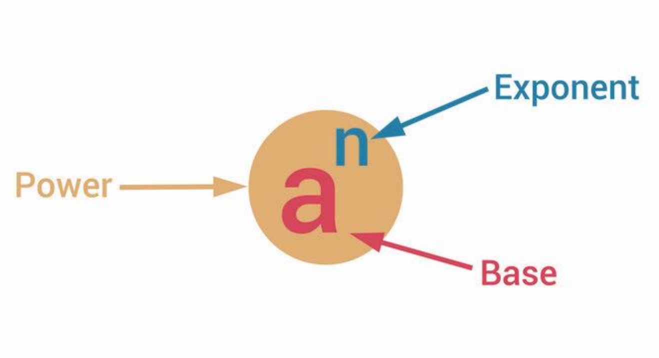
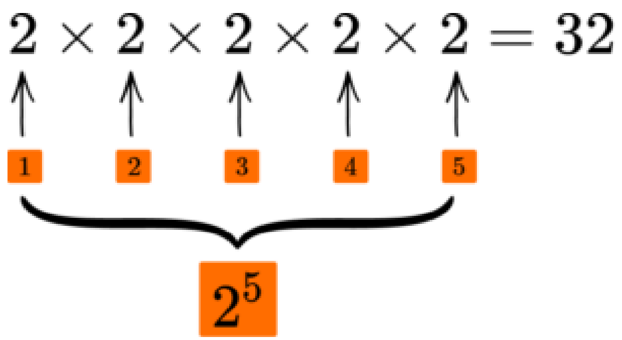


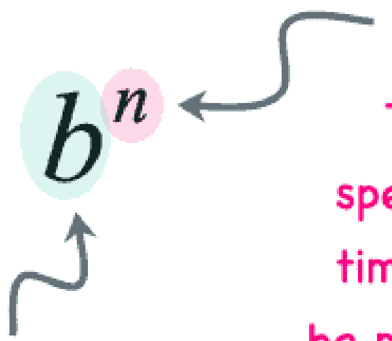
Lecture 5:

- Powers
- Logarithms

Exponents / Powers

$$\underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}_{8\ times} = 5^8$$





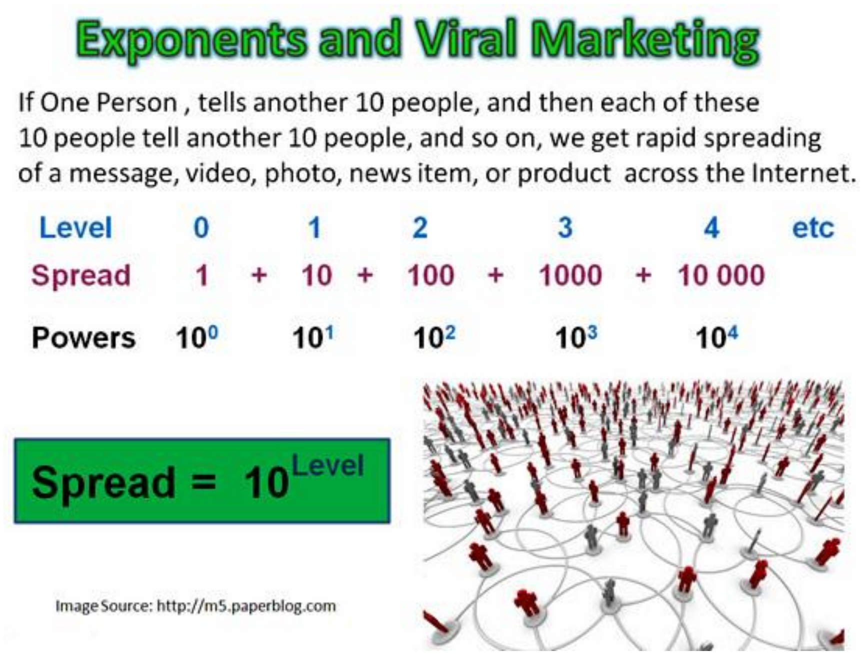
Exponent

The value that specifies how many times the base will be multiplied by itself

Base

The number or variable that is being multiplied repeatedly in the expanded form

$$7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^9$$



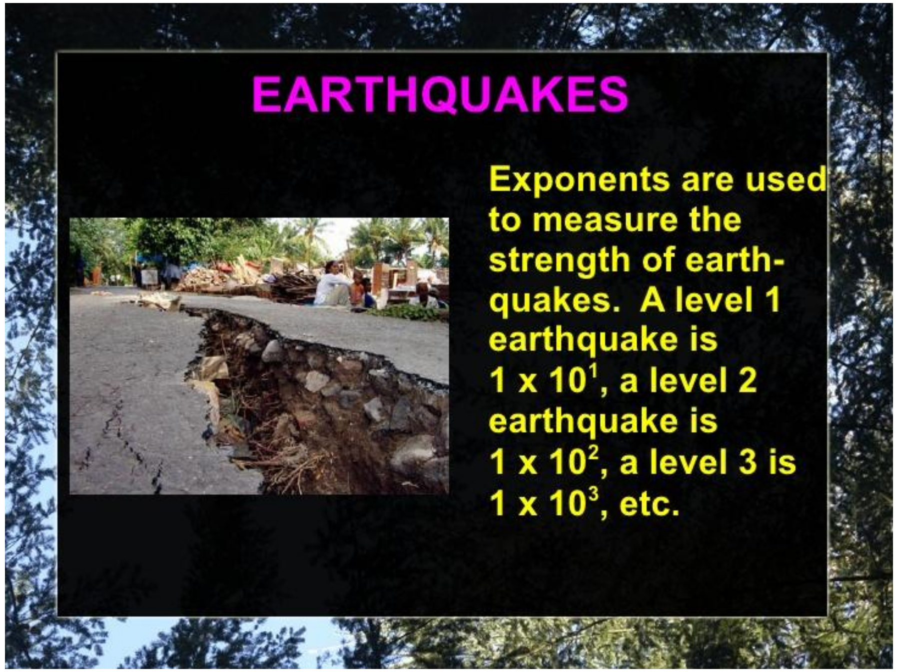
Exponents and Viral Marketing

If One Person , tells another 10 people, and then each of these 10 people tell another 10 people, and so on, we get rapid spreading of a message, video, photo, news item, or product across the Internet.

Level	0	1	2	3	4	etc
Spread	1 +	10 +	100 +	1000 +	10 000	
Powers	10^0	10^1	10^2	10^3	10^4	

