Linear Algebra

Matrices in MATLAB

LAB 2

My Rules

- After 10 minutes you cannot enter the class.
- No phone usage during class.
- No talking to each other about anything, if you have questions, ask me.
- Don’t use computer while explanation.

Creating and Accessing Matrices

```
>> M = [1 2; 3 4]    % 2x2 matrix
```

Accessing a single entry in the matrix

- M(2,1), the first index is the row and the second index is the column

Accessing a whole Row in the matrix

- M(2,:), the first index is the row and the second index is the column, to access all the elements of a row, we use : for the column index (second index).

Accessing a whole Column:

- M(:,2), the first index is the row and the second index is the column, to access all the elements of a column, we use : for the row index (first index).

Why Indexing?

Correcting any entry:

- You can correct any index you want in your matrix through indexing, ex:

```
>>M =[1 2; 3 4]
>>M(1 , 2) = 6
```

Functions in MATLAB

create E	*create u*	*change E*	*multiply Eu*
$E = \text{eye}(3)$	$u = E(:,1)$	$E(3,1) = 5$	$v = E * u$
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$

- The word eye stands for the identity matrix
- The submatrix u = E(:,1) picks out column 1
- The instruction E (3,1) = 5 resets the entry to 5
- The command E * u multiplies the matrices E and u

Functions in MATLAB

create A	*create b*	*invert A*	*solve $Ax = b$*
$A = \text{ones}(3) + \text{eye}(3)$	$b = A(:,3)$	$C = \text{inv}(A)$	$x = A\backslash b$ or $x = C * b$
$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$	$\begin{bmatrix} .75 & -.25 & -.25 \\ -.25 & .75 & -.25 \\ -.25 & -.25 & .75 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

- The matrix of all ones was added to eye
- b is its third column
- Inv(A) produces the inverse matrix normally in decimals
- The system Ax = b is solved by x = inv(A) * b

Creating sub-matrix

```
>>A(2:3,1:2)
```

$$A = \begin{pmatrix} 2 & 1 & 4 \\ \boxed{6} & \boxed{9} & 3 \\ \boxed{5} & \boxed{7} & 8 \end{pmatrix}$$

- To create a sub-matrix, you can use the command above (A box).

$$A = \begin{pmatrix} 2 & 1 & 4 \\ \boxed{6} & 9 & \boxed{3} \\ \boxed{5} & 7 & \boxed{8} \end{pmatrix}$$

```
>>A(2:3,[1,3])
```

Removing a Row or Column

- To remove the third row, use the following command:

```
>>A (3,:) = []
```

- To remove 2 column, use the following command:

```
>>A (:,2) = []
```

Transposing a Matrix

- To transpose a matrix in MATLAB, you use ‘, ex:

```
>>A’
```

- The transpose of m × n real matrix A is the n × m matrix that results from interchanging the rows and columns of A. The transpose matrix is denoted A^T.

Solving linear Equations

- One of the problems encountered most frequently in scientific computation is the solution of systems of simultaneous linear equations. With matrix notation, a system of simultaneous linear equations is written as Ax=b.
- In linear algebra we learn that the solution to Ax = b can be written as $x = A^{-1} b$, where A^{-1} is the inverse of A.
- For example, consider the following system of linear equations

$$\begin{cases} x + 2y + 3z &= 1 \\ 4x + 5y + 6z &= 1 \\ 7x + 8y &= 1 \end{cases}$$

- The coefficient matrix A is $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ and the vector $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

- With matrix notation, a system of simultaneous linear equations is written as Ax = b
- There are typically two ways to solve for x in MATLAB:

Solving linear Equations

1. The first way to solve linear equations is to use inverse (inv):

```
>>A = [1 2 3; 4 5 6; 7 8 0]
>>b = [1; 1; 1] % what is another way to do create this?
>>x = inv(A)*b
```

2. The second one is to use the backslash (\) operator. The numerical algorithm behind this operator is computationally efficient. This is a numerically reliable way of solving system of linear equations by using a well-known process of Gaussian elimination.

```
>>A = [1 2 3; 4 5 6; 7 8 0]
>>b = [1; 1; 1]
>>x = A\b
```

Finding cofactor

Example: Find the cofactor matrix of $\mathbf{A}$ given that $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$.

Solution: First find the cofactor of each element.

$$A_{11} = \begin{vmatrix} 4 & 5 \\ 0 & 6 \end{vmatrix} = 24 \qquad A_{12} = -\begin{vmatrix} 0 & 5 \\ 1 & 6 \end{vmatrix} = 5 \qquad A_{13} = \begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix} = -4$$

$$A_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 6 \end{vmatrix} = -12 \qquad A_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 6 \end{vmatrix} = 3 \qquad A_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 2$$

$$A_{31} = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = -2 \qquad A_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = -5 \qquad A_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} = 4$$

The cofactor matrix is thus $\begin{bmatrix} 24 & 5 & -4 \\ -12 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix}$.

Finding cofactor in MATLAB

- First, you find the determinant of the matrix
- Second you find the inverse of the matrix.
- then multiplication of the inverse and determinant
- Then lastly transpose of the multiplication

```
>>A = [1 2 3; 0 4 5; 1 0 6]
>>detA = det(A)
>>invA = inv(A)
>>cofactorA = transpose(detA*invA)
```

- detA*invA, gives the transpose of cofactor.