slope

- Positive
- Negative
- Zero
- Infinite



The y-intercept and the x-intercept

Intercepts are where lines on graphs cross axes.

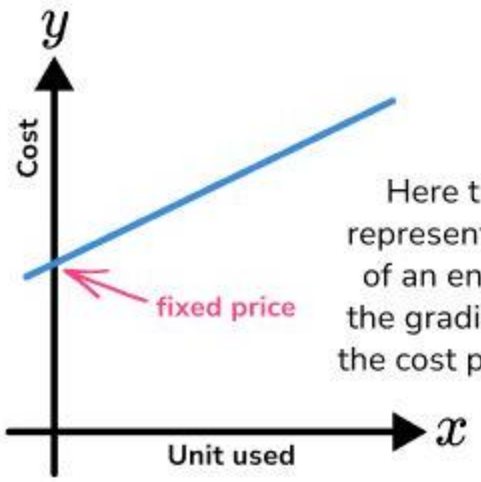
The y-intercept is the value of y when the x coordinate is 0

The x-intercept is the value of x when the y coordinate is 0

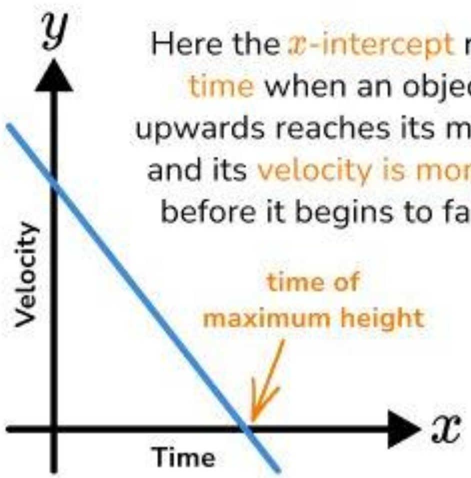
To find the y-intercept, substitute $x = 0$

To find the x-intercept, substitute $y = 0$

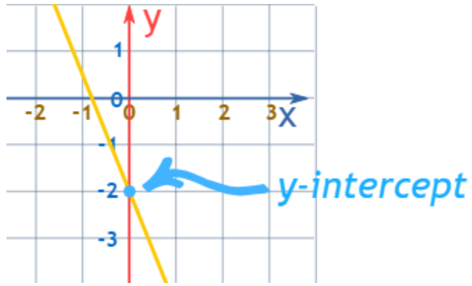
Examples



Here the y-intercept represents the fixed price of an energy tariff, with the gradient representing the cost per unit of energy.



Here the x-intercept represents the time when an object projected upwards reaches its maximum height and its velocity is momentarily zero, before it begins to fall back down.



Find the equation of line L passing through P (2, 3) and Q (7, 13)

1) find a slope.

2) put coordinates of any of given points and find constant ***k***.

1. $m = \frac{13-3}{7-2} = \frac{10}{5} = 2 \quad (slope)$

2. $y = mx + k$ – *line equation*

- $y = 2x + k \qquad m = 2$
- *P(2,3):* $3 = 2 \times 2 + k$

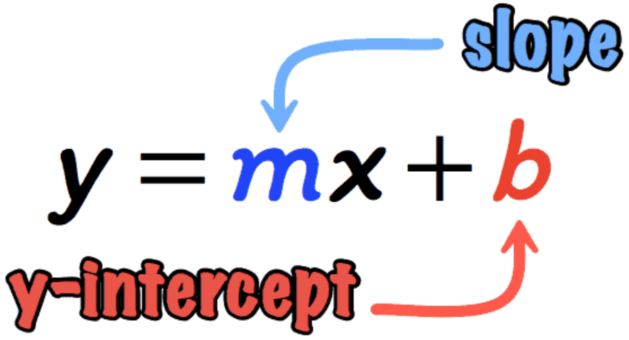
$$3 = 4 + k$$

$$k = 3 - 4$$

$$k = -1$$

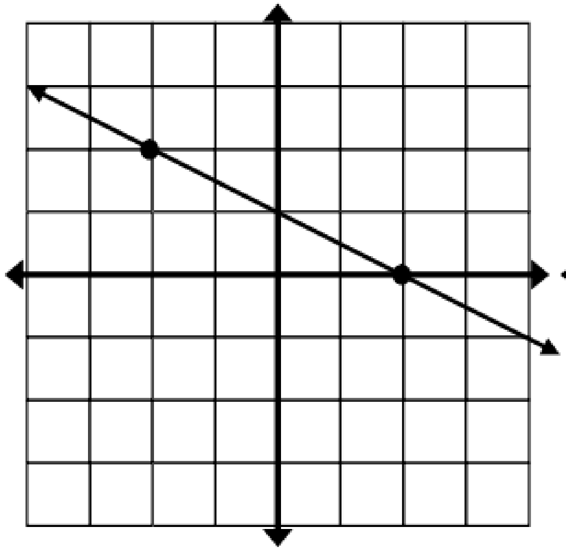
- $m = 2, k = -1$

$y = 2x - 1$

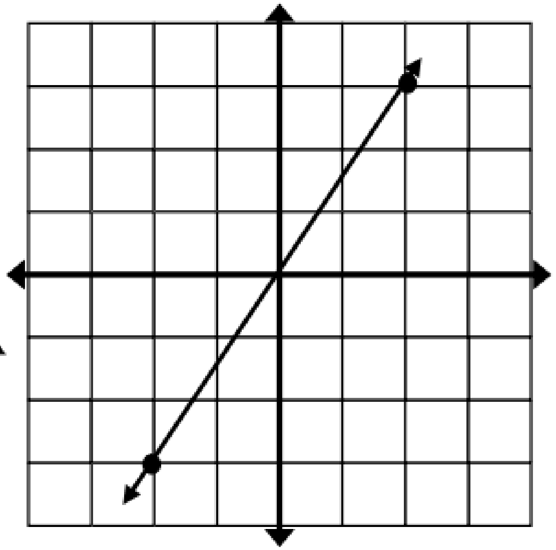


1) **<u>For each graph: Write the equation of the line in SLOPE-INTERCEPT FORM.</u>**