$$\text{Spread} = 10^{\text{Level}}$$

Image Source: http://m5.paperblog.com

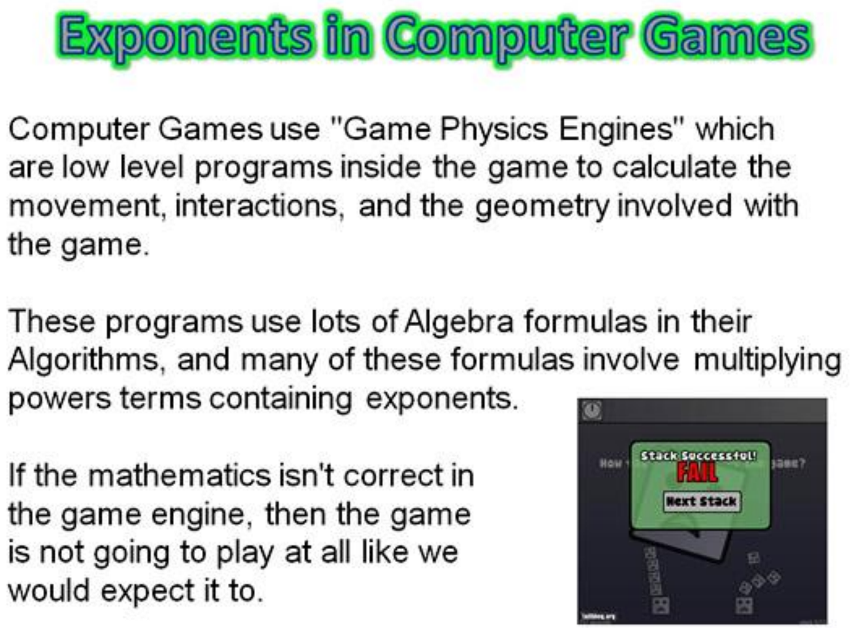


EARTHQUAKES

Exponents are used to measure the strength of earthquakes. A level 1 earthquake is 1×10^1, a level 2 earthquake is 1×10^2, a level 3 is 1×10^3, etc.

Smart Phone Uptake and Sales

At first only a few people had smart phones, then within only a few years, it seems that everybody has an iPhone or similar. Eg. The Growth in Smart Phone usage has been Exponential.



Exponents in Computer Games

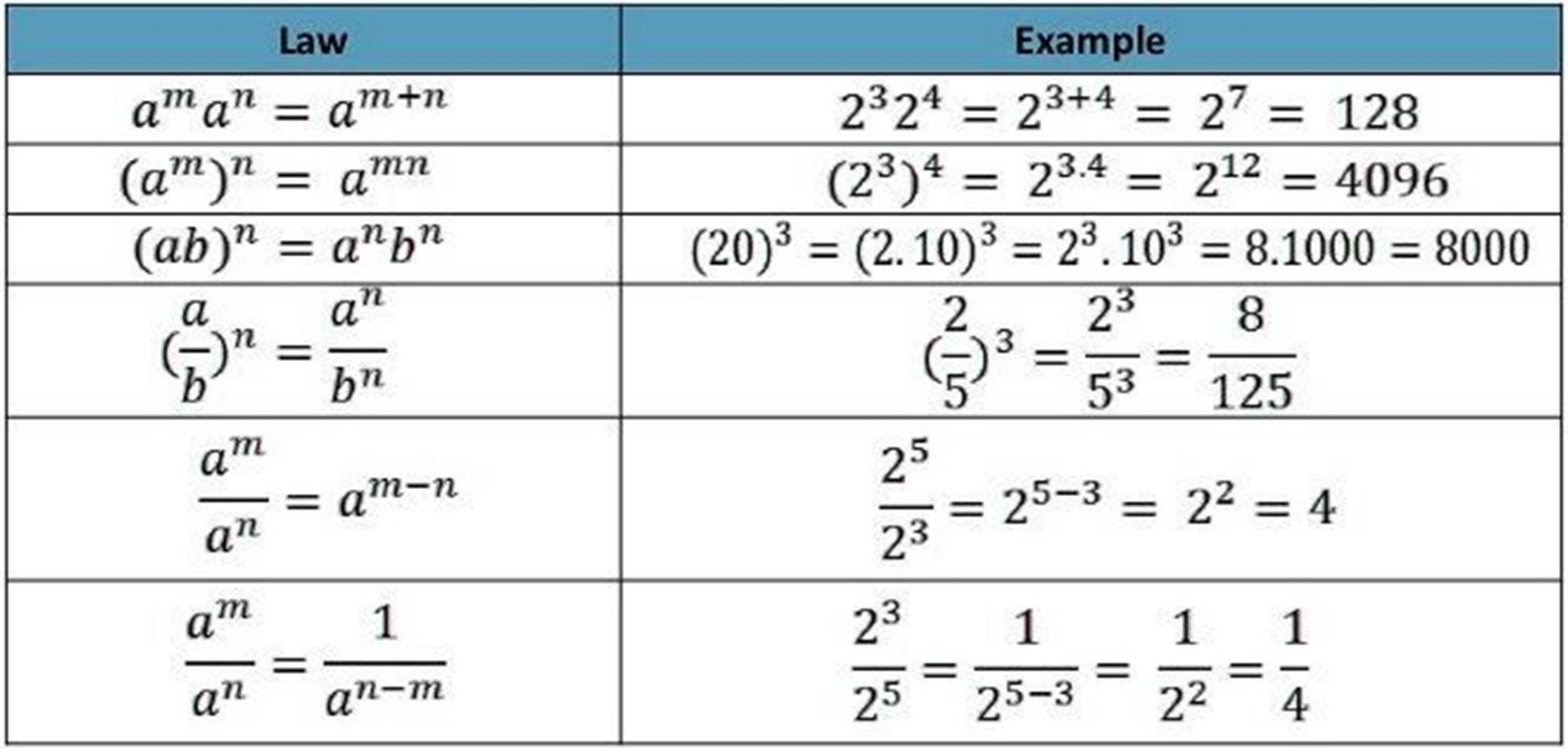
Computer Games use "Game Physics Engines" which are low level programs inside the game to calculate the movement, interactions, and the geometry involved with the game.

These programs use lots of Algebra formulas in their Algorithms, and many of these formulas involve multiplying powers terms containing exponents.

If the mathematics isn't correct in the game engine, then the game is not going to play at all like we would expect it to.

Exponents/Powers Properties

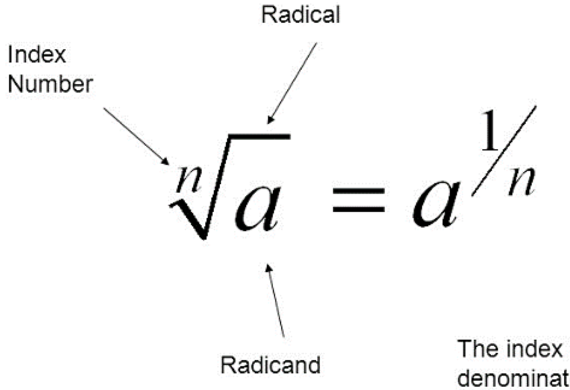
RULES



Law	Example
$a^m a^n = a^{m+n}$	$2^3 2^4 = 2^{3+4} = 2^7 = 128$
$(a^m)^n = a^{mn}$	$(2^3)^4 = 2^{3.4} = 2^{12} = 4096$
$(ab)^n = a^n b^n$	$(20)^3 = (2.10)^3 = 2^3.10^3 = 8.1000 = 8000$
$(\frac{a}{b})^n = \frac{a^n}{b^n}$	$(\frac{2}{5})^3 = \frac{2^3}{5^3} = \frac{8}{125}$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$
$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{2^3}{2^5} = \frac{1}{2^{5-3}} = \frac{1}{2^2} = \frac{1}{4}$

Exponents/Powers with Roots

RULES



$$\sqrt[n]{a} = a^{1/n}$$

$n > 1$

The index number becomes the denominator of the exponent.

$nth\ root$ $function\ or\ fractional\ power$:

$$\sqrt{2} = 2^{\frac{1}{2}}$$

$$\sqrt[3]{7} = 7^{\frac{1}{3}}$$

$$\sqrt[4]{5^3} = 5^{\frac{3}{4}}$$

…

Negative Exponents

$$a^{-n} = \frac{1}{a^n}$$

For $a \neq 0$

a^{-n} is a reciprocal of a^n

Example:

$$3^{-2} = \frac{1}{3^2}$$

$$\left(\frac{2}{5}\right)^{-6} = \left(\frac{5}{2}\right)^6$$

Logarithms

Logarithms play a crucial role in various aspects of computer science, from algorithms and data structures to cryptography and information theory.

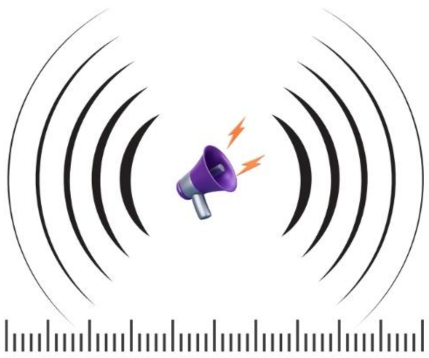
Logarithms

- In computer science, we almost always work with logarithms base 2, because we work with bits
- $\log_2 n$ (or we can just write $\log n$) tells us how many bits we need to represent n possibilities
  - Example: To represent 10 digits, we need $\log 10 = 3.322$ bits
- Logarithms also tell us how many times we can cut a positive integer in half before reaching 1
  - Example: $16/2=8$, $8/2=4$, $4/2=2$, $2/2=1$, and $\log 16 = 4$
  - Example: $10/2=5$, $5/2=2.5$, $2.5/2=1.25$, and $\log 10 = 3.322$

Data compression

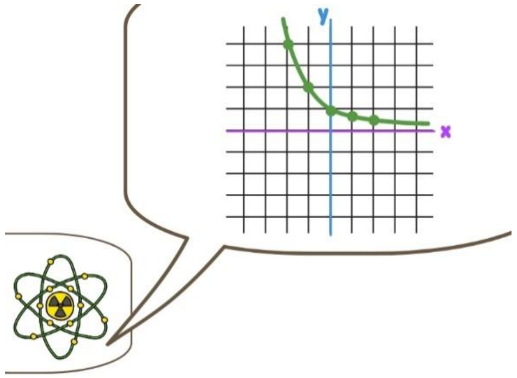
- Ever wondered how such large data obtained from various organizations and governments is stored and used? Storing such large data requires various storage devices, which will still be insufficient. In that case, the data compression technique is used, which uses logarithms to simplify and compress the data.

- The **data is compressed** using different coding processes, such as arithmetic, transform, Huffman, Delta, entropy, Shannon-Fano, run-length encoding, etc., based on logarithm and its application.



Measuring the sound intensity

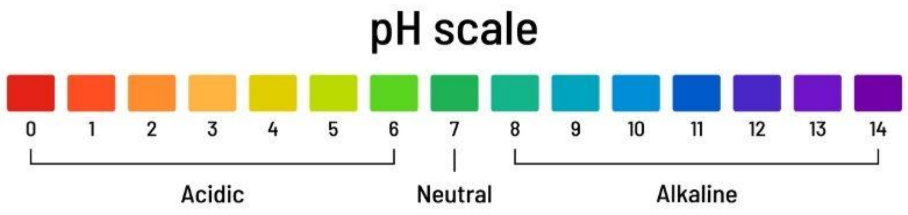
Stock market analysis



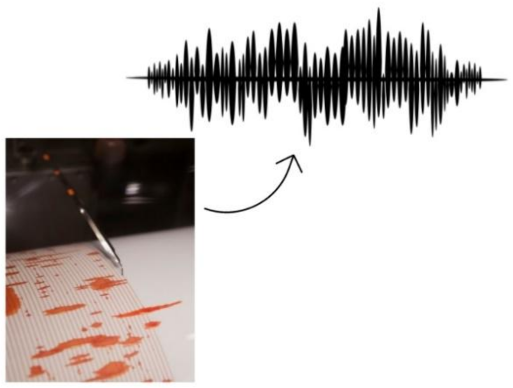
Studying the process of decay of radioactive elements

Analyzing drug concentrations in medicines

Calculating the growth of the human species or other living species



Measuring pH levels of chemicals



Assessing the magnitude of earthquakes using the Richter scale

Logarithmic Function

exponent

$$\log_a x = y \quad \text{means} \quad a^y = x$$

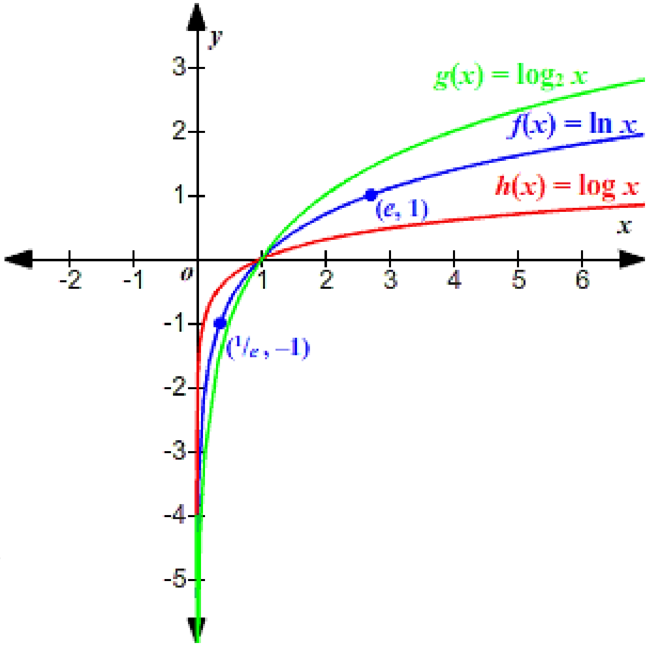
base

$$a > 0,\ a \neq 1, y \neq 0$$

Example:

$$\log_2 8 = 3 \quad \text{means} \quad 2^3 = 8$$

Graph of logarithmic functions



List of logarithm rules

$$
\begin{aligned}
\log_a(bc) &= \log_a(b) + \log_a(c) \\
\log_a(b^c) &= c\log_a(b) \\
\log_a(1/b) &= -\log_a(b) \\
\log_a(1) &= 0 \\
\log_a(a) &= 1 \\
\log_a(a^r) &= r \\
\log_{1/a}(b) &= -\log_a(b) \\
\log_a(b)\log_b(c) &= \log_a(c) \\
\log_b(a) &= \frac{1}{\log_a(b)} \\
\log_{a^m}(a^n) &= \frac{n}{m}, \qquad m \neq 0
\end{aligned}
$$

Natural Logarithm

A special form of logarithms in which **the base is mathematical constant e**

$$\log_e x = \ln x$$

$e \approx 2.718281828459045$

Write the following equalities in exponential form.

(1) $\log_3 81 = 4$

(2) $\log_7 7 = 1$

(3) $\log_{\frac{1}{2}} \frac{1}{8} = 3$

(4) $\log_3 1 = 0$

(5) $\log_4 \frac{1}{64} = -3$

(6) $\log_6 \frac{1}{36} = -2$

(7) $\log_x y = z$

(8) $\log_m n = \frac{1}{2}$

(1) $3^4 = 81$

(2) $7^1 = 7$

(3) $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

(4) $3^0 = 1$

(5) $4^{-3} = \frac{1}{64}$

(6) $6^{-2} = \frac{1}{36}$

(7) $x^z = y$

(8) $m^{\frac{1}{2}} = n$

Write the following equalities in logarithmic form.