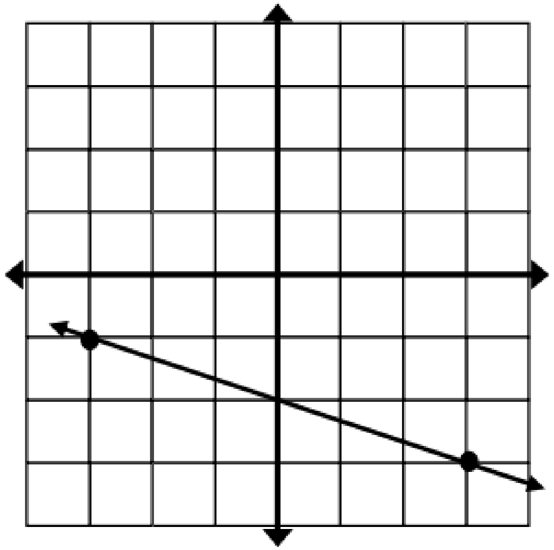
a.



b.



c.



a.

Slope = ___________

y-intercept = __________

equation: _____________

b.

Slope = __________

y-intercept = __________

equation: _____________

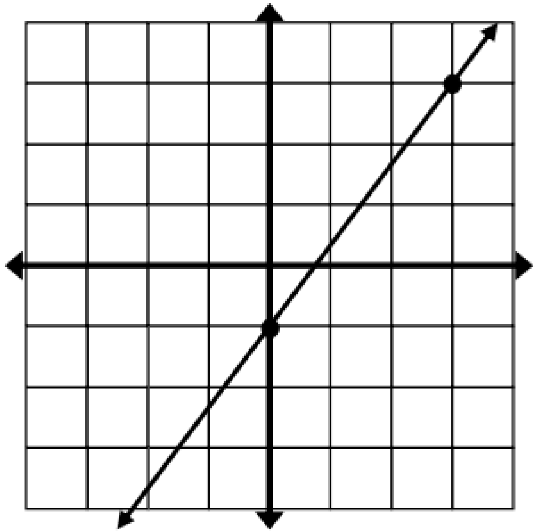
c.

Slope = __________

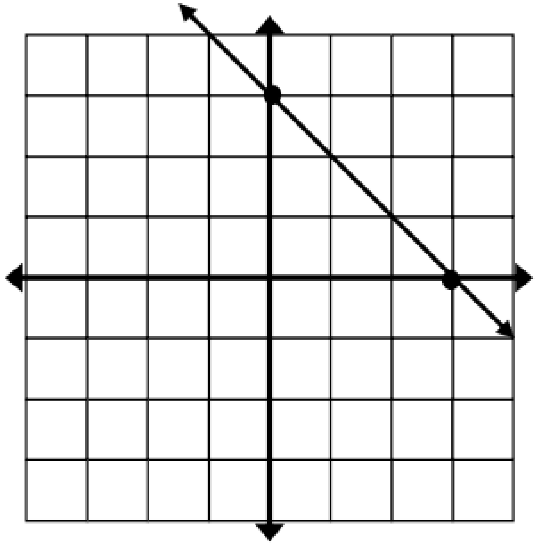
y-intercept = __________

equation: _____________

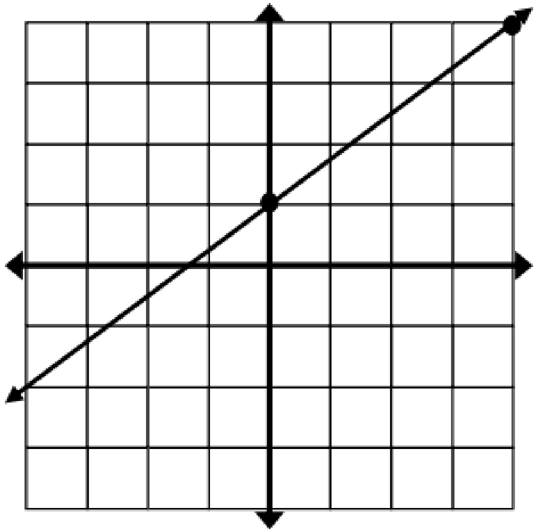
d.



e.



f.



Slope = __________ Slope = __________ Slope = __________

y-intercept = __________ y-intercept = __________ y-intercept = __________

equation: ____________ equation: ____________ equation: ____________

Practice Problems

1. Find the equation of the line that passes through the point (1, 4) and has a slope of 12.

2. Find the equation of the line that passes through the point (1, 4) and has a slope of 2.

3. Find the equation of the line that passes through the point (27, 4) and has a slope of $\frac{-2}{9}$.

4. Find the equation of the line that passes through the point $(-11, 2)$ and has a slope of $\frac{-5}{11}$.

5. Find the equation of the line that passes through the point (10, 6) and has a slope of $\frac{1}{5}$. What is the y-intercept of the line?

6. Find the equation of the line that passes through the point (3, 29) and has a slope of 6. What is the y-intercept of the line?