(1) $8^2 = 64$

(2) $10^3 = 10000$

(3) $4^{-2} = \frac{1}{16}$

(4) $3^{-4} = \frac{1}{81}$

(5) $\left(\frac{1}{2}\right)^{-5} = 32$

(6) $\left(\frac{1}{3}\right)^{-3} = 27$

(7) $x^{2z} = y$

(8) $\sqrt{x} = y$

(1) $\log_8 64 = 2$

(2) $\log_{10} 10000 = 3$

(3) $\log_4 \frac{1}{16} = -2$

(4) $\log_3 \frac{1}{81} = -4$

(5) $\log_{\frac{1}{2}} 32 = -5$

(6) $\log_{\frac{1}{3}} 27 = -3$

(7) $\log_x y = 2z$

(8) $\log_x y = \frac{1}{2}$

1. Find the value of y.

(1) $\log_5 25 = y$ (2) $\log_3 1 = y$ (3) $\log_{16} 4 = y$ (4) $\log_2 \frac{1}{8} = y$

(5) $\log_5 1 = y$ (6) $\log_2 8 = y$ (7) $\log_7 \frac{1}{7} = y$ (8) $\log_3 \frac{1}{9} = y$

(9) $\log_y 32 = 5$ (10) $\log_9 y = -\frac{1}{2}$ (11) $\log_4 \frac{1}{8} = y$ (12) $\log_9 \frac{1}{81} = y$

2. Evaluate.

(1) $\log_3 1$ (2) $\log_4 4$ (3) $\log_7 7^3$ (4) $b^{\log_b 3}$ (5) $\log_{25} 5^3$ (6) $16^{\log_4 8}$

1. (1) 2

(2) 0

(3) $\frac{1}{2}$

(4) -3

(5) 0

(6) 3

(7) -1

(8) -2

(9) 2

(10) $\frac{1}{3}$

(11) $-\frac{3}{2}$

(12) -2

2. (1) 0

(2) 1

(3) 3

(4) 3

(5) $\frac{3}{2}$

(6) 64

True or False?

(1) $\log\left(\frac{x}{y^3}\right) = \log x - 3\log y$

(2) $\log(a-b) = \log a - \log b$

(3) $\log x^k = k \cdot \log x$

(4) $(\log a)(\log b) = \log(a+b)$

(5) $\frac{\log a}{\log b} = \log(a-b)$

(6) $(\ln a)^k = k \cdot \ln a$

(7) $\log_a a^a = a$

(8) $-\ln\left(\frac{1}{x}\right) = \ln x$

(9) $\ln_{\sqrt{x}} x^k = 2k$

(1) True
(2) False
(3) True
(4) False
(5) False
(6) False
(7) True
(8) True

Solve the following logarithmic equations.

(1) $\ln x = -3$

(2) $\log(3x-2) = 2$

(3) $2\log x = \log 2 + \log(3x-4)$

(4) $\log x + \log(x-1) = \log(4x)$

(5) $\log_3(x+25) - \log_3(x-1) = 3$

(6) $\log_9(x-5) + \log_9(x+3) = 1$

(7) $\log x + \log(x-3) = 1$

(8) $\log_2(x-2) + \log_2(x+1) = 2$

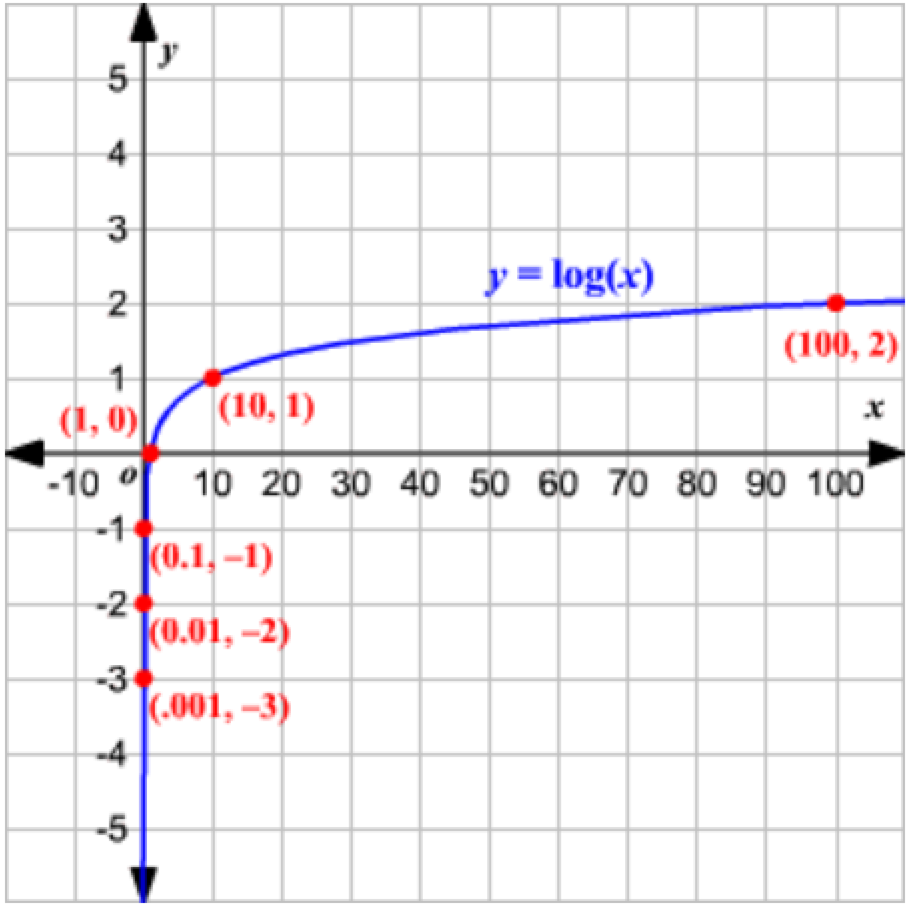
(1) $S = \{e^{-3}\}$
(2) $S = \{34\}$
(3) $S = \{2, 4\}$
(4) $S = \{5\}$
(5) $S = \{2\}$
(6) $S = \{6\}$
(7) $S = \{5\}$
(8) $S = \{3\}$

Graphing Logarithmic Functions

- When no base is written, assume that the log is *base* 10.

$$y = log x$$

x	$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000
$y = \log x$	-3	-2	-1	0	1	2	3

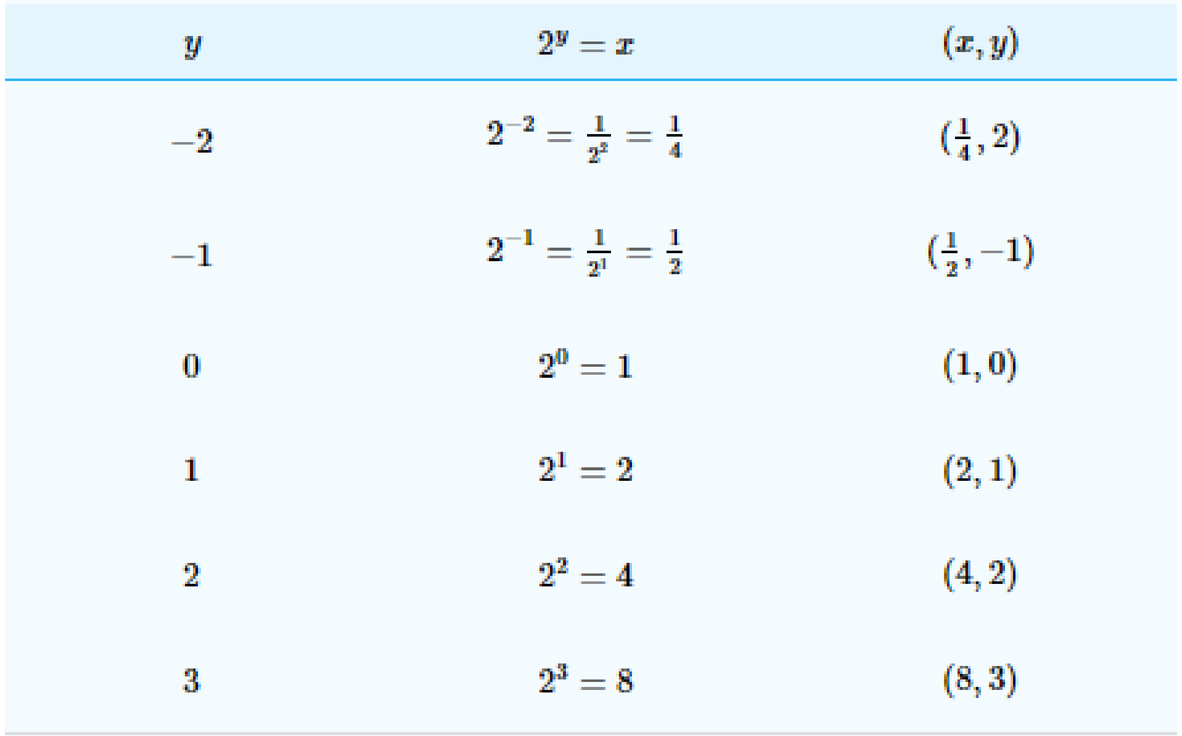


- The **domain** of log function $y = \log x$ is $x > 0$ (or) $(0, \infty)$.
- The **range** of any log function is the set of all real numbers (R)

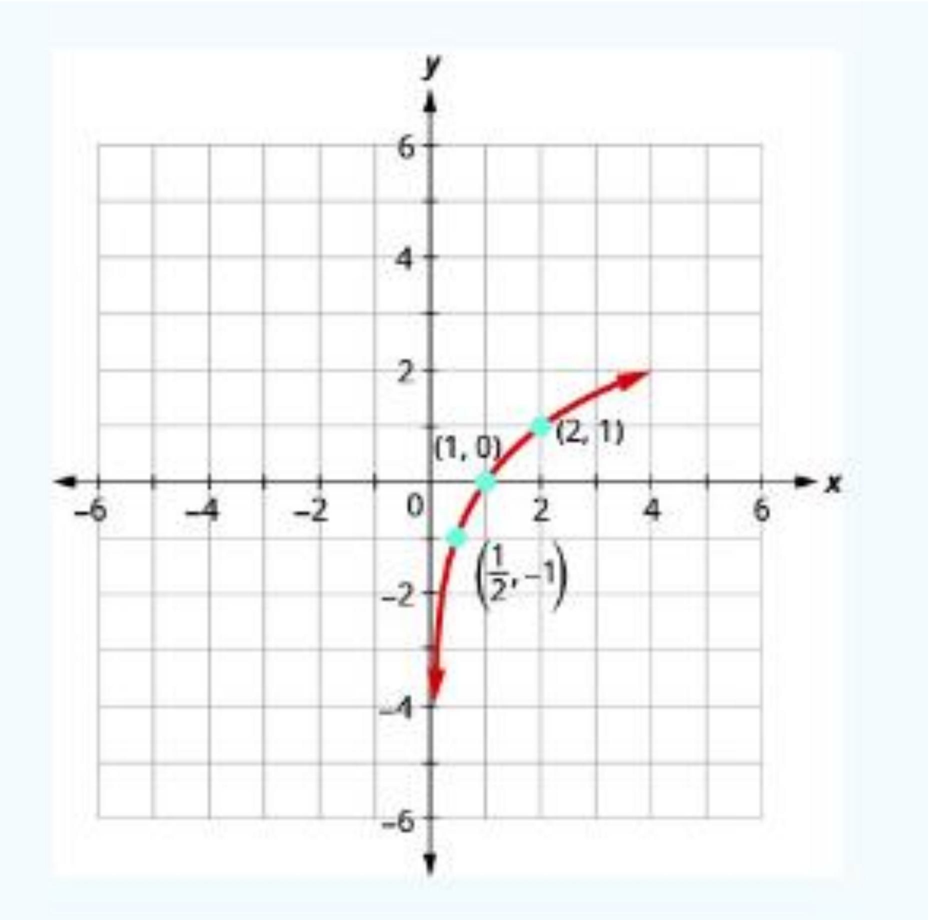
The graph of the function $y = \log_2 x$.

$$\downarrow$$

$$x = 2^y$$



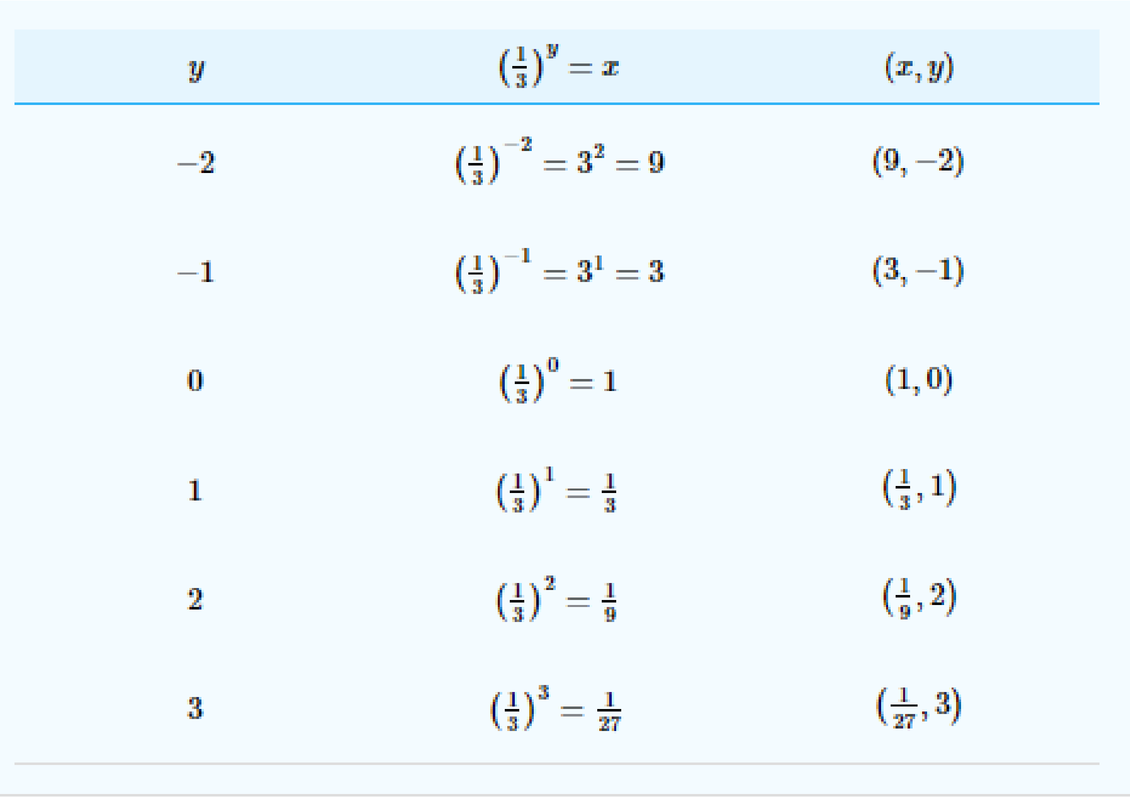
y	$2^y = x$	(x, y)
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$	$(\frac{1}{4}, 2)$
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$	$(\frac{1}{2}, -1)$
0	$2^0 = 1$	$(1, 0)$
1	$2^1 = 2$	$(2, 1)$
2	$2^2 = 4$	$(4, 2)$
3	$2^3 = 8$	$(8, 3)$



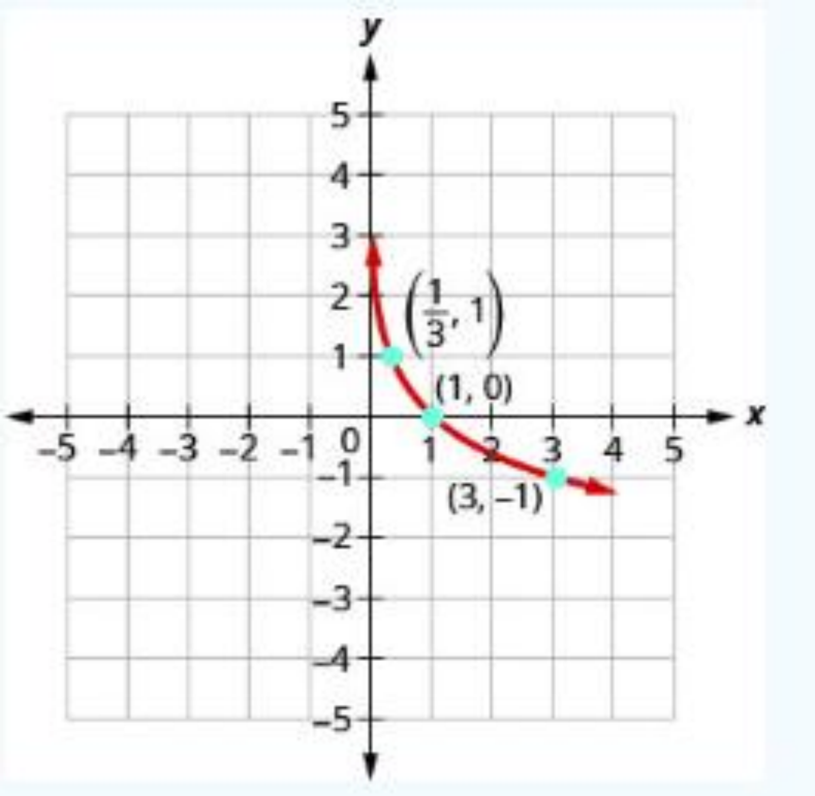
The graph of the function $y = \log_{\frac{1}{3}} x$.

↓

$$x = \frac{1^y}{3}$$



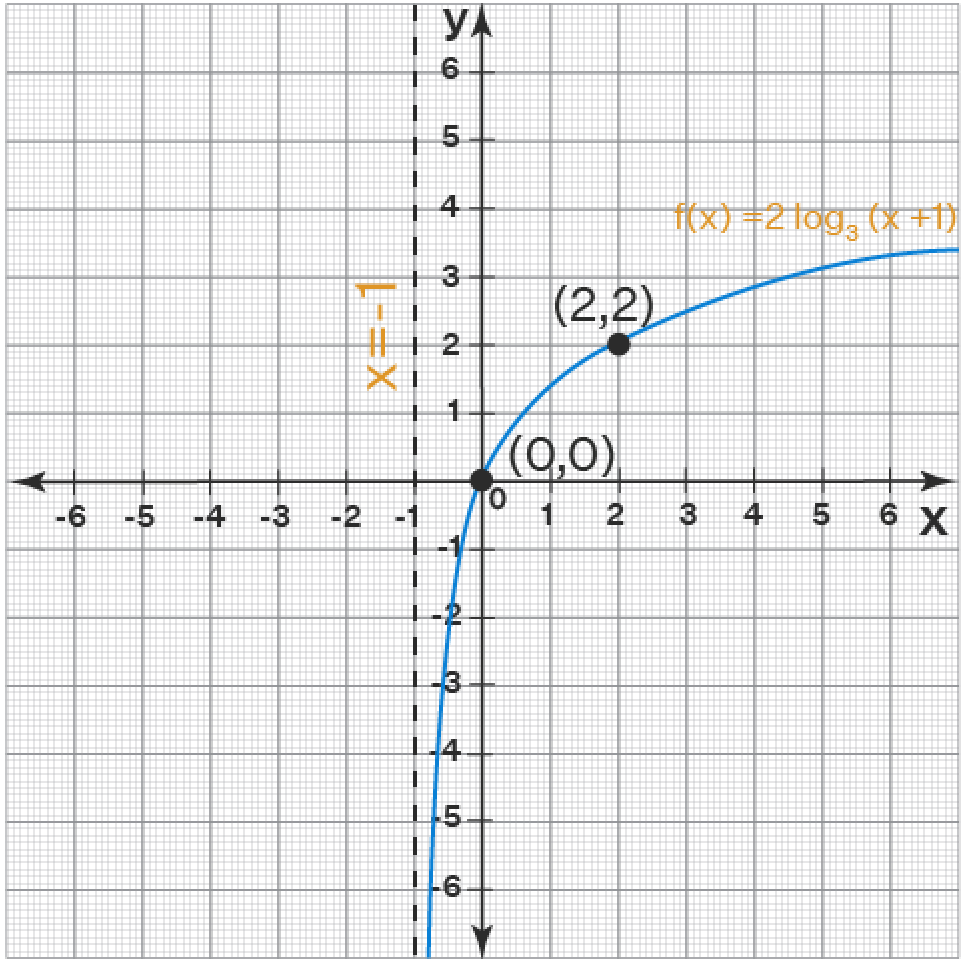
y	$\left(\frac{1}{3}\right)^y = x$	(x, y)
-2	$\left(\frac{1}{3}\right)^{-2} = 3^2 = 9$	$(9, -2)$
-1	$\left(\frac{1}{3}\right)^{-1} = 3^1 = 3$	$(3, -1)$
0	$\left(\frac{1}{3}\right)^0 = 1$	$(1, 0)$
1	$\left(\frac{1}{3}\right)^1 = \frac{1}{3}$	$\left(\frac{1}{3}, 1\right)$
2	$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$	$\left(\frac{1}{9}, 2\right)$
3	$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$	$\left(\frac{1}{27}, 3\right)$



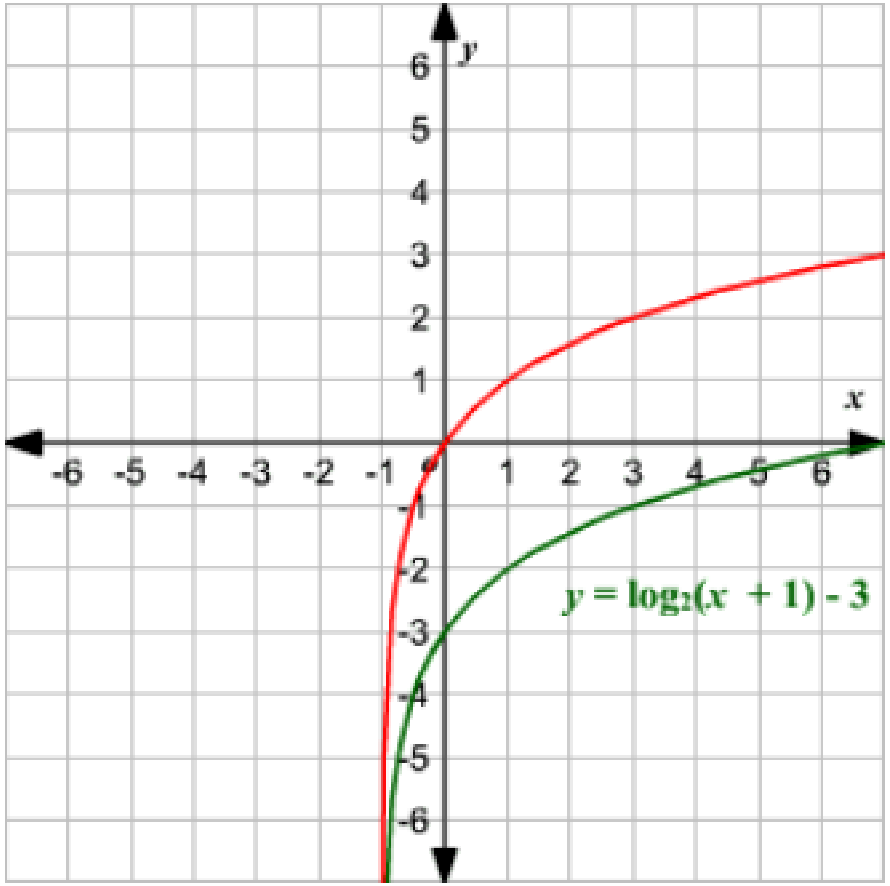
Graph the logarithmic function $f(x) = 2\log_3(x+1)$.

For domain: $x + 1 > 0 \Rightarrow x > -1$. So domain $= (-1, \infty)$.
Range $= R$.
Vertical asymptote is $x = -1$.

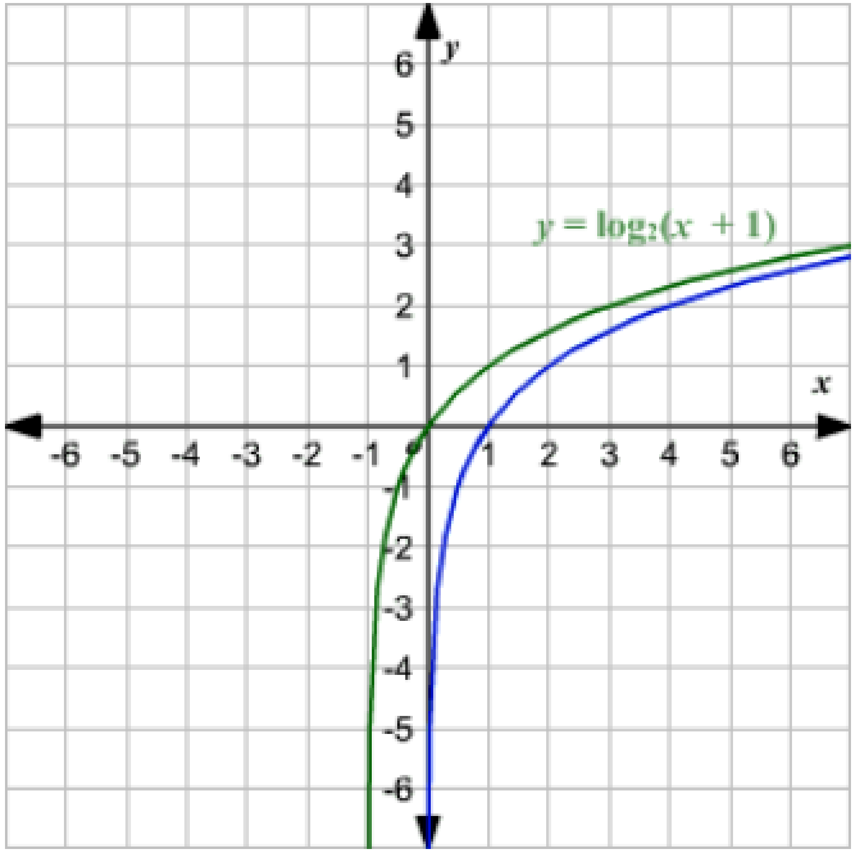
- At $x = 0, y = 2\log_3(0+1) = 2\log_3 1 = 2(0) = 0$
- At $x = 2, y = 2\log_3(2+1) = 2\log_3 3 = 2(1) = 2$



The graph of the function
$y = \log_2(x + 1) - 3.$

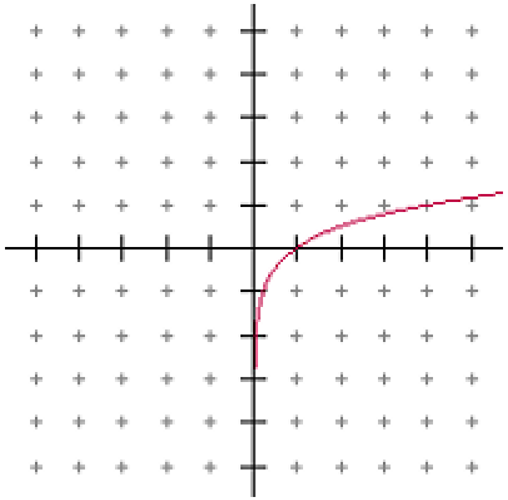


The graph of the function
$y = \log_2(x + 1).$



Match the graphs with their appropriate equation below.

7



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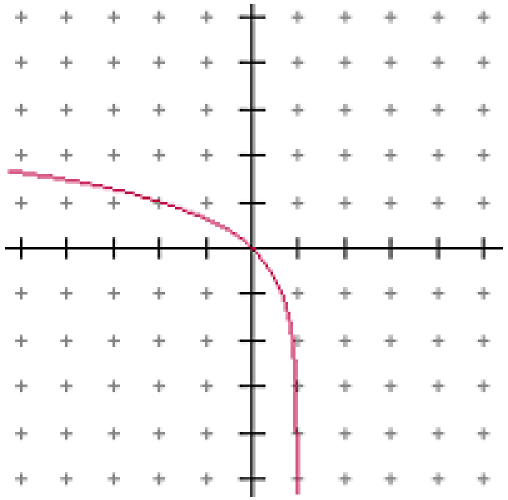

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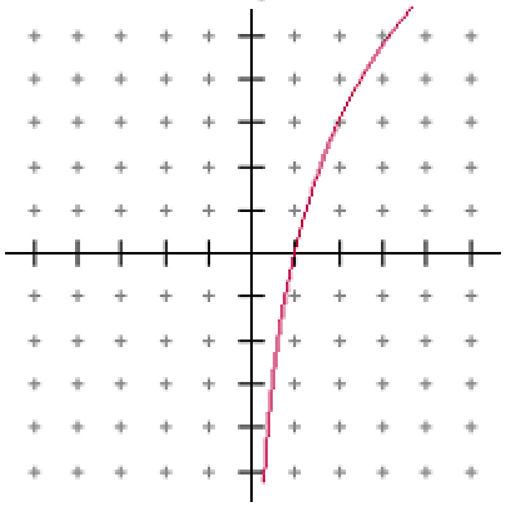

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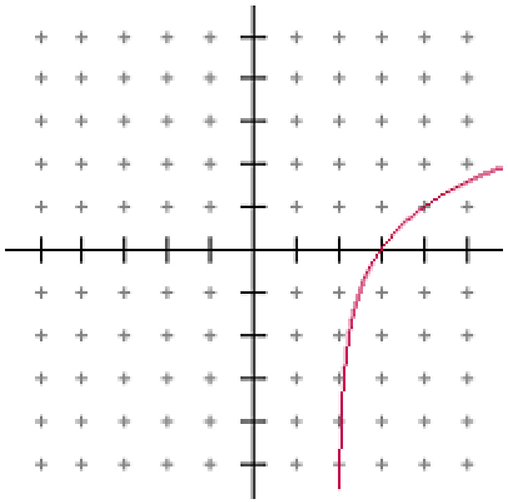

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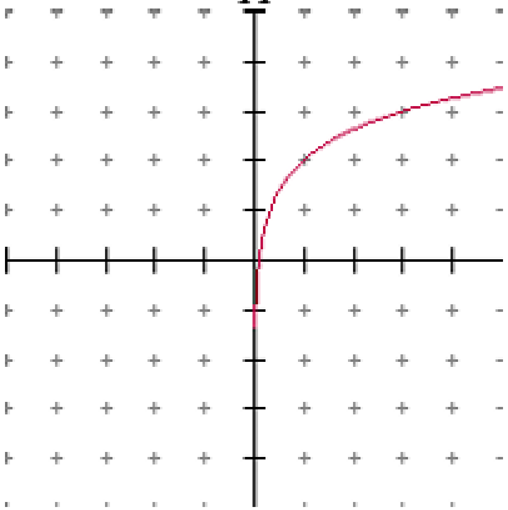

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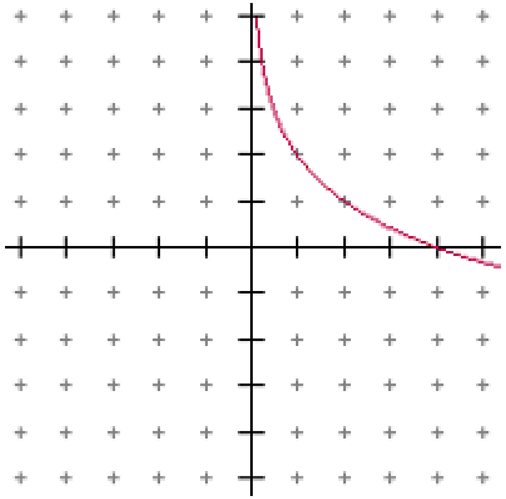


A) $f(x) = \log_2(x - 2)$

B) $f(x) = \log_3(1 - x)$

C) $f(x) = -\log_2 x + 2$

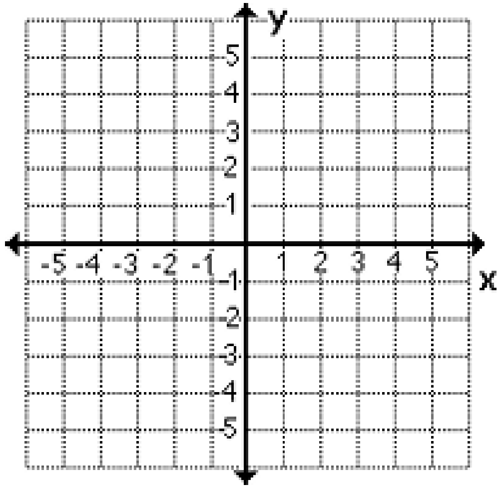
D) $f(x) = \log_3 x + 2$

E) $f(x) = \frac{1}{2}\log_2 x$

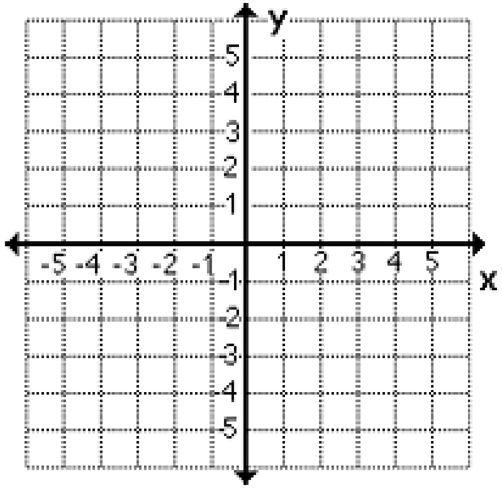
F) $f(x) = 3\log_2 x$

Graph each of the following logarithmic functions. Label the key point for each.

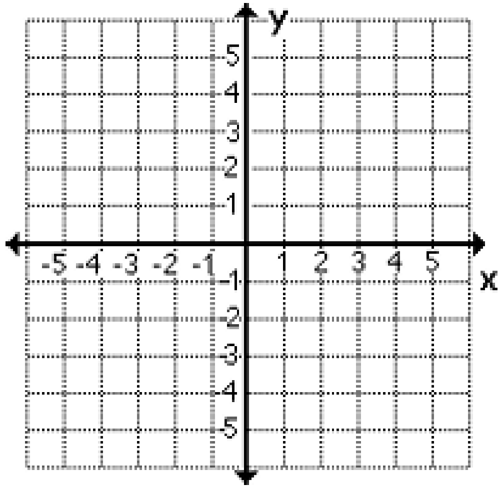
13 $f(x)=\log_2 x$



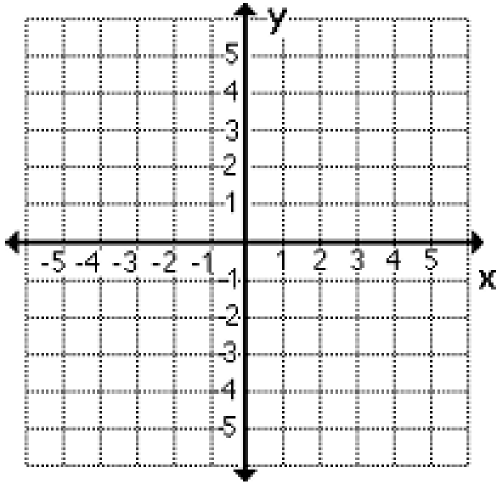
14 $f(x)=\log_4 x$



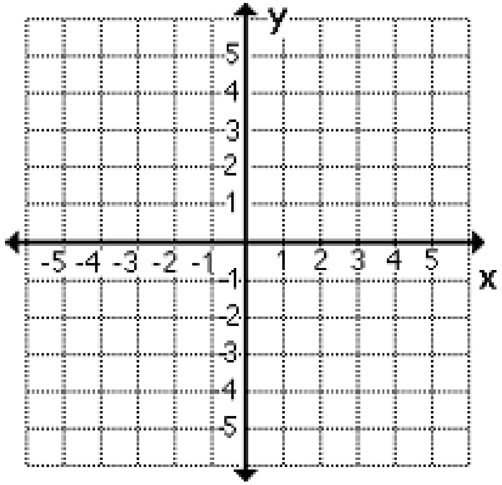
15 $f(x)=\log_4(x-3)$



16 $f(x)=\log_2(x+2)$



17 $f(x)=\log_3 x+2$



18 $f(x)=\log_5(x-3)+2$